1. **[CJC 19 MYE]**
The functions f and g are defined by

$$f : x \mapsto x^2 + 2, \qquad x \in \mathbb{R},$$
$$g : x \mapsto -2x + 5, \qquad x \in \mathbb{R}.$$

State a sequence of transformations which transform the graph of $y = f(x)$ to the graph of $y = fg(x)$. [3]

2. **[NYJC 19 MYE (modified)]**
A curve C has equation $ax^2 - 4y^2 - 2abx + 24y - 36 = 0$, where a and b are positive constants. C passes through $(0, 3)$ and has an oblique asymptote with equation $y = -\frac{5}{2}x + 8$.

(a) Find the values of a and b. [5]

(b) Find the equation of the other oblique asymptote. [2]

(c) Sketch C, stating clearly the coordinates of the vertices. [3]

3. **[CJC 19 MYE]**
The functions p and q are defined by

$$p : x \mapsto (x + 3)^2 - 1, \qquad x \in \mathbb{R}, x \geq -4.$$
$$q : x \mapsto \frac{1}{x - 5}, \qquad x \in \mathbb{R}, x > 5.$$

(a) Only one of the composite functions pq and qp exists. Give a definition (including the domain) of the composite that exists, and explain why the other composite does not exist. [3]

(b) Find the range of the composite function that exists from part (a). [2]

4. **[RJC 19 MYE]**
Functions f and g are defined by

$$f\colon x \mapsto \frac{1}{(x-1)^2} \qquad \text{for } x \in \mathbb{R}, x \neq 1,$$
$$g\colon x \mapsto e^x, \qquad \text{for } x \in \mathbb{R}.$$

(a) Show that f does not have an inverse. [1]

(b) If the domain of f is restricted to $x > k$, state the least value of k for which the function f^{-1} exists. [1]

In the rest of the question, the domain of f is $[2, \infty)$.

(c) Determine whether there are solutions to the equation $f^{-1}f(x) = ff^{-1}(x)$, showing your working clearly. [2]

(d) Explain why only one of the composite functions fg and gf exists. Find the range of the composite function that exists. [4]

5. **[EJC MYE 19 (modified)]**
The curve C has parametric equations

$$x = -\frac{1}{2}\cos 2\theta, \quad y = \sqrt{2}\sin\theta - 1, \quad \text{where } -\pi < \theta < \pi.$$

(a) Sketch the curve C, giving the coordinates of its vertex, endpoints and any points where C crosses the x- and y- axes. [5]

(b) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ, leaving your answer in the simplest form as a single trigonometric function. [3]

(c) The line $y = 2x - 1$ intersects C. Find the point(s) of intersection. [3]

6. **[VJC MYE 19]**

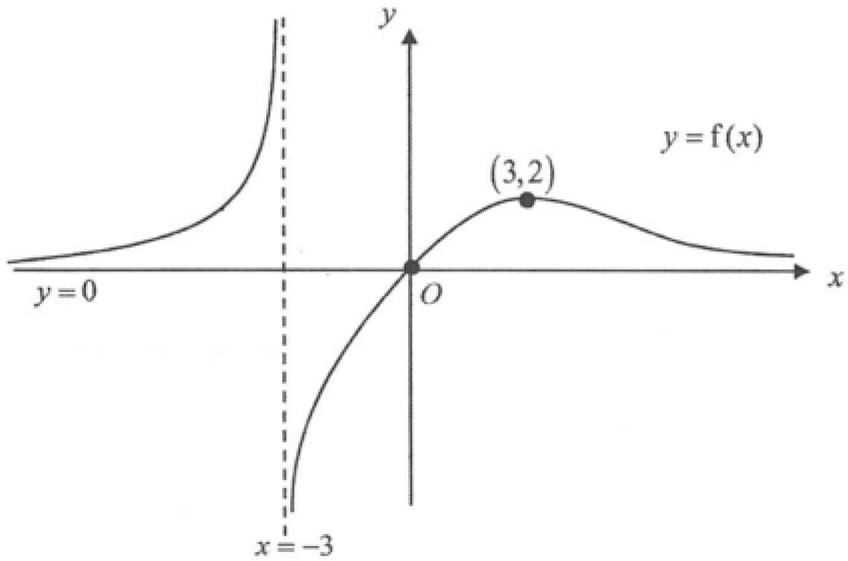
(a) Given that $y = \ln\sqrt{\frac{1-x}{1+x^2}}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Given that $\cos^{-1}(\sin x) + \frac{x}{y} = 4$, $\frac{\pi}{2} < x < \pi$, show that that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{x}\left(\frac{1}{y} + k\right)$, where k is a constant to be determined. [4]

(c) A curve has equation $y = 3^{1-\sec\left(2x+\frac{\pi}{3}\right)}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Hence, without using a graphing calculator, find the equations of the two tangents to the curve which are parallel to the x-axis. [5]

7. **[CJC 19 MYE]**
The diagram shows that graph of $y = f(x)$.



On separate diagrams, indicating clearly the equations of any asymptotes, the coordinates of turning points, and the coordinates of any points of intersection with the x- and y-axes, sketch the graphs of

(a) $y = f(\frac{1}{2}x) - 1$, [4]

(b) $y = f(|x|)$. [3]

(c) $y = \dfrac{1}{f(x)}$. [4]

Answers

1. 1. Translate by 5 units in the negative x-direction.
   2. Reflect about the y-axis.
   3. Scale by a factor of $\frac{1}{2}$ parallel to the x-axis.

   (Other answers are possible too: check with me.)

2. (a) $a = 25, b = 2$.

   (b) $y = \frac{5}{2}x - 2$.

3. (a) qp does not exist because $R_p = [-1, \infty) \nsubseteq D_q = (5, \infty)$.
   $pq : x \mapsto (\frac{1}{x+5} + 3)^2 - 1, x \in \mathbb{R}, x > 5$.

   (b) $R_{pq} = (8, \infty)$.

4. (b) Least $k = 1$.

   (c) There are no solutions.

   (d) $R_g = (0, \infty) \nsubseteq [2, \infty) = D_f$ so fg does not exist.
   $R_f = (0, 1] \subseteq (-\infty, \infty) = D_g$ so gf exists.
   $R_{gf} = (1, e]$.

5. (a) $(0, -2), (0, 0), (-\frac{1}{2}, -1), (\frac{1}{2}, \sqrt{2} - 1), (\frac{1}{2}, -\sqrt{2} - 1)$.

   (b) $(-0.309, -1.62)$.

   (c) $\frac{\sqrt{2}}{2} \csc \theta$.

6. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(\frac{-1}{1-x} - \frac{2x}{1+x^2})$.

   (b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{x}(\frac{1}{y} + 1)$.

   (c) $\frac{\mathrm{d}y}{\mathrm{d}x} = (-2 \ln 3)(3^{1-\sec(2x+\frac{\pi}{3})})(\sec(2x + \frac{\pi}{3}))(\tan(2x + \frac{\pi}{3}))$.
   $y = 1$ or $y = 9$.