1. **[CJC 19 MYE]**
The functions f and g are defined by

$$f : x \mapsto x^2 + 2, \qquad x \in \mathbb{R},$$
$$g : x \mapsto -2x + 5, \qquad x \in \mathbb{R}.$$

State a sequence of transformations which transform the graph of $y = f(x)$ to the graph of $y = fg(x)$. [3]

2. **[JPJC 19 MYE]**
The curve $y = g(x)$ undergoes the transformations A, B and C in succession:

- A: a translation of 1 unit in the positive x-direction,
- B: a scaling parallel to the x-axis with scale factor $\frac{1}{2}$, and
- C: a translation of 3 units in the positive y-direction.

Find an expression for $g(x)$ if the equation of the resulting curve is $y = 3 - \dfrac{1}{2x-1}$. [3]

3. **[TMJC 19 MYE]**
A curve undergoes transformations in the following sequence:

- Stretch by factor 2 parallel to the y-axis.
- Translation of 1 unit in the positive x-direction.
- Reflection in the y-axis.

The resulting curve has equation $y = 3(x+1)^2 - 10 - \dfrac{2}{x+1}$.

Determine the equation of the original curve, showing your steps clearly. [3]

4. **[YJC 19 MYE]**
A curve has equation $y = \dfrac{2x-7}{x-3}$.

(a) Express the equation of C in the form $y = A + \frac{B}{x-3}$, where A and B are constants to be found. [1]

(b) Sketch the graph of C, giving the equations of any asymptotes and the coordinates of any points of intersection with x- and y-axes. [2]

(c) Describe a sequence of transformations which transforms the graph of C on to the graph of $y = -\dfrac{1}{x}$. [2]

5. **[EJC 19 MYE]**
The graph of $y = f(x)$ undergoes the following sequence of transformations

- A: Stretch with scale factor $\frac{2}{3}$ parallel to the x-axis
- B: Reflect about the x-axis
- C: Translate 4 units in the negative x-direction

Given that the equation of the resulting curve is $y = -\dfrac{1}{3x+13}$, find the equation of the curve before the 3 transformations were effected. [4]

6. **[EJC 19 MYE]**
The curve $y = f(x)$ has asymptotes $x = -1$ and $y = 4$. State the equations of asymptotes of the curve $y = 2f(-x) - 3$. [3]

7. **[NJC 19 MYE]**
Describe a pair of transformations which transforms the curve with equation $\frac{x^2}{6^2} + \frac{(y+3)^2}{2^2} = 1$ on to the circle with centre at the origin and radius 6 units. [3]

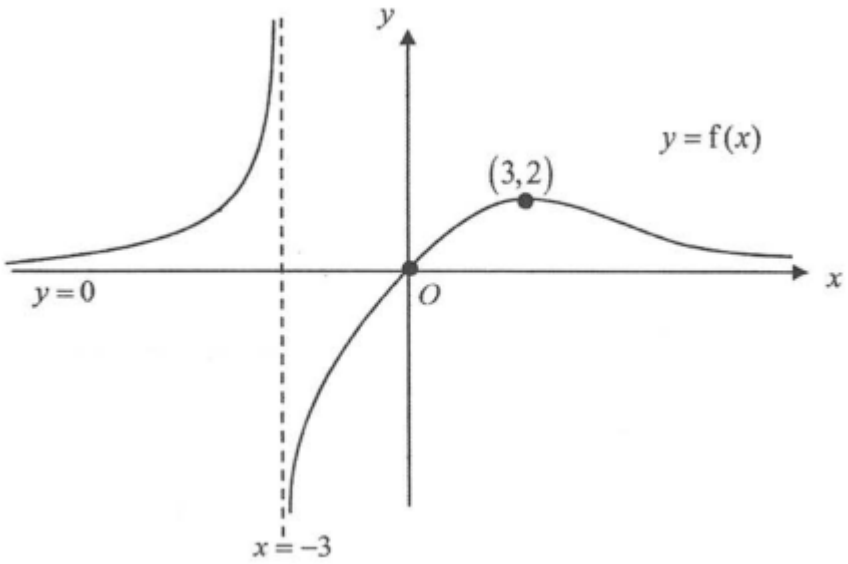
8. **[SAJC 19 MYE]**
(a) Find the exact value of a such that $\mathrm{e}^{-ax} = 3^{-x}$. hence show that

$$3^{3-x}\left(x\ln 3 + \frac{1}{3}\right) = 9\mathrm{e}^{-(\ln 3)x}(3x\ln 3 + 1).$$

[2]

(b) Describe a series of transformations that maps the graph of $y = \mathrm{e}^{-x}(3x+1)$ onto the graph of $y = 3^{3-x}\left(x\ln 3 + \frac{1}{3}\right) + 1$. [3]

9. **[CJC 19 MYE]**
The diagram shows that graph of $y = f(x)$.



On separate diagrams, indicating clearly the equations of any asymptotes, the coordinates of turning points, and the coordinates of any points of intersection with the x- and y-axes, sketch the graphs of

(a) $y = f(\frac{1}{2}x) - 1$, [4]

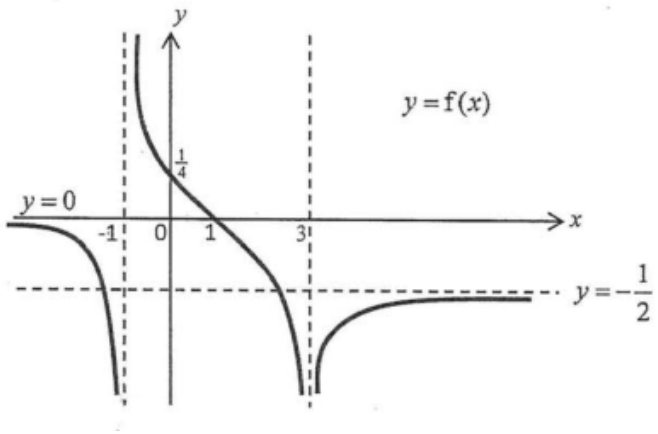
(b) $y = f(|x|)$, [3]

(c) $y = \dfrac{1}{f(x)}$, [4]

(d) $y = f'(x)$. [3]

10. **[AJC 18 MYE]**

The diagram shows that graph of $y = f(x)$.



On separate diagrams, indicating clearly the equations of any asymptotes, the coordinates of turning points, and the coordinates of any points of intersection with the x- and y-axes, sketch the graphs of

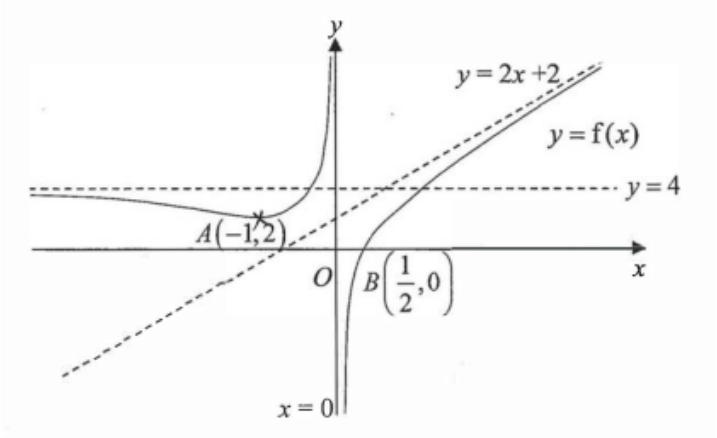
(a) $y = f(1 - 2x)$, [3]

(b) $y = \dfrac{1}{f(x)}$, [3]

(c) $y = f'(x)$. [3]

11. **[RJC 18 MYE]**

The diagram shows that graph of $y = f(x)$.



On separate diagrams, indicating clearly the equations of any asymptotes, the coordinates of turning points, and the coordinates of any points of intersection with the x- and y-axes, sketch the graphs of

(a) $y = 3 - 2f(x)$, [3]

(b) $y = \dfrac{1}{f(x)}$, [3]

(c) $y = f'(x)$. [3]

Answers

1. 1. Translate by 5 units in the negative x-direction.
   2. Reflect about the y-axis.
   3. Scale by a factor of $\frac{1}{2}$ parallel to the x-axis.
   (Other answers are possible too: check with me)

2. $g(x) = -\frac{1}{x}$.

3. $y = \frac{3}{2}x^2 - 5x + \frac{1}{x}$.

4. (a) $y = 2 + \frac{-1}{x-3}$.

   (c) 1. Translation of 2 units in the negative y-direction.
   2. Translation of 3 units in the negative x-direction.
   (For this question, the order can be swapped.)

5. $y = \frac{1}{2x+1}$.

6. $y = 5, x = 1$.

7. 1. Translate in the positive y-direction by 3 units.
   2. Scale parallel to the y-axis by a factor of 3.

8. (a) $a = \ln 3$.

   (b) 1. Scale by factor $\frac{1}{\ln 3}$ parallel to the x-axis.
   2. Scale by factor 9 parallel to the y-axis.
   3. Translation of 1 unit in the positive direction of the y-axis.