Answers: Paper 1

1. $-1.6 < x < 8$.

2. $p = 2\mathrm{e}^{-\frac{\pi}{\sqrt{3}}}$.

3. $\pi(\pi - 2)$.

4. (a) Any horizontal line $y = k, k \in \mathbb{R}$ cuts the curve of $y = f(x)$ at most once. Hence f is one-one and f has an inverse.

   (b) $f^{-1}(x) = \sqrt{\frac{2-x}{1-x}}$.
   $D_{f^{-1}} = (-\infty, 1) \cup [2, \infty)$.

   (c) ff does not exist since $R_f = (-\infty, 1) \cup [2, \infty) \not\subset [0, 1) \cup (1, \infty) = D_f$.
   (We will skip the "hence" part as there is some problem with the question).

5. $A = 1, B = -1$.

   (a) $\frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$.

   (b) As $n \to \infty$, $\frac{1}{2n+1} \to 0$ so $\frac{1}{2}\left(1 - \frac{1}{2n+1}\right) \to \frac{1}{2}$.
   Hence $\displaystyle\sum_{r=1}^{\infty} \frac{1}{4r^2 - 1}$ converges and $\displaystyle\sum_{r=1}^{\infty} \frac{1}{4r^2 - 1} = \frac{1}{2}$.

   (c) $\frac{1}{2}\left(\frac{1}{5} - \frac{1}{2n+5}\right)$.

6. (b) $y = 2x + \sqrt{2}$.

   (c) $a = 2$, $\left(\frac{2}{3}\sqrt{\frac{2}{3}}, \frac{4}{\sqrt{3}}\right)$.

   (d) $b = \frac{b}{\sqrt{2}}$.

7. (a) $z = -2 + 2i, w = 2 - i$.

   (b) $1024\mathrm{e}^{-3\frac{\pi}{4}}$. (Remarks: we have modified the question such that $0 < \theta \leq \pi$).

   (c) Smallest positive integers $n = 1, 5, 9$.

8. (b) \$71.37.

   (c) 20.

   (d) \$206.

9. (a) $l : \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$.

   (b) $\sqrt{14}$.

   (c) $60^\circ, \frac{2}{3}\sqrt{42}$.

   (d) $3x - 4y - 4z = 2$.

10. (a) i. $\frac{5}{12}\sqrt{6}$.
    ii. Minimum $T = 17.9$ J.

    (b) 2.65 rad/hr.

Answers: Paper 2

1. (a) $\left(\frac{a-1}{2}, 3b\right)$.
   (b) Translate 6 units in the positive x-axis direction followed by scaling of factor 4 parallel to the y-axis.

2. (a) $1 + 3x + \frac{5}{2}x^2 + \ldots$
   (b) $a = \frac{4}{3}, n = \frac{9}{4}$.
   $\frac{5}{18}$.

3. (a) $(1-\lambda)\mathbf{a} + \lambda\mathbf{b}$.
   (b) $\lambda = \frac{1}{6}, \mu = \frac{1}{2}$.
   (c) $1.2\mathbf{j} - 0.6\mathbf{k}$.

4. (b) $t = \frac{10}{3}\ln\left(\frac{4-2x}{x+1}\right)$.
   (c) 4.62 minutes.
   (d) $x = \frac{4-\mathrm{e}^{\frac{3t}{10}}}{2+\mathrm{e}^{\frac{3t}{10}}}$, 0.345 g.

5. (a) 0.0235.
   (b) 0.553.

6. (a) $P(X=0) = \frac{3}{8}$, $P(X=1) = \frac{1}{3}$, $P(X=2) = \frac{1}{4}$, $P(X=4) = \frac{1}{24}$.
   (b) $E(X) = 1$.
   (c) $E(Y) = 3, Var(Y) = 1$.

7. (a) $\frac{1}{5}$.
   (b) $\frac{4}{5}$.
   (c) $\frac{4}{5}$.
   (d) $\frac{1}{8}$.
   (e) $P(A' \cap B \cap C) \leq \frac{1}{8}$.
   (For a 3-4 mark version of the question, $\frac{3}{40} \leq P(A' \cap B \cap C) \leq \frac{1}{8}$).

8. (a) The probability that a chocolate is misshapen is the same for each chocolate. The event that a chocolate is misshapen is independent from the event that another chocolate is misshapen.
   (b) Misshapen chocolate may be clustered together during manufacturing. Hence whether one chocolate is misshapen is not independent from the other chocolates in the packet.
   (c) 0.590.
   (d) 0.00508.
   (e) 0.379.
   (f) The event in part (iv) is a subset of the event in part (v).
   (g) 8.

9. (a) 86 400.
   (b) 172 800.
   (c) i. 1 209 600.
       ii. 2 983 680.
   (d) 28 800.

10. (a) 300.
    (b) 0.675.
    (c) 0.535.
    (d) $\mu = 214.29, \sigma = 1.17$.