1. **[ACJC 11 Prelims (modified)]**
The functions f and g are defined by

$$f : x \mapsto \sqrt{5 - ax^2}, \qquad x \in \mathbb{R}, 0 \leq x \leq \sqrt{\frac{5}{a}},$$
$$g : x \mapsto 1 + e^{-x}, \qquad x \in \mathbb{R}, x \geq 0.$$

(a) Show that f^{-1} exists and define f^{-1} in similar form. [4]

(b) It is given that $f^2(x) = x$ for all $x \in \mathbb{R}, 0 \leq x \leq \sqrt{\frac{5}{a}}$. Using your answer to (a) or otherwise, show that $a = 1$. [2]

(c) Show that fg exists and find its corresponding range. [3]

2. **[DHS 11 Prelims (modified)]**
The functions g and h are defined as follows:

$$g : x \mapsto \ln x, \qquad x \geq 1,$$
$$h : x \mapsto x^2 - 2x + 2, \qquad x > 0.$$

(a) Explain why gh exists and define gh in similar form. Also find the range of gh. [4]

(b) Sketch, on the same diagram, the graphs of g, g^{-1} and $g^{-1}g$. [3]

(c) State the range of values of x satisfying the equation $g^{-1}g(x) = gg^{-1}(x)$. [1]

3. **[SRJC 14 MYE]**
A graph with equation $y = f(x)$ undergoes in succession, the following transformations:

A : A translation of 3 units in the direction of the negative x-axis
B : A reflection about the y-axis
C : A scaling parallel to the x-axis by a factor of 2

The equation of the resulting curve is given by $y = \dfrac{x-4}{2x-18}$.

Find the equation of $y = f(x)$. [3]

4. **[RI 15 Prelims (modified)]**
If $f(x) = \sin 2x$, describe a sequence of transformations which would transform the graph of $y = f(x)$ to the graph of $y = f''(x)$. [2]

5. **[TJC 11 Prelims]**
A curve has parametric equations given by

$$x = 2\theta - \sin 2\theta \quad \text{and} \quad y = 2 - \cos 2\theta$$

for $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}$.

(a) Sketch the curve, labelling the coordinates of the end points. [3]

(b) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \cot\theta$. [3]

(c) Find the coordinates of the point where the tangent is parallel to the y-axis. [3]

6. [**NJC 18 Prelims (modified)**]
It is given that $f(x) = \dfrac{x^2+x+a}{x+b}$, where a and b are constants.
   (a) Given that the graph of $y = f(x)$ has a vertical asymptote $x = 3$ and passes through $(0, 2)$, find the values of a and b. [2]
   (b) On separate diagrams, sketch the graphs of
      i. $y = f(x)$, [3]
      ii. $y = f(|x|)$. [2]

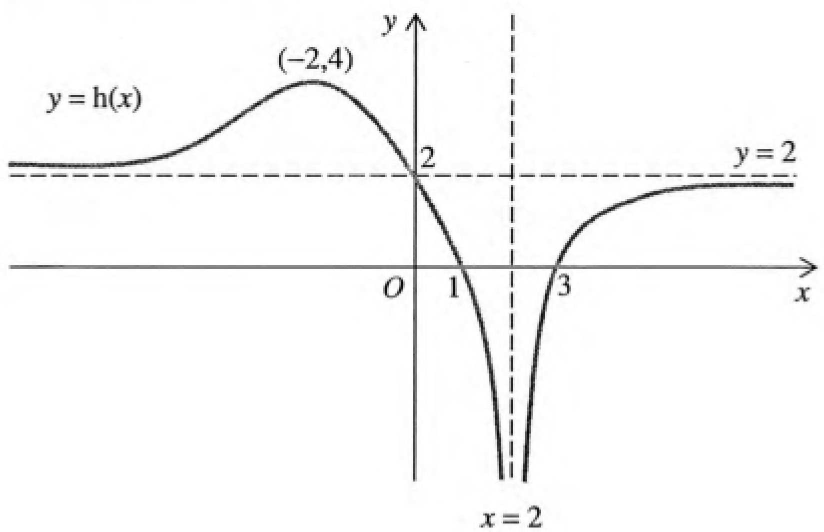
   (Your graphs should label clearly any axial intercepts and linear asymptotes.)

7. [**DHS 11 Prelims (modified)**]
Given that $\ln y = \tan^{-1} x$, show that

$$(1+x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}x}.$$

[3]

8. [**RI 15 Prelims (modified)**]
The diagram shows that graph of $y = h(x)$.



On separate diagrams, indicating clearly the equations of any asymptotes, the coordinates of turning points, and the coordinates of any points of intersection with the x- and y-axes, sketch the graphs of
   (a) $y = h(3x-2)$, [3]
   (b) $y = h'(x)$, [3]
   (c) $y = \dfrac{1}{h(x)}$, [3]

Answers

1. (a) Since any horizontal line $y = k, k \in \mathbb{R}$ cuts the graph of $y = f(x)$ at most once, f is a one-one function. Hence f^{-1} exists.
   $f^{-1} : x \mapsto \sqrt{\frac{5-x^2}{a}}, 0 \leq x \leq \sqrt{5}$.

   (c) fg exists since $R_g = (1, 2] \subseteq [0, \sqrt{5}] = D_f$.
   $R_{fg} = [1, 2)$.

2. (a) gh exists since $R_h = [1, \infty) \subseteq [1, \infty) = D_g$.
   $gh : x \mapsto \ln(x^2 - 2x + 2), x > 0$.
   $R_{gh} = [0, \infty)$.

   (c) $x \geq 1$.

3. $y = \frac{x-1}{2x+3}$.

4. 1. Scale by a factor of 4 parallel to the y-axis.
   2. Reflect about the x-axis.

5. $(2\pi, 1)$.

6. $a = -6, b = -3$.