1. **[MJC 17 Promos (modified)]**
A curve has parametric equations

$$x = \frac{2}{t} - 3,\ y = (2t-3)^2 \text{ where } t > 0.$$

(a) Sketch the curve, showing clearly the intersection between the curve and the axes. [2]

(b) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [2]

(c) Find the coordinates of the point where the tangent is parallel to the line $y = x - 7$. [3]

2. **[JPJC 19 MYE]**
The curve $y = g(x)$ undergoes the transformations A, B and C in succession:

- A: a translation of 1 unit in the positive x-direction,
- B: a scaling parallel to the x-axis with scale factor $\frac{1}{2}$, and
- C: a translation of 3 units in the positive y-direction.

Find an expression for $g(x)$ if the equation of the resulting curve is $y = 3 - \dfrac{1}{2x-1}$. [3]

3. **[NJC 19 MYE (modified)]**

(a) Describe a pair of transformations which transforms the curve C with equation $\dfrac{x^2}{6^2} + \dfrac{(y+3)^2}{2^2} = 1$ on to the circle with centre at the origin and radius 6 units. [3]

(b) Sketch the curve C. [2]

[3]

4. **[EJC 19 MYE]**
The function f is defined by

$$f : x \mapsto \sqrt{x+1} - \frac{1}{2}, \qquad x \in \mathbb{R}, x > -1.$$

(a) Sketch the graph of $y = f(x)$. Your sketch should state the coordinates of any points of intersection with the axes and endpoints. [2]

(b) Find $f^{-1}(x)$, stating the domain of f^{-1}. [3]

(c) On the same diagram as in part (a), sketch the graph of $y = f^{-1}(x)$. [1]

(d) Write down the equation of the line in which the graph of $y = f(x)$ must be reflected in order to obtain the graph of $y = f^{-1}(x)$, and hence find the exact solution of the equation $f(x) = f^{-1}(x)$. [4]

(e) Use (c) and (d) to deduce the solution set of $f(x) \geq f^{-1}(x)$. [2]

5. **[EJC MYE 19 (modified)]**
Functions f and g are defined by

$$f(x) = \begin{cases} x(2-x) & \text{for } 0 \leq x < 1, \\ 2-x & \text{for } 1 \leq x < 2, \end{cases}$$
$$g(x) = e^x, \quad x \in \mathbb{R}, x \geq 0.$$

(a) Sketch the graph of $y = f(x)$ for $0 \leq x < 2$. [3]

(b) Explain why the composite function gf exists and give a definition (including the domain) of gf and find its range. [4]

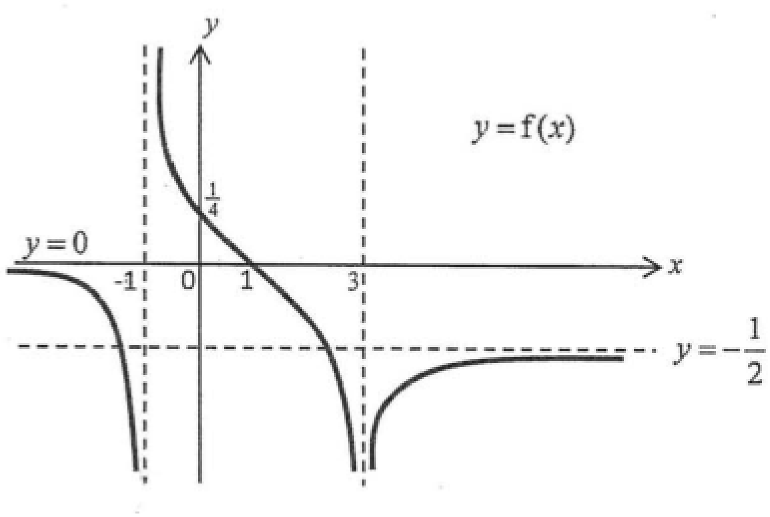
(c) Given further that $f(x-2) = f(x)$ for all real values of x, find the exact value of $f\left(\frac{101}{4}\right)$. [2]

6. **[EJC 17 Promos]**
If $y = e^{\sin^{-1} x}$, show that

$$(1-x^2)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} - y = 0.$$

[4]

7. **[AJC 18 MYE (modified)]**
The diagram shows that graph of $y = f(x)$.



On separate diagrams, indicating clearly the equations of any asymptotes, the coordinates of turning points, and the coordinates of any points of intersection with the x- and y-axes, sketch the graphs of

(a) $y = 1 - 2f(x)$, [3]

(b) $y = f'(x)$, [3]

(c) $y = \frac{1}{f(x)}$, [3]

Answers

1. (b) $-2t^2(2t-3)$.

   (c) $(1,4)$.

2. $g(x) = -\frac{1}{x}$.

3. 1. Translate in the positive y-direction by 3 units.
   2. Scale parallel to the y-axis by a factor of 3.

4. (b) $f^{-1}(x) = (x+\frac{1}{2})^2 - 1, D_{f^{-1}} = (-1,\infty)$.

   (d) $x = \frac{\sqrt{3}}{2}$.

   (e) $\{x \in \mathbb{R}, -0.5 < x \leq \frac{\sqrt{3}}{2}\}$.

5. (b) $R_f = [0,1] \subseteq [0,\infty) = D_g$.

   $$gf(x) = \begin{cases} e^{x(2-x)} & \text{for } 0 \leq x < 1, \\ e^{2-x} & \text{for } 1 \leq x < 2. \end{cases}$$

   $R_{gf} = [1,e]$.

   (c) $\frac{3}{4}$