Lateral surface area = $\pi r\ell$

$= \frac{1}{2}\pi d\ell$

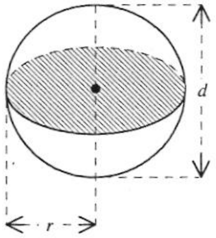
Total surface area = $\pi r(r + \ell)$

$= \frac{\pi d}{2}(\frac{d}{2} + \ell)$

Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{12}\pi d^2 h$

2. **Sphere**



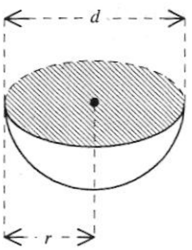
Surface area = $4\pi r^2$

$= \pi d^2$

Volume = $\frac{4}{3}\pi r^3$

$= \frac{1}{6}\pi d^3$

3. **Hemisphere**



Surface area = $3\pi r^2$

$= \frac{3}{4}\pi d^2$

Volume = $\frac{2}{3}\pi r^3$

$= \frac{1}{12}\pi d^3$

PAST EXAMINATION QUESTIONS

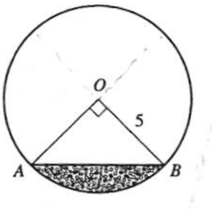
1. (a) The volume of a cube is 200 cubic centimetres. Find the length of an edge of the cube, correct to the nearest centimetre.

   (b) The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. Find, correct to one significant figure, the volume of a sphere with radius 10 cm. (N99/1/8`

2. **[The value of π is 3.142 correct to three decimal places.]**

   Earth is excavated to make a railway tunnel. The tunnel is a cylinder of radius 5 m and lengt 450 m.

   (a) Calculate the volume of earth removed.
   A level surface is laid inside the tunnel to carry the railway lines. The diagram shows the circular cross-section of the tunnel. The level surface is represented by AB, the centre of the circle is O and $A\hat{O}B = 90°$. The space below AB is filled with rubble.

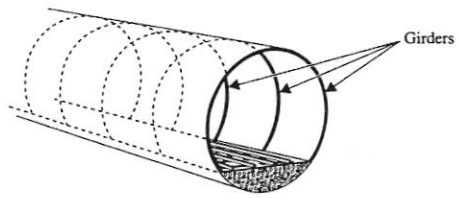


(b) Calculate

  (i) the area of triangle AOB,

  (ii) the volume of rubble used in the 450 m length of tunnel.

(c) Steel girders are erected above the tracks to strengthen the tunnel. Some of these are shown in the diagram above. The girders are erected at 6 m intervals along the length of the tunnel, with one at each end.



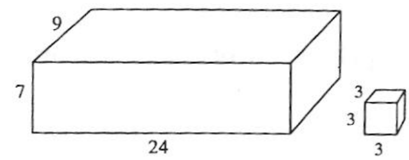
  (i) How many girders are erected?

  (ii) Calculate the length of each girder.

  (iii) Calculate the total length of steel required in the 450 m length of tunnel. (N99/2/7)

3. A rectangular block of wood has dimensions 24 cm by 9 cm by 7 cm. It is cut up into children's bricks.

Each brick is a cube of side 3 cm.

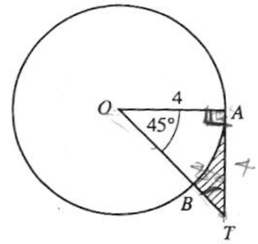


(a) Find the largest number of bricks that can be cut from the block.

(b) Find the volume of wood that is left. (N2000/1/3)

4. [The value of π is 3.14 correct to three significant figures.]

In the diagram, the circle, centre O passes through A and B. The radius of the circle is 4 cm and $A\hat{O}B = 45°$.



(a) Find the area of the minor sector AOB.

(b) The tangent at A meets OB produced at T. Find the shaded area.

(N2000/1/13)

5. [The value of π is 3.142, correct to three decimal places.]

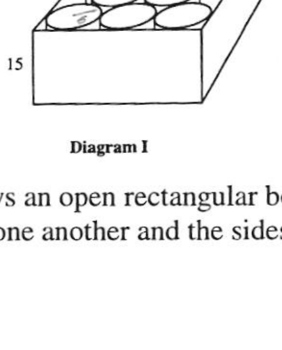


Diagram I

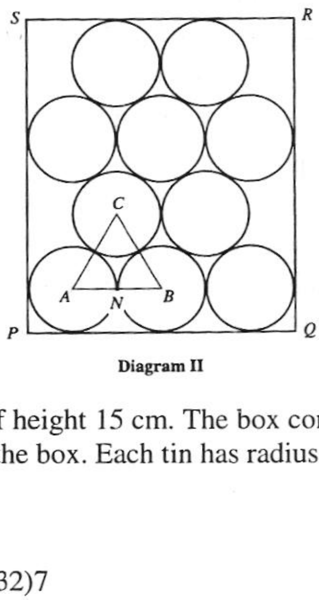
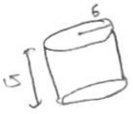


Diagram II

Diagram I shows an open rectangular box of height 15 cm. The box contains 10 cylindrical tins. The tins touch one another and the sides of the box. Each tin has radius 6 cm and height 15 cm.



(a) (i) Each tin has a label wrapped round it, which exactly covers its vertical surface. Calculate the area of paper needed for one label.

(ii) Calculate the volume of one tin.

(b) Diagram II shows the view of the box and the tins from above. $PQRS$ is the rectangular cross-section of the box. The points A, B and C are the centres of the circular tops of three adjacent tins which touch one another. The midpoint of AB is N.

(i) Write down the length of AC.

(ii) Calculate

(a) the length of CN,

(b) the length of PS.

(c) The height of the box is also 15 cm. Calculate the volume of the space in the box which is **not** occupied by the tins. (N2000/2/7)

6. **[The value of π is 3.142 correct to three decimal places.]**

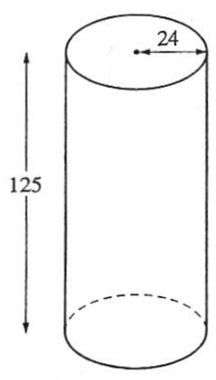


Diagram I

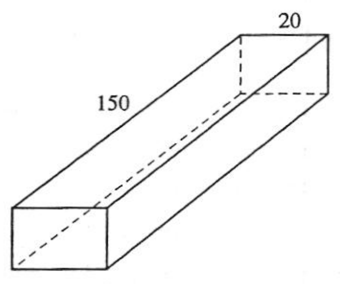


Diagram II

A water tank, shown in Diagram I, is a circular cylinder of radius 24 cm and height 125 cm. It is open at one end and full of water.

(a) Calculate

(i) the volume, in litres, of water in the tank,

(ii) the **total** area, in square metres, of the outside of the open tank.

(b) Diagram II shows a rectangular trough of length 150 cm and width 20 cm. The trough wa completely filled with 48 000 cm^3 of water from the tank.

Calculate the depth of the trough.

(c) After the trough had been filled, water started to leak from the tank. In 2 hours 30 minu it was found that 20 000 cm^3 ran out of the tank.

Calculate the rate at which the level of water in the tank was falling. Express your ans in centimetres per hour. (N2001

7. **[The value of π is 3.142 correct to three decimal places.]**

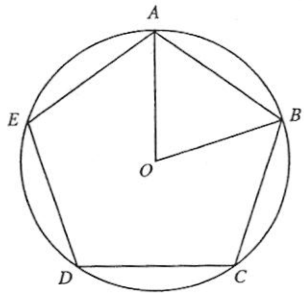


Diagram I

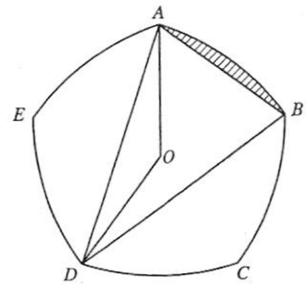


Diagram II

(a) Explain why $A\hat{O}B = 72°$.

(b) Calculate the area of the pentagon *ABCDE*.

(c) Diagram II shows a design for a new coin. The vertices of the regular pentagon *ABCDE* are joined by circular arcs whose centres are the opposite vertices. For example, the arc *AB* has centre *D* and radius *DA*.

(i) Explain why $A\hat{D}B = 36°$.

(ii) Show that the length of *DA* is approximately 2.85 cm.

(iii) Calculate the area of triangle *DAB*.

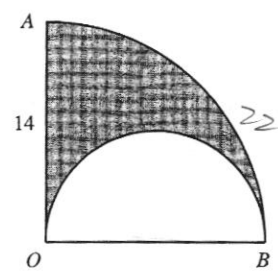
(iv) Calculate the area of the segment shaded in Diagram II.

(v) Calculate the area of the face *ABCDE* of the coin. (N2001/2/10)

8. In the diagram, *OAB* is a quadrant of a circle, radius 14 cm. A semicircle is drawn on *OB* as diameter. Taking π to be $\frac{22}{7}$, calculate

(a) the arc length *AB*,

(b) the perimeter of the shaded part of the diagram.



(N2002/1/13)

9.

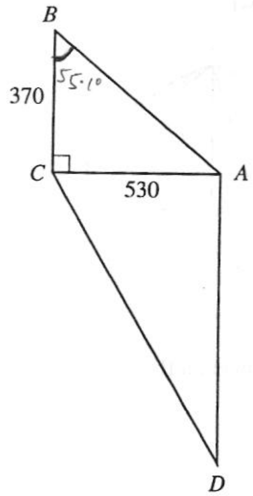


Diagram I

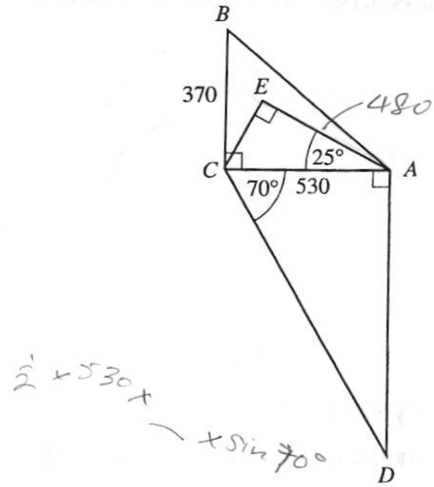


Diagram II

Diagram I shows a path, AC, in a park $ABCD$.
It is given that $AC = 530$ m, $BC = 370$ m and that AC is perpendicular to BC.

(a) Calculate angle ABC.

(b) Diagram II shows two other paths, AE and CE, in the park.
Given that angle $CAE = 25°$ and angle $AEC = 90°$, calculate the length of AE.

(c) Given also that angle $ACD = 70°$ and angle $CAD = 90°$, calculate

(i) the length of CD,

(ii) the area of the park $ABCD$. (N2002/2/1)

10. [The area of the curved surface of a cone of radius r and slant height l is πrl.
The volume of a cone is $\frac{1}{3}$ × base area × height.]

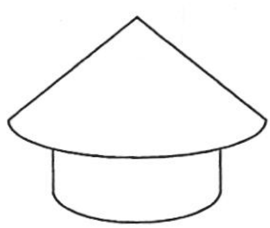


Diagram I

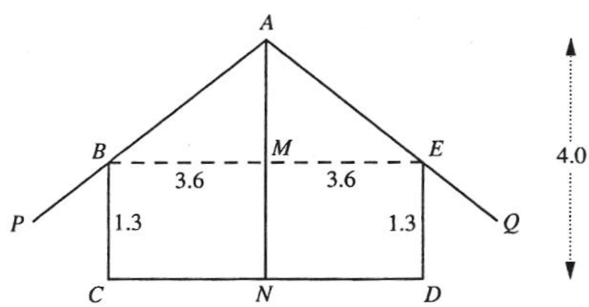


Diagram II

Diagram I shows a traditional hut which consists of a circular cylinder with an overhanging roof. The roof is the curved surface of a cone and is supported by a central vertical pole.

Diagram II shows a vertical cross-section of the hut.
BE and CD are horizontal.

$AN = 0.4$ m, $BM = ME = 3.6$ m and $BC = DE = 1.3$ m.

(a) Show that $AB = 4.5$ m.

(b) Calculate

(i) the volume of the inside of the hut,

(ii) the **total** surface area of the inside of the hut (including the floor).

(c) The sun is directly overhead. The shadow of the overhanging section of the roof on the ground is a circular ring around the hut.

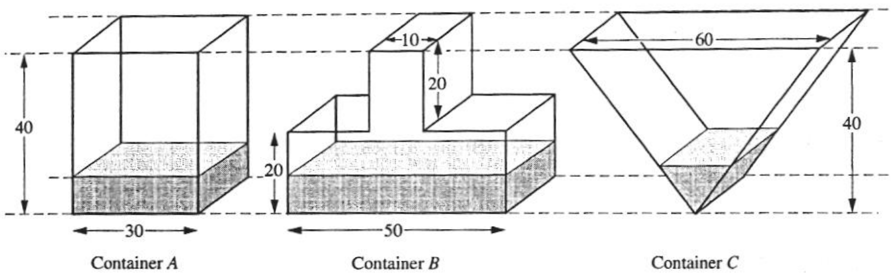
$AP = AQ = 5.5$ m.

Calculate

(i) PQ,

(ii) the area of the circular ring of shadow **outside** the hut. (Ignore the thickness of the walls.) (N2002/2/9)

11.



Each of the containers shown in the diagrams has a height of 40 cm. Their other dimensions are as shown. All three containers have uniform cross-sections. The containers are being filled to the brim with water which flows into each one at the same constant rate. It takes 12 minutes to fill each container.

(a) Find the time taken for the water to reach a depth of 20 cm in

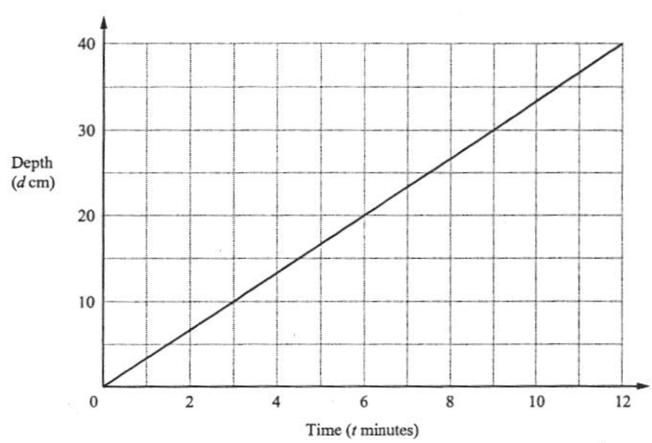
(i) container B,

(ii) container C.

(b) A graph is drawn showing the relationship between the depth of the water, d cm, and the time, t minutes, as each container is being filled. The graph shown in the answer space is that for container A. On the same diagram, sketch the graph for

(i) container B,

(ii) container C.



(N2003/1/23)

12.

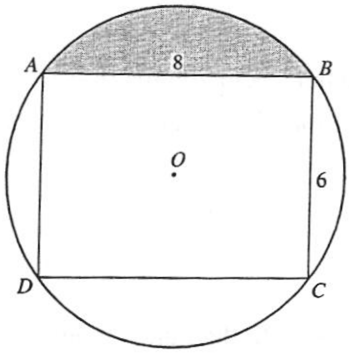


Diagram I

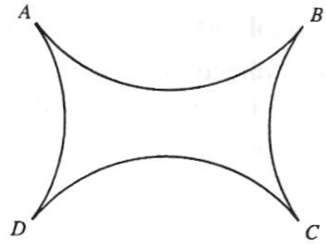


Diagram II

$ABCD$ is a rectangle in which $AB = 8$ cm and $BC = 6$ cm.
A circular piece of wire, centre O, passes through the vertices of the rectangle as shown in Diagram I.

(a) Show that the radius of the circular wire is 5 cm.

(b) Show that angle $AOB = 106.3°$, correct to 1 decimal place.

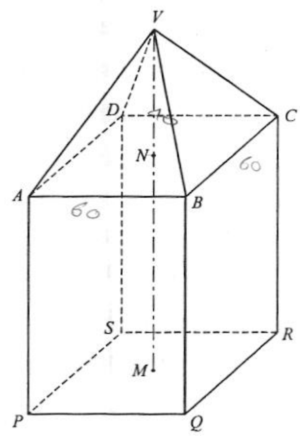
(c) Calculate the area of the shaded segment.

(d) The circular wire is cut at A, B, C and D, and the four pieces are joined to form the shape shown in Diagram II.
Calculate the area enclosed by the wires in Diagram II. (N2003/2/6)

13. **[The volume of a pyramid = $\frac{1}{3}$ × base area × height.]**

The diagram shows a solid traffic bollard.
It consists of a square-based pyramid, $VABCD$, attached to a cuboid, $ABCDPQRS$.
The vertical line, VNM, passes through the centres, N and M, of the horizontal squares $ABCD$ and $PQRS$.

$AB = BC = 60$ cm and $VN = 40$ cm.



(a) Calculate

(i) VA,

(ii) angle VAN,

(iii) angle VAP.

(b) Given also that $AP = BQ = CR = DS = 80$ cm, calculate

(i) the volume of the bollard,

(ii) the total surface area of the sides and top of the bollard.

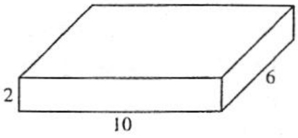
(c) The highway authority needs to paint the sides and tops of 17 of these bollards.
The paint is supplied in tins, each of which contains enough paint to cover 8 m^2.
Find the number of tins of paint needed. (N2003/2/7)

14. A block of wood is a cuboid, 10 cm by 6 cm by 2 cm.

Find

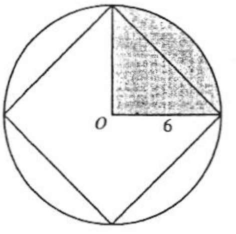
(a) its volume,

(b) its surface area.



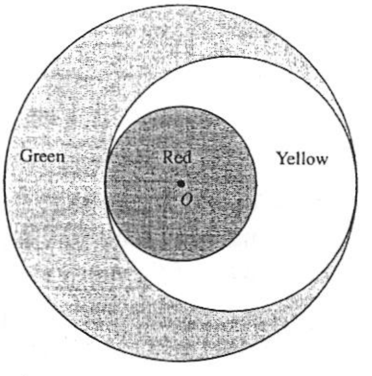
(N2004/1/7)

15. The diagram shows a square inscribed in a circle, centre O.
The radius of the circle is 6 cm.
The perimeter of the shaded region can be written as $(p\pi + q)$ centimetres.
Find the values of p and q.



(N2004/1/17b)

16. The diagram shows the design of a company symbol.
It consists of three circles.
The smallest circle has centre O and radius $2x$ centimetres.
The largest circle has centre O and radius $2y$ centimetres.
The third circle touches both the other two circles as shown.
The three regions formed are coloured red, yellow and green as shown.



(a) Explain fully why the radius of the third circle is $(x + y)$ centimetres.

(b) Write down, in terms of π, x and y, expressions for the area of the region that is coloured

(i) yellow,

(ii) green.

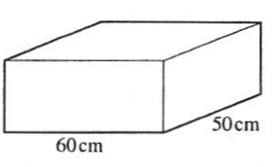
(c) The area of the green region is twice the area of the yellow region.
Use this information to write down an equation involving x and y, and show that it can be simplified to $y^2 - 6xy + 5x^2 = 0$.

(d) (i) Factorise $y^2 - 6xy + 5x^2$.

(ii) Solve the equation $y^2 - 6xy + 5x^2 = 0$, expressing y in terms of x.

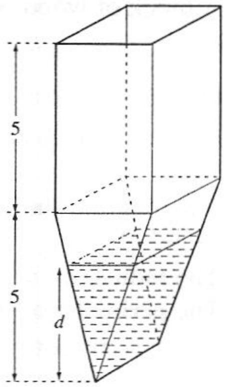
(e) Calculate the fraction of the design that is coloured yellow. (N2004/2/8)

17. A cuboid is shown in the diagram.
The volume of the cuboid is 90 000 cm^3.
Find the height of the cuboid.



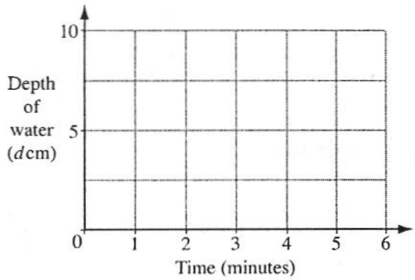
(N2005/1/7)

18. The container shown in the diagram is a **prism**.
The cross-section consists of a rectangle and a triangle.
The heights of both the rectangle and the triangle are 5 cm.
Water is poured into the empty container at a constant rate and fills it in 6 minutes.



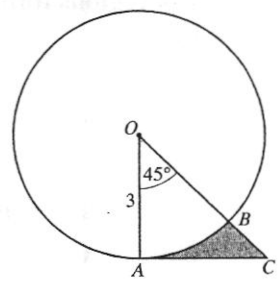
(a) After how many minutes will the **triangular** prism be full?

(b) On the axes in the answer space, sketch the graph showing how the depth of the water, d centimetres, in the container varies over the six minutes.



(N2005/1/14)

19. In the diagram, the circle, centre O, passes through A and B.
The tangent at A meets OB produced at C.
The radius of the circle is 3 cm and $A\hat{O}B = 45°$.



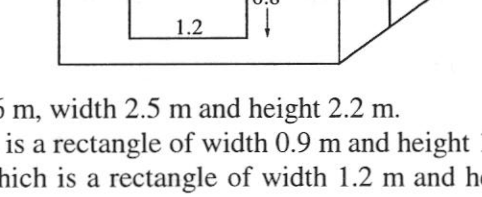
(a) The area of the shaded region can be written as $(p - q\pi)$ cm^2.
Find the values of p and q.

(b) The perimeter of the shaded region can be written as $(v\pi + \sqrt{t})$ cm.
Find the values of v and t.

(N2005/1/16)

Find perimeterin the form $(v\pi + \sqrt{t})$, where v t +ve integers

20.



A room has length 3.6 m, width 2.5 m and height 2.2 m.
It has one door which is a rectangle of width 0.9 m and height 1.9 m.
It has one window which is a rectangle of width 1.2 m and height 0.8 m, with a semicircle on one of its longer sides.

(a) (i) Calculate the area of the window.

(ii) Show that the area of the walls, correct to three significant figures, is 23.6 m^2.

(b) Tiles are to be fixed to the walls inside the room.
Eileen estimated the number of tiles needed to cover the walls inside the room in the following way.
She first increased the area, 23.6 m^2, by 12% and then calculated the number of tiles that she needed to cover this total area.
Each tile is a square of side 25 cm.

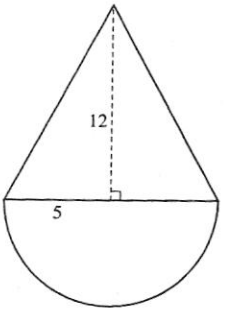
(i) Find the number of tiles that she needed.

(ii) The tiles are sold in boxes, each containing 20 tiles.
Each box of tiles costs $15.
Calculate the cost of the boxes of tiles that she bought.

(iii) When the shopkeeper sold the tiles at $15 per box, he made a profit of 20%.
Calculate the profit that he made on each box. (N2005/2/7)

21. **[The surface area of a sphere = $4\pi r^2$.]**
[The volume of a sphere = $\frac{4}{3}\pi r^3$.]
[The area of the curved surface of a cone of radius r and slant height l is πrl.]
[The volume of a cone = $\frac{1}{3}\times$ base area $\times$ height.]

A solid cone has a base radius of 5 cm and height 12 cm.
A solid hemisphere has a radius of 5 cm.
A metal toy is formed by joining the plane faces of the cone and the hemisphere.



(a) Show that the length of the slant edge of the cone is 13 cm.

(b) Calculate

(i) the surface area of the toy,

(ii) the volume of the toy.

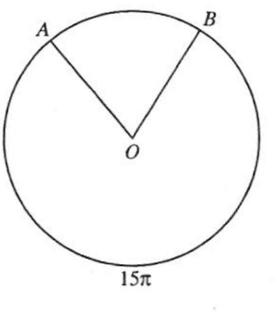
(c) A solid metal cylinder has a radius of 1.5 m and height 2 m.
The cylinder was melted down and all of the metal was used to make a large number of these toys. Calculate the number of toys that were made. (N2005/2/9)

22. The points A and B lie on the circumference of a circle, centre O.
The area of the circle is 81 π cm^2.
The length of the major arc AB is 15π cm.
Calculate

(a) the radius of the circle,

(b) the length of the minor arc, giving your answer in the form $k\pi$,

(c) the acute angle AOB.



(N2006/1/15)

23. The triangle with vertices $A(4, 4)$, $B(-2, -6)$ and $C(4, -1)$ is shown in the diagram. Find

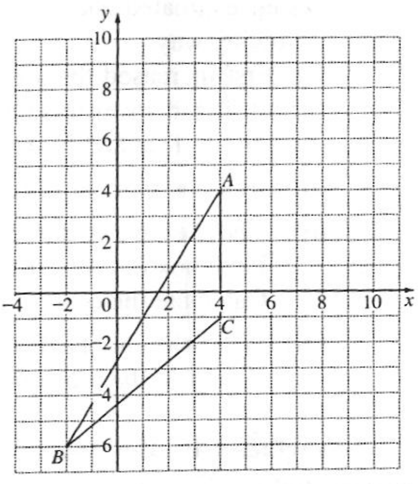
(a) (i) the area of ΔABC,

(ii) the coordinates of the point P such that $ABCP$ is a parallelogram,

(iii) the area of the parallelogram $ABCP$,

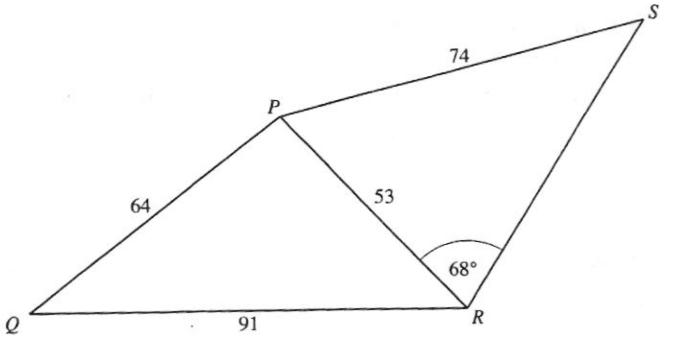
(iv) $\tan B\hat{A}C$.

(b) It is given that the length of $BC = k$ units. Write down $\cos B\hat{C}A$, giving your answer in terms of k.



(N2006/1/22)

24.



The diagram shows a footpath PR across a park $PQRS$.
$PQ = 64$ m, $PR = 53$ m, $PS = 74$ m and $QR = 91$ m.
Angle PRS = 68°.

Calculate

(a) $Q\hat{P}R$,

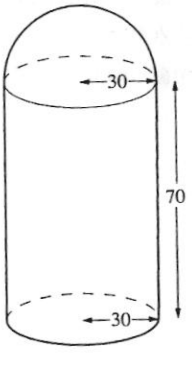
(b) $R\hat{P}S$

(c) the area of triangle PRS.

(N2006/2/3)

25. **[Surface area of a sphere = $4\pi r^2$]**
[Volume of a sphere = $\frac{4}{3}\pi r^3$]

A hot water tank is made by joining a hemisphere of radius 30 cm to an open cylinder of radius 30 cm and height 70 cm.



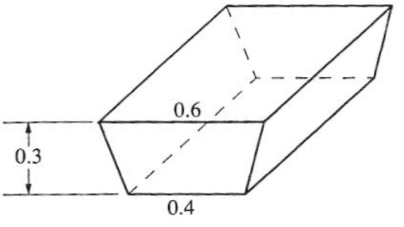
(a) Calculate the total surface area, including the base, of the outside of the tank.

(b) The tank is full of water.

(i) Calculate the number of litres of water in the tank.

(ii) The water drains from the tank at a rate of 3 litres per second.
Calculate the time, in minutes and seconds, to empty the tank.

(iii)



All of the water from the tank runs into a bath, which it just completely fills.

The bath is a prism whose cross-section is a trapezium.

The lengths of the parallel sides of the trapezium are 0.4 m and 0.6 m.

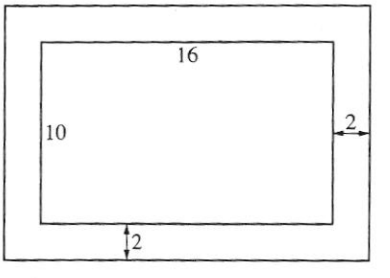
The depth of the bath is 0.3 m.

Calculate the length of the bath. (N2006/2/7)

26. A rectangular picture is 16 cm long and 10 cm wide.
It has a 2 cm border around it, as shown in the diagram.
Calculate the area of the border, giving your answer in

(a) square centimetres,

(b) square millimetres.



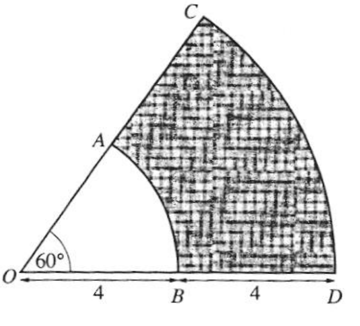
(N2007/1/9)

27. The diagram shows the arcs AB and CD of two circles, centre O, with radii 4 cm and 8 cm respectively.
OAC and OBD are straight lines and angle $COD = 60°$.
Calculate

(a) the perimeter of the shaded region, giving your answer in the form $a + b\pi$,

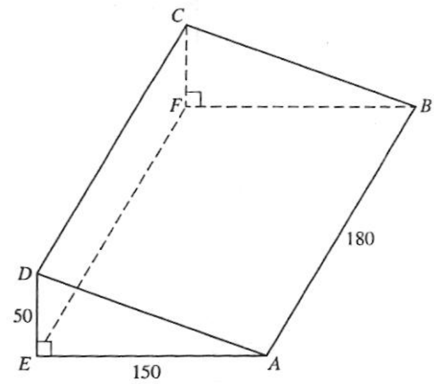
(b) the area of the shaded region, giving your answer as a multiple of π,

(c) the ratio of the shaded area to the area of the sector AOB.
Give your answer in the form $m : n$ where m and n are integers.



(N2007/1/23)

28. The diagram shows a concrete slab, $ABCDEF$, in the form of a right-angled triangular prism. $DE = 50$ cm, $AE = 150$ cm and $AB = 180$ cm.



(a) Calculate the volume, in cubic metres, of the block.

(b) The concrete is a mixture of cement, sand and stones in the ratio 3 : 5 : 6 by volume.
Show that, correct to three significant figures, the volume of cement needed to make the slab is $0.145\ m^3$.

(c) The mass of 1 cubic metre of cement is 1420 kg.
Calculate the mass of cement needed to make the slab.

(d) Cement is sold in bags, each of which contains 25 kg of cement.
How many bags of cement should be bought to make the block?

(e) The price of each bag of cement is $8.50, which includes 5% Goods and Services Tax.
Calculate, correct to the nearest cent, the **total** tax paid for the cement bought to make the slab. (N2007/2/6)