

§1 Equations and inequalities extra problems

February 25, 2017

1. Show that $\cos(n) \sin 1$ can be expressed as $A(\sin P - \sin Q)$, where A, P and Q are constants (possibly in terms of n) to be determined.

Hence find

$$\cos 1 \sin 1 + \cos 2 \sin 1 + \cos 3 \sin 1 + \cos 4 \sin 1 + \cdots + \cos 100 \sin 1.$$

2. Solve the following inequalities

(a) $\frac{2x}{x-1} \leq \frac{1}{3},$

(b) $x \geq \frac{4}{x},$

(c) $\frac{1}{x+2} > \frac{2}{x}.$

3. [SRJC 16 Prelims]

It is given that $y = \frac{x^2 - x - 1}{x + 1}$ for $x \in \mathbb{R}, x \neq -1$. Without using a graphic calculator, find the set of values that y cannot take. [3]

4. [ACJC 16 Prelims]

Without the use of a calculator, solve the inequality

$$\frac{2w - 5}{w^2 - 3} > 0.$$

[2]

Hence solve

$$\frac{(2|y| - 5) \sin x}{y^2 - 3} \leq 0,$$

given that $\pi < x \leq \frac{3\pi}{2}$.

[3]

5. [HCI 16 Prelims (part)]

Solve the inequality

$$\frac{2}{4(x+1)^2 + 1} > 1.$$

[2]

6. [IJC 16 Prelims]

A theme park sells tickets at different prices according to the age of the customer. The age categories are: senior citizen, adult and child. Four tour groups visited the theme park on the same day. The numbers in each category for three of the groups, together with the total cost of the tickets for each of these groups, are given in the following table.

Group	Senior Citizen	Adult	Child	Total cost
<i>A</i>	2	19	9	\$1982
<i>B</i>	0	10	3	\$908
<i>C</i>	1	7	4	\$778

Find the total cost of tickets for Tour Group *D*, which consists of four senior citizens, five adults and one child. [4]

7. [IJC 16 Prelims]

Without using a calculator, solve the inequality

$$\frac{3}{4x+3} \leq \frac{x}{x+1}.$$

[5]

Hence or otherwise solve the inequality

$$\frac{3}{4e^x+3} > \frac{e^x}{e^x+1}.$$

[2]

8. [JJC 16 Prelims]

In the finals of a quiz, a team is required to answer 25 questions. Each question that is correctly answered scored 5 points, while a question that is wrongly answered is deducted 3 points. If the answer is partially correct, the team scores 2 points.

After 24 questions, a team has a correct answers, b partially correct answers and c wrong answers for a total of 79 points. If the team answers the last question wrongly, then the total number of questions answered correctly and partially correct is four times the number of questions answered wrongly. By forming a system of linear equations, find the values of a , b and c . [4]

9. [NJC 16 Prelims]

(a) By using an algebraic method, solve the inequality

$$\frac{3x^2 + 14}{(x + 1)(x + 2)} \geq 2.$$

[4]

(b) Hence, showing all your working clearly, solve the inequality

$$\frac{3x^2 + 14}{(|x| - 1)(|x| - 2)} \geq 2.$$

[2]

10. [PJC 16 Prelims]

The police wish to crack a 3-digit passcode. The sum of the digits is 14. When the digits in the number are reversed, the new number becomes 495 more than the original number. The digit in the tens position is 3 more than the digit in the hundreds position. What is the passcode? [4]

11. [SAJC 16 Prelims]

Solve the inequality

$$\frac{x^2 + 6x + 8}{x - 1} \geq 0.$$

Hence, by completing the square, solve the inequality

$$\frac{y^2 + 2y + 15}{|y + 1| - 1} \geq -6.$$

[6]

12. [TJC 16 Prelims]

Solve the inequality

$$\frac{x^2 - 2a^2}{x} < a,$$

giving your answer in terms of a , where a is a positive real constant. [3]

Hence solve

$$\frac{x^2 - 2a^2}{|x|} < a.$$

[2]

13. [VJC 16 Prelims]

Without using a calculator, solve the inequality

$$\frac{4 - 7x}{x - 3} \geq x.$$

[4]

14. [VJC 16 Prelims]

In 2016, Edwin, his father and his grandfather have an average age of 53. In the same year, the sum of one-half of his grandfather's age, one-third of his father's age and one-fourth of Edwin's age is 65. Twenty-two years ago, his grandfather's age was twice the sum of his father's age and his age. What are their respective ages in 2016? [3]
(You may assume that Edwin's age in 2016 is more than 22.)

15. [YJC 16 Prelims]

Solve the inequality

$$\frac{2}{x + 2} \geq \frac{x + 1}{3}.$$

[3]

Hence, find the range of values of x for which

$$\frac{2}{x + 3} \geq \frac{2 + 2}{3}.$$

[2]

Answers

- $\sin 99 + \sin 100 - \sin 1.$
- $-\frac{1}{5} \leq x < 1.$
 $-2 \leq x < 0$ or $x \geq 2.$
 $-4 < x < -2$ or $x > 0.$
- $(-5, -1).$
- $-\sqrt{3} < w < \sqrt{3}$ or $w > \frac{5}{2}.$
 $-\sqrt{3} < y < \sqrt{3}$ or $y \geq \frac{5}{2}$ or $y \leq -\frac{5}{2}.$
- $-\frac{3}{2} < x < -\frac{1}{2}.$
- \$570.
- $x < -1$ or $-\frac{\sqrt{3}}{2} \leq x < -\frac{3}{4}$ or $x \geq \frac{\sqrt{3}}{2}.$
 $x < \ln \frac{\sqrt{3}}{2}.$
- $a = 17, b = 3, c = 4.$
- $x < -2$ or $x > -1.$
 $x < -2$ or $-1 < x < 1$ or $x > 2.$
- 257.
- $-4 \leq x \leq -2$ or $x > 1.$
 $y < -2$ or $y > 0.$
- $x < -a$ or $0 < x < 2a.$
 $-2a < x < 2a, x \neq 0.$
- $x \leq -2 - 2\sqrt{2}$ or $-2 + 2\sqrt{2} \leq x < 3.$
- 24, 51, 84.
- $x \leq -4$ or $-2 < x \leq 1.$
 $x \leq -5$ or $-3 < x \leq 0.$