1. **[EJC 18 MYE]**
The Hand Foot Mouth Disease is a disease that is present all year round in Singapore with seasonal outbreaks every year. The mean duration of a child's infectious period is 150 hours. The Health Promotion Institution is concerned over the recent spike of Hand Foot Mouth Disease cases in Singapore and launched an investigation to check whether the mean duration of a child's infectious period has changed. A random sample of 100 infected children is selected and the duration of each child's infectious period is recorded. The results are summarized as follows.

$$\sum(x-150)=450 \quad \text{and} \quad \sum(x-150)^2=42005$$

Test at the 5% level of significance whether the mean duration of a child's infectious period has changed. State, giving a reason, whether it is necessary to assume that the duration of a child's infectious period follows a normal distribution for the test to be valid. [6]

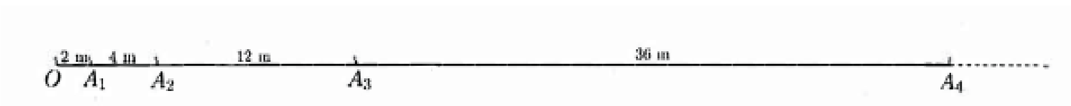
2. **[MI 18 MYE (modified)]**
A charity has a large number of collection boxes at a number of locations. They are emptied on a regular basis and it is claimed the mean amount collected per box is \$85.40. A random sample of 50 boxes is emptied and the contents, \$x, are summarised by $\sum x = 4399$ and $\sum x^2 = 392\ 264$.

(a) Test, at the 5% significance level, whether the population mean amount collected per box is \$85.40. [4]

Assume now that the population standard deviation of the amount collected per box is \$10. A new sample of 50 boxes is emptied and the mean of this sample if \$$k$. A test at the 10% significance level indicates that the mean amount collected is more than \$85.40.

(b) Find the range of values of k, giving your answer correct to 2 decimal places. [3]

3. **[NJC 18 MYE]**
A robot sweeper operates in stages by travelling from a starting point O to and from a series of points $A_1, A_2, A_3, \ldots$, increasingly far away in a straight line. In stage 1, the robot sweeper travels from O to A_1 and back at O. In stage 2, the robot sweeper travels from O to A_2 and back at O, and so on.



The distances between the points are such that $OA_1 = 2$ m, $A_1A_2 = 4$ m, $A_2A_3 = 12$ m and $A_nA_{n+1} = 3A_{n-1}A_n$ (see figure above).

(a) Find an expression for the total distance travelled by the robot just after completing n stages. [2]

(b) Hence, find the distance between the robot sweeper from O after it has travelled a total distance of 1200 m. [5]

4. **[SAJC 18 MYE (modified)]**
Two credit cards, HMiles and TRewards, provide a rewards programme in terms of points for their customers for every dollar charged to the card.
HMiles reward 50 points for every dollar charged to the card for the first ten dollars. For the eleventh dollar onwards, the number of points awarded will be 5 more than the number of points awarded for the previous dollar, so that 55 points will be awarded for the eleventh dollar, 60 points for the twelfth dollar, and so on.
TRewards 20 points for the first dollar charged to the card and the number of points awarded increases by 5% for each additional dollar charged to the card.

(a) Find an expression in terms of n for the number of points accumulated if a customer charged $\$n (n \geq 10)$ to the HMiles credit card. [3]

(b) What is the minimum amount a customer needs to charge on a HMiles credit card if he wants to redeem a gift which costs 35000 points? [3]

(c) Find the minimum amount a customer needs to charge to the TRewards card such that its programme will earn more points compared to that of the HMiles card. [5]

Answers

1. p-value $= 0.025138$. Reject H_0.
   There is sufficient evidence at 5% level of significance that the mean number of hours a person remains infectious has changed.
   It is not necessary to assume that the number of infectious hours follow a normal distribution since the sample size $n = 100$ is large. Thus by CLT, $\overline{X}$ is normally distributed approximately.

2. (a) p-value$= 0.077 > 0.05$. Do not reject H_0.
   The is insufficient evidence at 5% level of significance to conclude that the population mean amount collected per box is not \$85.40.

   (b) $k > 87.21$.

3. (a) $2(3^n - 1)$ m.

   (b) 256 m.

4. (a) $500 + \frac{n-10}{2}[100 + 5(n - 11)]$.

   (b) \$118.

   (c) 74.