

1. [SRJC 18 MYE]

The lines l_1 and the plane p_1 have equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} + t \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}, t \in \mathbb{R} \quad \text{and} \quad p_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 2.$$

It is given that the point A has the position vector $3\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$.

(a) Find the acute angle between l_1 and p_1 [2]

(b) Find the coordinates of the foot of the perpendicular from point A to p_1 . [4]

The plane p_2 has equation $-2x + z = 6$. Given that the plane p_1 meets the plane p_2 at the line l_2 ,

(c) find the equation of l_2 . [1]

2. [TPJC 18 MYE]

Referred to the origin O , points A, B and C have position vectors \mathbf{a}, \mathbf{b} and $\frac{5}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ respectively. The point P on AB is such that $AP : PB = \lambda : 1 - \lambda$ and the point P on OC is such that $OP : PC = \mu : 1 - \mu$.

(a) Express \overrightarrow{OP} in terms of λ, \mathbf{a} and \mathbf{b} . [1]

(b) By expressing \overrightarrow{OP} in terms of μ, \mathbf{a} and \mathbf{b} , find the values of λ and μ . Hence show that P is the midpoint of OC . [4]

(c) It is given that the position vectors of the points A and B are $2\mathbf{j} - \mathbf{k}$ and $-6\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}$ respectively. The point Q lies on OA such that PQ is perpendicular to OA . Find the position vector of the point Q . [6]

3. [JJC 18 MYE (modified)]

A mine contains several underground tunnels beneath a hillside. All the tunnels are straight and the width of the tunnels may be neglected. A coordinate system is chosen with the z -axis pointing vertically upwards. The hillside contains points $A(10, -65, 15)$ and $B(-80, 95, 35)$.

The tunnel T_A starts at A and goes in the direction of the vector $15\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$.

(a) Write down a vector equation of T_A and find the shortest distance from B to T_A . [4]

(b) Another tunnel T_B starts at B and passes through the point $D(13, 133, p)$. T_A and T_B meets at the point Q . Find the coordinates of Q . [4]

4. [CJC H1 18 Prelims (modified)]

A government introduces the carbon tax to encourage companies to reduce carbon dioxide emission and lessen the effect of global warming. A researcher takes a sample of 60 companies and the mean amount of annual carbon dioxide emission recorded is k tonnes. It is given that the population standard deviation is 2000 tonnes. A test at the 1% significance level indicates that the population mean amount of annual carbon dioxide emission does not differ from 25000 tonnes.

Find the set of values of k , giving your answer correct to the nearest integer. [5]

Answers

- (a) 6.3° .

(b) $(1, -4, 4)$.

(c) $\mathbf{r} = \begin{pmatrix} -3 \\ 8 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \mu \in \mathbb{R}$.
- (a) $(1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$.

(b) $\lambda = \frac{1}{6}, \mu = \frac{1}{2}$.

(c) $1.2\mathbf{j} - 0.6\mathbf{k}$.
- (a) 180 units.

(b) $Q(385, 285, -35)$.
- $\{k \in \mathbb{Z} : 24335 \leq k \leq 25665\}$.