1. Find the equations of the tangents to the curve $x^2 + y^2 = \dfrac{1+y}{x}$ that are parallel to the y-axis. [6]

2. A curve is defined by the parametric equations

$$x = \frac{t}{1+2t},\ y = \frac{t}{1-2t},\ \text{where } t \in \mathbb{R}, t \neq \pm\frac{1}{2}.$$

Show that the tangent to the curve at any point with parameter t has equation $(1-2t)^2 y = (1+2t)^2 x - 4t^2$. [4]

3. By using the substitution $x = \frac{\cos\theta}{a}$, show that $\displaystyle\int_0^a \sqrt{1-a^2x^2}\,\mathrm{d}x = \frac{\pi}{4a}$. [3]

4. The roots of the equation $z^2 + 5 + 12i = 0$ are z_1 and z_2. Find z_1 and z_2 in cartesian form $x + yi$, showing your working. [4]

5. The complex numbers u and v are given by $\sqrt{3} + i$ and $-k + ki$ respectively, where k is a positive real number. Find $\dfrac{v^7}{(u^*)^3}$ in the form $re^{i\theta}$, where r is a positive constant in terms of k and $-\pi < \theta \leq \pi$. [5]

6. **[AJC 18 MYE (modified)]**

   (a) Show that $[(k+2)!]\,k - (k!)(k-2) = (k!)(k^3 + 3k^2 + k + 2)$. [1]

   (b) Hence find $\displaystyle\sum_{k=3}^{n} \left[(k!)(k^3 + 3k^2 + k + 2)\right]$ in terms of n. [3]

   (c) Hence find $\displaystyle\sum_{k=n+1}^{2n} \left[(k!)(k^3 + 3k^2 + k + 2)\right]$ in terms of n. [2]

7. **[EJC 18 MYE (modified)]** The amount of salt dissolved in a tank, S (in grams), at time t minutes is described by the differential equation $\dfrac{\mathrm{d}S}{\mathrm{d}t} = 10 - \dfrac{S}{t+10}$.

   (a) Use the substitution $Q = (t+10)S$ to simplify the differential equation into the form $\dfrac{\mathrm{d}Q}{\mathrm{d}t} = a(t+10)$, where a is a constant to be determined. [2]

   (b) Hence find S in terms of t, given that there was initially 300 grams of salt dissolved. [2]