

## Questions

1. [2015/NYJC/II/6]

Chickens sold by a supermarket are graded 'small', 'medium' or 'large'. The weights of the chicken have a normal distribution. Chickens with weight less than 1 kg are graded as 'small', chickens with weight greater than 1.8 kg are graded as 'large' and the rest are graded as 'medium'. Given that 20% of chicken are small and 15% are large, find the mean and standard deviation of the distribution. [4]

2. [2015/TPJC/II/7]

A hair salon offers two hair care treatment packages, namely the Herbal Treatment and the Wonder Treatment. The length of time taken for one Herbal Treatment session has a normal distribution with mean  $\mu$  minutes and standard deviation 5 minutes. The length of time taken for one Wonder Treatment session has an independent normal distribution with mean 50 minutes and standard deviation 7 minutes.

- (i) Alice went for a session of Herbal Treatment while Betty went for a session of Wonder Treatment. The probability that Alice has to wait for Betty for at least five minutes after her own session is 0.05. Assume that they started their sessions at the same time, show that the value of  $\mu$  is 59.1, correct to 3 significant figures. [3]
- (ii) Using the value of  $\mu$  shown in part (i), find the probability that the total time taken for 3 sessions of the Herbal Treatment exceeds four times the time taken for 1 session of Wonder Treatment. State clearly the mean and variance of any normal distribution you use in your calculation. [3]

3. The masses, in g, of Grade A and Grade B mangoes sold by a wholesaler are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard deviation
Grade A	450	18
Grade B	375	22

Grade A mangoes are sold at \$6 per kg and Grade B mangoes are sold at \$4 per kg.

- (i) Find the probability that a randomly chosen Grade B mango has a mass between 360 g and 390 g. [1]
- (ii) Find the probability that the total selling price of a randomly chosen Grade A mango and a randomly chosen Grade B mango is more than \$4.25. [3]

A box of Grade A mangoes contains 4 mangoes. A box of Grade B mangoes contains 5 mangoes. Assume that the boxes are of negligible weight.

- (iii) Find the probability that a randomly chosen box of Grade B mangoes weighs more than a randomly chosen box of Grade A mangoes. [3]

## Normal distribution and hypothesis testing

4.

It has been suggested that the optimal pH value for shampoo should be 5.5, to match the pH level of healthy scalp. Any pH value that is too low or too high may have undesirable effects on the user's hair and scalp. A shampoo manufacturer wants to investigate if the pH level of his shampoo is at the optimal value, by carrying out a hypothesis test at the 10% significance level. He measures the pH value,  $x$ , of  $n$  randomly chosen bottles of shampoo, where  $n$  is large.

(a) In the case where  $n = 30$ , it is found that  $\sum x = 178.2$  and  $\sum x^2 = 1238.622$ .

- (i) Find unbiased estimates of the population mean and variance, and carry out the test at the 10% significance level. [6]

5.

The power consumption of a randomly chosen Effixion laptop has a normal distribution. The salesman at Elf Superstore claims that the average power consumption of an Effixion laptop is 100 watts. The power consumption,  $w$  watts, is measured for a random sample of 50 Effixion laptops. The results are summarised as follows.

$$\sum (w-100) = 26 \quad \sum (w-100)^2 = 273$$

Test whether this data provides evidence at the 3% level of significance, that the salesman has made an understatement. [6]

6.

The Kola Company receives a number of complaints that the volume of cola in their cans are less than the stated amount of 500 ml. A statistician decides to sample 50 cola cans to investigate the complaints. He measures the volume of cola,  $x$  ml, in each can and summarised the results as follows:

$$\sum x = 24730, \quad \sum x^2 = 12242631.$$

- (i) Find unbiased estimates of the population mean and variance correct to 2 decimal places and carry out the test at the 1% level of significance. [6]

## Answers

1.  $\mu = 1.36$  and  $\sigma = 0.426$
2. **(ii)** 0.219
3. (i) 0.505  
(ii) 0.360  
(iii) 0.891
4. (ai) p-value = 0.334. Do not reject  $H_0$ . Insufficient evidence.
5. P-value = 0.0550. Do not reject  $H_0$ . Insufficient evidence.
6. P-value = 0.00572. Reject  $H_0$ . Sufficient evidence.