1. **[EJC 17 Promos]**
Given that $\displaystyle\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find $\displaystyle\sum_{r=1}^{n}(r+1)(2r+1)$. [4]

2. **[EJC 17 Promos]**
(a) Verify that $\dfrac{1}{r^2-1} = \dfrac{1}{2(r-1)} - \dfrac{1}{2(r+1)}$. [1]
(b) Prove by method of differences that
$$\sum_{r=2}^{n} \frac{1}{r^2-1} = \frac{3}{4} - \frac{1}{2n} - \frac{1}{2(n+1)}$$
and state $\displaystyle\sum_{r=2}^{\infty} \frac{1}{r^2-1}$. [4]
(c) Hence find $\displaystyle\sum_{r=2}^{n-1} \frac{1}{r(r+2)}$. [3]

3. **[RI 17 Promos]**
A piece of paper in the form of a semi-circle of radius r is cut into twelve sectors such that the areas are in arithmetic progression, and the area of the biggest sector is three times that of the smallest sector. Find the exact area of the smallest sector in terms of r. [2]

4. **[RI 17 Promos]**
An arithmetic series A has first term a and common difference d, where a and d are non-zero. A convergent geometric series G has common ratio r.

The first three terms of G are equal to the first, eleventh and seventeenth terms of A, respectively.

(a) Find r. [4]
(b) Using your answer in part (a), find the exact ratio of the sum to infinity of G to the sum of the first four terms of G. [2]

5. **[VJC 17 Promos]**

(a) i. A particular bond is issued at \$100 per unit with a 5% annual coupon rate. The bondholder will receive a fixed amount of $\$(100 \times 5\%)$ as coupons for 1 bond that he holds, at the end of every year until the bond matures after 30 years.

At the start of 2016, Mr Reech purchased 10 units of this particular bond. At the start of each subsequent year, he purchases another 10 units of the same bond. Assuming that Mr Reech receives the coupons as cash at the end of each year, show that the total amount of cash he receives over a period of 20 years is \$10,500. [2]

ii. Mr Puwer, on the other hand, puts his money in a savings account that pays him $k\%$ compound interest at the end of every year. At the start of 2016, Mr Puwer deposits \$800 in the savings account. At the start of each subsequent year, he deposits \$800 into the same account. Mr Puwer does not withdraw any money from this account. Show that the total amount of money Mr Puwer has in his savings account at the end of n years is $\$\left(\frac{800(100+k)}{k}\left[\left(\frac{100+k}{100}\right)^n - 1\right]\right)$. [3]

iii. Hence, find the range of values of k such that at the end of 20 years, Mr Reech receives more cash in coupons than what Mr Puwer receives in total interest paid. [4]

(b) Mr Soo has \$100,000 in his savings account, and the prevailing interest rate is 2% per annum. Mr Soo does not deposit any more money in his account. Interest paid at the end of every year is withdrawn by him from the account, but the annual interest rate is halved every year. Find the minimum number of years it takes for the total interest withdrawn by Mr Soo to be at most \$20 less than the theoretical maximum total interest payable by the bank. [4]

Answers

1. $\frac{n}{3}(n+1)(2n+1)+\frac{n}{2}(3n+5)$.
2. (b) $\frac{3}{4}$.

   (c) $\frac{5}{12}-\frac{1}{2n}-\frac{1}{2(n+1)}$.
3. $\frac{\pi}{48}r^2$.
4. (a) $r=\frac{3}{5}$.

   (b) 625:544.
5. (a) i. $10 500.

   iii. $0<k<4.59$.

   (b) 8.