1. **[TPJC Prelims 17]**
The town council is investigating the mass of rubbish in domestic dustbins. In 2016, the mean mass of rubbish in domestic dustbins was 20.0 kg per household per week. The town council starts a recycling initiative and wishes to determine whether there has been a reduction in the mass of rubbish in domestic dustbins.

The mass of rubbish in a domestic dustbin is denoted by x kg. A random sample of 50 domestic dustbins is selected and the results are summarised as follows.

$$n = 50 \qquad \sum x = 924.5 \qquad \sum x^2 = 18249.2$$

(a) Explain what is meant in this context by the term 'a random sample'. [2]

(b) Explain why the town council is able to carry out a hypothesis test without knowing anything about the distribution of the mass of rubbish in domestic dustbins. [2]

(c) Find the unbiased estimates of the population mean and variance and carry out the test at 1% level of significance for the town council. [6]

(d) Use your results in part (c) to find the range of values of n for which the result of the test would be that the null hypothesis is rejected at the 1% level of significance. [3]

2. **[CJC 19 MYE]**
Two lines l_1 and l_2 are represented by cartesian equations $\dfrac{x-1}{2} = -z-1, y = 2$ and $x+3 = 4-y = z+2$ respectively.

(a) Show that l_1 and l_2 intersect, and state the point of intersection. [4]

(b) Find the acute angle between l_1 and l_2. [2]

(c) Find the position vector of the foot of perpendicular from point $(1, 2, -1)$ to l_2. [3]

(d) Given that $(1, 2, -1)$ lies on l_1, find a vector equation for the line of reflection of l_1 in l_2. [3]

3. **[YJC 18 MYE]**
The points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, where O is the origin and $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. The point C lies on OB such that $\overrightarrow{OC} = \lambda\overrightarrow{OB}$ where λ is a real non-zero constant. The point D lies on AC such that $AD : DC = 2 : 3$ and the point E lies on AB such that $AE : EB = 2 : 5$.

(a) Find $\overrightarrow{OD}$ and $\overrightarrow{OE}$ in terms of $\mathbf{a}$,$\mathbf{b}$ and λ. [2]

(b) Given that the points O, D and E are collinear, show that $\lambda = \dfrac{3}{5}$. [3]

(c) Hence, find the area of triangle OAD in the form $k|\mathbf{a} \times \mathbf{b}|$ where k is a constant to be determined. [2]

Answers

1. (a) Every dustbin has an equal probability of being selected and the selections are made independently.

   (b) Since $n = 50$ is large, by the Central Limit Theorem, the man mass of rubbish will be approximately normally distributed.

   (c) $\overline{x} = 18.49, s^2 = 23.6$.
   $p = 0.013937$. Do not reject H_0.

   (d) $n \geq 56, n \in \mathbb{Z}$.

2. (a) $(-1, 2, 0)$.

   (b) $75.0°$.

   (c) $\frac{1}{3}(-2\mathbf{i} + 5\mathbf{j} + \mathbf{k})$.

   (d) $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} 4 \\ 2 \\ -5 \end{pmatrix}, \nu \in \mathbb{R}$.

3. (a) $\overrightarrow{OD} = \frac{3\mathbf{a}+2\lambda\mathbf{b}}{5}, \overrightarrow{OE} = \frac{5\mathbf{a}+2\mathbf{b}}{7}$.

   (c) $k = \frac{3}{25}$.