1. **[CJC 19 MYE]**
The functions f and g are defined by

$$f : x \mapsto x^2 + 2, \qquad x \in \mathbb{R},$$
$$g : x \mapsto -2x + 5, \qquad x \in \mathbb{R}.$$

State a sequence of transformations which transform the graph of $y = f(x)$ to the graph of $y = fg(x)$. [3]

2. **[JPJC 19 MYE]**
The curve $y = g(x)$ undergoes the transformations A, B and C in succession:

- A: a translation of 1 unit in the positive x-direction,
- B: a scaling parallel to the x-axis with scale factor $\frac{1}{2}$, and
- C: a translation of 3 units in the positive y-direction.

Find an expression for $g(x)$ if the equation of the resulting curve is $y = 3 - \dfrac{1}{2x-1}$. [3]

3. **[CJC 18 Prelims]**
(a) Let $y = \ln(e^x + 1)$.
Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - \dfrac{\mathrm{d}y}{\mathrm{d}x} + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 0.$ [2]

(b) By further differentiation of the result in part (a), find the first four non-zero terms in the Maclaurin series for y. [5]

(c) Hence, expand $\dfrac{\ln(e^x+1)}{4-x^2}$ in ascending powers of x up to and including the term in x^3. Leave the coefficients of the series in exact form. [3]

4. **[ACJC 18 Prelims]**
In the triangle ABC, $AB = 2$, $BC = 3$ and angle $ABC = \dfrac{\pi}{3} - \theta$ radians. Given that θ is a sufficiently small angle, show that

$$AC \approx \left(7 - 6\sqrt{3}\theta + 3\theta^2\right)^{\frac{1}{2}} \approx a + b\theta + c\theta^2,$$

where a, b and c are to be determined in exact form. [7]

5. **[MI 18 Prelims]**
(a) Given that $\ln y = \sin 2x$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y\cos 2x$. Hence find the values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x = 0$. [3]

(b) Write down the first three non-zero terms in the Maclaurin series for y. [1]

(c) It is given that the second and third non-zero terms in part (b) are equal to the first and second non-zero terms in the series expansion of $e^{px}\ln(1+qx)$ respectively. Using appropriate expansions from the List of Formulae (MF26), find the values of the constant p. [3]

(d) Hence state the range of values of x for which the series expansion of $e^{px}\ln(1+qx)$ is valid. [1]

Answers

1. 1. Translate by 5 units in the negative x-direction.
   2. Reflect about the y-axis.
   3. Scale by a factor of $\frac{1}{2}$ parallel to the x-axis.

   (Other answers are possible too: check with me)

2. $g(x) = -\frac{1}{x}$.

3. (b) $y = \ln 2 + \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{192}x^4 + \dots$

   (c) $y = \frac{1}{4}\ln 2 + \frac{1}{8}x + (\frac{1}{32} + \frac{1}{16}\ln 2)x^2 + \frac{1}{32}x^3 + \dots$

4. $a = \sqrt{7}, b = -\frac{3\sqrt{21}}{7}, c = -\frac{3\sqrt{7}}{49}$.

5. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2, \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4$.

   (b) $y = 1 + 2x + 2x^2 + \dots$

   (c) $p = 2, q = 2$.

   (d) $-\frac{1}{2} < x \leq \frac{1}{2}$.