1. **[ACJC 18 CA2]**
   One root of the equation $w^3 - 9w^2 + 20w - 37 - b^2 = 0$, where b is real and positive, is $w = 1 + bi$.

   By expressing $w^3 - 9w^2 + 20w - 37 - b^2$ as a product of a linear factor and a quadratic factor, find the value of b and the other roots. [4]

2. **[AJC 18 MYE]**
   The complex numbers p and q satisfy the following simultaneous equations

   $$p + (2 + i)q = 5 + i \quad \text{and} \quad ip - 2q = 4.$$

   Find p and q in the form $x + yi$, where x and y are real. [4]

3. **[SAJC 18 MYE]**
   The complex number z is given by $z = re^{i\theta}$, where $r > 0$ and $0 \leq \theta \leq \pi$, and the complex number $w = \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) z$.

   (a) Find $|w|$ in terms of r and $\arg w$ in terms of θ, giving both your answers in exact form. [2]

   (b) Given that $\frac{z^6}{w^*}$ is purely imaginary, find the three smallest values of θ in exact form, leaving your answers in terms of π. [5]

Answers

1. $b = \sqrt{5}$, $w = 7, 1 \pm i\sqrt{5}$.
2. $p = 6 - 6i, q = 1 + 3i$.
3. (a) $|w| = r, \arg w = -\frac{\pi}{6} + \theta$.
   (b) $\frac{2\pi}{21}, \frac{5\pi}{21}, \frac{8\pi}{21}$.