1. **[CJC 19 MYE]**

   Two lines l_1 and l_2 are represented by cartesian equations $\dfrac{x-1}{2} = -z-1, y = 2$ and $x+3 = 4-y = z+2$ respectively.

   (a) Show that l_1 and l_2 intersect, and state the point of intersection. [4]

   (b) Find the acute angle between l_1 and l_2. [2]

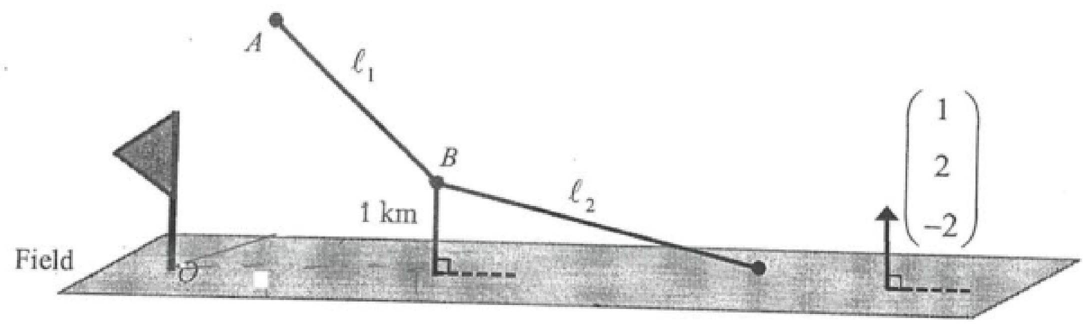
   (c) Find the position vector of the foot of perpendicular from point $(1, 2, -1)$ to l_2. [3]

   (d) Given that $(1, 2, -1)$ lies on l_1, find a vector equation for the line of reflection of l_1 in l_2. [3]

2. **[CJC 19 MYE]**

   In a skydiving performance, skydivers jump from an aircraft and perform manoeuvres in the air before landing on a field by parachute. Points (x, y, z) are defined relative to a point $O(0, 0, 0)$, the base of flag pole, where units of distance are measured in kilometres.

   A skydiver descends from $A(6.5, 6, 4)$ at uniform wind condition and constant speed, in a straight path l_1 along the direction $\begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix}$. At point B, a distance of 1km perpendicular to the field, he opens his parachute, and subsequently descends in a straight path l_2 along the direction $\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$. The field on which the skydiver lands can be modelled by the equation $x + 2y - 2z = 0$ (see diagram).



   (a) Find the acute angle between the path along which the skydiver lands and the field. [2]

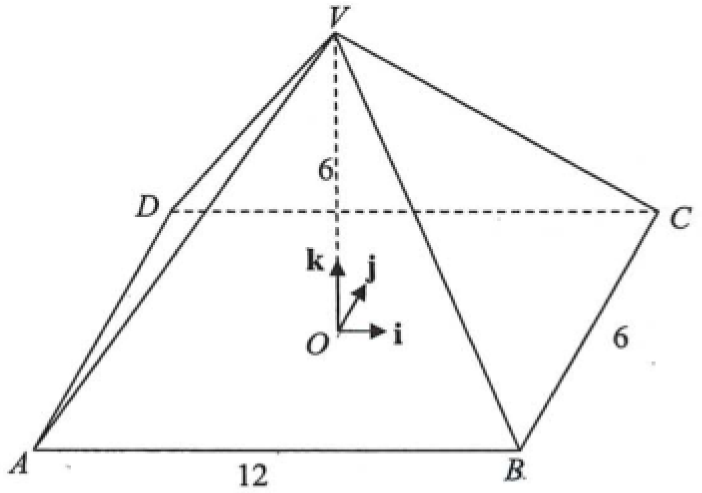
   (b) Find the perpendicular distance from A to the field. [2]

   (c) Find the coordinates of B. [5]

   The skydiver drops his action camera on the field in his descent and decides to search within 0.5 km of his landing point (a, b, c).

   (d) By considering the distance between (a, b, c) and a point (x, y, z) on the field in the skydiver's search region, or otherwise, show that the skydiver's search region satisfies $(2z - 2y - a)^2 + (y - b)^2 + (z - c)^2 < 0.25$. [3]

3. **[JPJC 19 MYE]**
In the diagram, O is the centre of the rectangular base $ABCD$ of a right pyramid with vertex V. Perpendicular unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are parallel to AB, BC and OV respectively. The lengths of AB, BC and OV are 12 cm, 6 cm and 6 cm respectively. The point O is taken as the origin for position vectors.



(a) Find a vector equation of the line BV. [2]

A line m has cartesian equation $\dfrac{x+1}{-5} = y+1 = \dfrac{t-z}{-4}$.

(b) If the line m intersects the line BV at X, find the value of t and the position vector of X. [5]

(c) Find the acute angle between BV and the line m. [3]

(d) Using your results in (b) and (c), find the perpendicular distance from B to the line m. [3]

4. **[JPJC 19 MYE]**
The line l passes through the points $A(1,-2,3)$ and $B(-4,-4,7)$. The plane Π contains a point $C(-2,4,3)$ and a line m with vector equation
$\mathbf{r} = (3+\alpha)\mathbf{j} + (2+2\alpha)\mathbf{k}, \alpha \in \mathbb{R}$.

(a) Show that a cartesian equation of the plane Π is $x + 4y - 2z = p$, where p is a constant to be determined. [3]

The line l meets the plane Π at the point D. Find

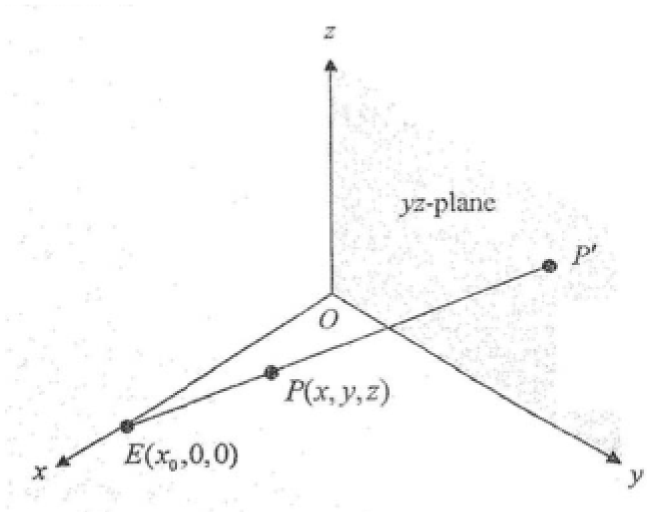
(b) the position vector of D, [3]

(c) the acute angle between the line l and the plane Π. [3]

(d) Given that the coordinates of the foot of perpendicular from the point A to the plane Π is $(2,2,1)$, find a vector equation of the line of reflection of the line l in the plane Π. [4]

5. **[RJC 19 MYE]**

In computer graphics and perspective drawing, objects seen by the eye in the three-dimensional space are represented as images on a two-dimensional plane. Referred to the origin O, the eye at E and the point P have position vectors given by $\overrightarrow{OE} = x_0\mathbf{i}$ and $\overrightarrow{OP} = x\mathbf{i} + y\mathbf{j} + z\mathbf{j}$ respectively. With a ray from E, the point P is projected onto the yz-plane as the point P'.



It is given that $\overrightarrow{OP'} = \dfrac{x_0 y}{x_0 - x}\mathbf{j} + \dfrac{x_0 z}{x_0 - x}\mathbf{k}$.

(a) What can be said about the point P' when

(a) $x = 0$,

(b) $x_0 \to \infty$? [2]

It is given that $\overrightarrow{OE} = 4\mathbf{i}$ and the points Q and R are such that $\overrightarrow{OQ} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OR} = 5\mathbf{j} + 5\mathbf{k}$. With a ray from E, the points Q and R are projected onto the yz-plane as the points Q' and R' respectively.

(c) Determine if the points O, Q' and R' are collinear. [2]

(d) A typical problem for graphics designers involves hidden lines.

The line QR intersects a triangular plate with vertices $A(1, 0, 5)$ and $B(0, 5, 2)$ and $C(3, 4, 4)$ at the point D. The portion of the line QR behind the plate, DR, is hidden from the designer's view.

Find the position vector of D and hence find the length of DR. [5]

6. **[RJC 19 MYE]**
The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} -2 \\ 5 \\ -4 \end{pmatrix} + t \begin{pmatrix} 8 \\ -5 \\ 1 \end{pmatrix} \text{ and } \mathbf{r} = s \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

respectively, where t and s are parameters.

(a) Find the acute angle between l_1 and l_2. [2]

With reference to the origin O, P is the point on l_1 when $t = 0$. It is given that l_1 and l_2 intersect at the point Q.

(b) Find the coordinates of Q. [2]

(c) Find the area of triangle OPQ. [3]

(d) It is given that the centroid of any triangle ABC has the position vector $\frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$.
Find a vector equation of the line that is perpendicular to both l_1 and l_2 which passes through the centroid of the triangle OPQ. [2]

Answers

1. (a) $(-1, 2, 0)$.
   (b) $75.0°$.
   (c) $\frac{1}{3}(2\mathbf{i} + 5\mathbf{j} + \mathbf{k})$.
   (d) $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} 4 \\ 2 \\ -5 \end{pmatrix}, \nu \in \mathbb{R}$.
2. (a) $20.8°$.
   (b) 3.5.
   (c) $B(3.5, 2.25, 2.5)$.
3. (a) $\mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}, \lambda \in \mathbb{R}$.
   (b) $t = 6, \overrightarrow{OB} = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$.
   (c) $12.2°$.
   (d) 0.636.
4. (a) $p = 8$.
   (b) $6\mathbf{i} - \mathbf{k}$.
   (c) $43.1°$.
   (d) $\mathbf{r} = \begin{pmatrix} 6 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, t \in \mathbb{R}$.
5. (a) i. P' is $(0, y, z)$ (which is the same as point P).
   ii. Point P' tends to the point $(0, y, z)$.
   (b) Not collinear.
   (c) $\overrightarrow{OD} = \begin{pmatrix} 1 \\ 3 \\ \frac{7}{2} \end{pmatrix}$.
   $\frac{\sqrt{29}}{2}$.
6. (a) $45°$.
   (b) $(6, 0, -3)$.
   (c) $\frac{45}{2}$.
   (d) $\mathbf{r} = \frac{1}{3} \begin{pmatrix} 4 \\ 5 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \lambda \in \mathbb{R}$.