1. **[TMJC 19 MYE]**
The equations of a line l and a plane π are

$$l : \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} + \lambda \begin{pmatrix} b \\ 2 \\ -1 \end{pmatrix}, \lambda \in \mathbb{R} \quad \text{and} \quad \pi : \mathbf{r} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}, \mu, \gamma \in \mathbb{R}$$

respectively, where a and b are constants.

   (a) Show that an equation of the plane π in scalar product form is $\mathbf{r} \cdot \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} = 3$. [2]

   (b) In the case where $a = 2$ and $b = 0$
      i. find the acute angle between l and π, [2]
      ii. find the coordinates of the point at which l and π intersect. [3]

   (c) Given instead that l and π do not intersect, what can be said about the values of a and b? [4]

2. **[TMJC 19 MYE]**
Last December, Mr. Stuart took his family on a vacation to Tamridian Island, where they embarked on a road trip. Points (x, y, z) are defined relative to the airport at $(0, 0, 0)$. They collected their vehicle at a car rental company whose location is represented by the coordinates $(1, 0, 2)$ and started driving on Pines Highway, a straight road in the direction $\begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}$. As they approached a junction located at $(-2, 1, 7)$, they exited the highway to continue on another straight road to a cafe located at $(-8, 8, 9)$.

   (a) Write down a vector equation of the line representing Pines Highway, which Mr. Stuart was driving on right after collecting the car. [1]

   (b) Find the exact length of projection of the path he took from the junction to the cafe onto Pines Highway. [3]

   (c) Hence find the exact shortest distance between the cafe and Pines Highway. [2]

According to the map, there is a nice waterfall whose location is a reflection of the location of the cafe in the line reprsenting Pines Highway. If Mr. Stuart and his family had decided to vist the waterfall instead and followed another straight road at the junction that leads to the waterfall,

   (d) find a vector equation of the line representing the road which he would have travelled on. [6]

3. **[NJC 19 MYE]**
The plane Π has equation $x - 2y + 2z = 9$.

   (a) Find the cartesian equations of the planes such that the perpendicular distance from each plane to Π is 8. [3]

The line L has equation $\dfrac{x-1}{6a} = \dfrac{y-2}{3a} = \dfrac{3-z}{3}$, where a is a constant.

   (b) If the angle between L and Π is $30°$, find the possible exact values of a. [3]

4. **[NJC 19 MYE]**
The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}), \lambda \in \mathbb{R}$. The plane p is parallel to l and contains the points A and B with coordinates $(1, -3, 1)$ and $(4, 3, -2)$ respectively.

   (a) Find a cartesian equation of p. [3]

   (b) Find the exact distance between l and p. [2]

   The plane q has equation $\alpha x - 3y + 2z = \beta$, where α and β are real constants.

   (c) Given that l and q do not meet, what can be said about the values of α and β? [3]

   (d) Given instead that l lies in q, find a vector equation of the line where p and q meet. [2]

5. **[NJC 19 MYE]**
In this question, the direction of the vector $\mathbf{k}$ is taken to be (vertically) upwards and the unit for distance is metres.

   A skier is skiing down a slope where his position vector at time t seconds, relative to the origin O, is given by $\mathbf{r} = 4\mathbf{i} + 105\mathbf{j} + 92\mathbf{k} + t(3\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}), t \geq 0$. While skiing, the skier spots a bear sleeping on the slope at the point $(14, 94, 81)$. It may be assumed that the slope is smooth and can be modelled after a plane.

   (a) Explain why the skier will not collide with the sleeping bear and show that a cartesian equation of the slope is $22x + 7y + 13z = 2019$. [4]

   (b) The skier encounters a rectangular ramp protruding out from the slope. The ramp is perpendicular to $4\mathbf{j} + 3\mathbf{k}$. Find the obtuse angle the ramp makes with the slope. [2]

   (c) The skier passes over a point on the slope that is 8 metres directly beneath a cable car. Find the shortest distance between the cable car and the slope at this instant. [3]

   (d) Unexpectedly, the sleeping bear wakes up. Being an intelligent animal, at $t = 6$, it runs in a straight line such that it successfully catches the skier after 7 seconds of running. Determine the average speed, in ms^{-1}, at which the bear was running, giving your answer correct to two decimal places. [3]

Answers

1. (b) i. 41.6°.
   ii. $(1, \frac{3}{2}, \frac{7}{4})$.
   (c) $b = 2, a \neq 1$.

2. (a) $l : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}, \lambda \in \mathbb{R}$.
   (b) $\sqrt{35}$.
   (c) $3\sqrt{6}$.
   (d) $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} + \mu cvec0{-5}8, \mu \in \mathbb{R}$.

3. (a) $x - 2y + 2z = 33$ and $x - 2y + 2 = -15$.
   (b) $a = \pm\sqrt{\frac{7}{45}}$.

4. (a) $2x + 3y + 8z = 1$.
   (b) $\frac{17}{\sqrt{77}}$.
   (c) $\alpha = -7$ and $\beta \neq -19$.
   (d) $\mathbf{r} = \begin{pmatrix} \frac{18}{5} \\ -\frac{31}{15} \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}, \mu \in \mathbb{R}$.

5. (a) $22x + 7y + 13z = 2019$.
   (b) 120.4°.
   (c) $\frac{104}{\sqrt{702}}$.
   (d) 7.49 ms^{-1}.