MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9740/01

Paper 1

October/November 2012

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 A cinema sells tickets at three different prices, depending on the age of the customer. The age categories are under 16 years, between 16 and 65 years, and over 65 years. Three groups of people, A, B and C, go to the cinema on the same day. The numbers in each age category for each group, together with the total cost of the tickets for each group, are given in the following table.

Group	Under 16 years	Between 16 and 65 years	Over 65 years	Total cost
A	9	6	4	\$162.03
B	7	5	3	\$128.36
C	10	4	5	\$158.50

Write down and solve equations to find the cost of a ticket for each of the age categories. [4]

2 **(i)** Find $\displaystyle\int \frac{x^3}{1+x^4}\,\mathrm{d}x$. [2]

(ii) Use the substitution $u = x^2$ to find $\displaystyle\int \frac{x}{1+x^4}\,\mathrm{d}x$. [3]

(iii) Evaluate $\displaystyle\int_0^1 \left(\frac{x}{1+x^4}\right)^2 \mathrm{d}x$, giving the answer correct to 3 decimal places. [1]

3 A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_1 = 2 \quad \text{and} \quad u_{n+1} = \frac{3u_n - 1}{6} \quad \text{for } n \geqslant 1.$$

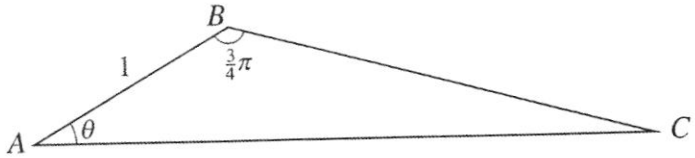
(i) Find the exact values of u_2 and u_3. [2]

(ii) It is given that $u_n \to l$ as $n \to \infty$. Showing your working, find the exact value of l. [2]

(iii) For this value of l, use the method of mathematical induction to prove that

$$u_n = \tfrac{14}{3}\left(\tfrac{1}{2}\right)^n + l.$$ [4]

4



In the triangle ABC, $AB = 1$, angle $BAC = \theta$ radians and angle $ABC = \frac{3}{4}\pi$ radians (see diagram).

(i) Show that $AC = \dfrac{1}{\cos\theta - \sin\theta}$. [4]

(ii) Given that θ is a sufficiently small angle, show that

$$AC \approx 1 + a\theta + b\theta^2,$$

for constants a and b to be determined. [4]

5 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ such that

$$\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \mathbf{b} = \mathbf{i} + 2\mathbf{j}.$$

The point C has position vector $\mathbf{c}$ given by $\mathbf{c} = \lambda\mathbf{a} + \mu\mathbf{b}$, where λ and μ are positive constants.

(i) Given that the area of triangle OAC is $\sqrt{(126)}$, find μ. [4]

(ii) Given instead that $\mu = 4$ and that $OC = 5\sqrt{3}$, find the possible coordinates of C. [4]

6 **Do not use a calculator in answering this question.**

The complex number z is given by $z = 1 + \mathrm{i}c$, where c is a non-zero real number.

(i) Find z^3 in the form $x + \mathrm{i}y$. [2]

(ii) Given that z^3 is real, find the possible values of z. [2]

(iii) For the value of z found in part (ii) for which $c < 0$, find the smallest positive integer n such that $|z^n| > 1000$. State the modulus and argument of z^n when n takes this value. [4]

7 A function f is said to be self-inverse if $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ for all x in the domain of f.

The function g is defined by

$$\mathrm{g} : x \mapsto \frac{x+k}{x-1}, \quad x \in \mathbb{R}, \ x \neq 1,$$

where k is a constant, $k \neq -1$.

(i) Show that g is self-inverse. [2]

(ii) Given that $k > 0$, sketch the curve $y = \mathrm{g}(x)$, stating the equations of any asymptotes and the coordinates of any points where the curve crosses the x- and y-axes. [3]

(iii) State the equation of one line of symmetry of the curve in part (ii), and describe fully a sequence of transformations which would transform the curve $y = \frac{1}{x}$ onto this curve. [4]

8 The curve C has equation

$$x - y = (x + y)^2.$$

It is given that C has only one turning point.

(i) Show that $1 + \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{2x + 2y + 1}$. [4]

(ii) Hence, or otherwise, show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\left(1 + \frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$. [3]

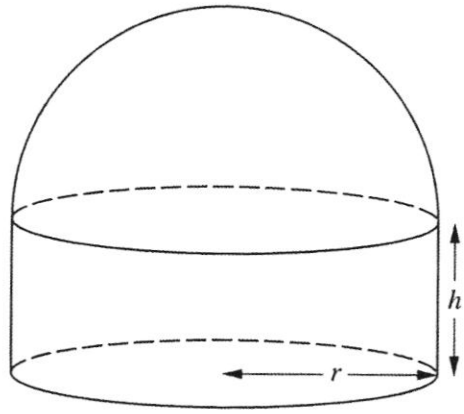
(iii) Hence state, with a reason, whether the turning point is a maximum or a minimum. [2]

9 **(i)** Find a vector equation of the line through the points A and B with position vectors $7\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$ and $-\mathbf{i} - 8\mathbf{j} + \mathbf{k}$ respectively. [3]

(ii) The perpendicular to this line from the point C with position vector $\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}$ meets the line at the point N. Find the position vector of N and the ratio $AN : NB$. [5]

(iii) Find a cartesian equation of the line which is a reflection of the line AC in the line AB. [4]

10 [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\frac{4}{3}\pi r^3$.]



A model of a concert hall is made up of three parts.

- The roof is modelled by the curved surface of a hemisphere of radius r cm.
- The walls are modelled by the curved surface of a cylinder of radius r cm and height h cm.
- The floor is modelled by a circular disc of radius r cm.

The three parts are joined together as shown in the diagram. The model is made of material of negligible thickness.

(i) It is given that the volume of the model is a fixed value $k\,\text{cm}^3$, and the external surface area is a minimum. Use differentiation to find the values of r and h in terms of k. Simplify your answers. [7]

(ii) It is given instead that the volume of the model is $200\,\text{cm}^3$ and its external surface area is $180\,\text{cm}^2$. Show that there are two possible values of r. Given also that $r < h$, find the value of r and the value of h. [5]

11 A curve C has parametric equations

$$x = \theta - \sin\theta, \quad y = 1 - \cos\theta,$$

where $0 \leqslant \theta \leqslant 2\pi$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\frac{1}{2}\theta$ and find the gradient of C at the point where $\theta = \pi$. What can be said about the tangents to C as $\theta \to 0$ and $\theta \to 2\pi$? [5]

(ii) Sketch C, showing clearly the features of the curve at the points where $\theta = 0$, π and 2π. [3]

(iii) Without using a calculator, find the exact area of the region bounded by C and the x-axis. [5]

(iv) A point P on C has parameter p, where $0 < p < \frac{1}{2}\pi$. Show that the normal to C at P crosses the x-axis at the point with coordinates $(p, 0)$. [3]

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MATHEMATICS

9740/02

Paper 2

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Section A: Pure Mathematics [40 marks]

1 **(a)** Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 16 - 9x^2,$$

giving your answer in the form $y = \mathrm{f}(x)$. [3]

(b) Given that u and t are related by

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 16 - 9u^2,$$

and that $u = 1$ when $t = 0$, find t in terms of u, simplifying your answer. [5]

2 The complex number z satisfies the equation $|z - (7 - 3\mathrm{i})| = 4$.

(i) Sketch an Argand diagram to illustrate this equation. [2]

(ii) Given that $|z|$ is as small as possible,

(a) find the exact value of $|z|$, [2]

(b) hence find an exact expression for z, in the form $x + \mathrm{i}y$. [2]

(iii) It is given instead that $-\pi < \arg z \leqslant \pi$ and that $|\arg z|$ is as large as possible. Find the value of $\arg z$ in radians, correct to 4 significant figures. [3]

3 It is given that $\mathrm{f}(x) = x^3 + x^2 - 2x - 4$.

(i) Sketch the graph of $y = \mathrm{f}(x)$. [1]

(ii) Find the integer solution of the equation $\mathrm{f}(x) = 4$, and prove algebraically that there are no other real solutions. [3]

(iii) State the integer solution of the equation $(x + 3)^3 + (x + 3)^2 - 2(x + 3) - 4 = 4$. [1]

(iv) Sketch the graph of $y = |\mathrm{f}(x)|$. [1]

(v) Write down two different cubic equations which between them give the roots of the equation $|\mathrm{f}(x)| = 4$. Hence find all the roots of this equation. [4]

4 On 1 January 2001 Mrs A put \$100 into a bank account, and on the first day of each subsequent month she put in \$10 more than in the previous month. Thus on 1 February she put \$110 into the account and on 1 March she put \$120 into the account, and so on. The account pays no interest.

(i) On what date did the value of Mrs A's account first become greater than \$5000? [5]

On 1 January 2001 Mr B put \$100 into a savings account, and on the first day of each subsequent month he put another \$100 into the account. The interest rate was 0.5% per month, so that on the last day of each month the amount in the account on that day was increased by 0.5%.

(ii) Use the formula for the sum of a geometric progression to find an expression for the value of Mr B's account on the last day of the nth month (where January 2001 was the 1st month, February 2001 was the 2nd month, and so on). Hence find in which month the value of Mr B's account first became greater than \$5000. [5]

(iii) Mr B wanted the value of his account to be \$5000 on 2 December 2003. What interest rate per month, applied from January 2001, would achieve this? [3]

Section B: Statistics [60 marks]

5 The probability that a hospital patient has a particular disease is 0.001. A test for the disease has probability p of giving a positive result when the patient has the disease, and equal probability p of giving a negative result when the patient does not have the disease. A patient is given the test.

(i) Given that $p = 0.995$, find the probability that

(a) the result of the test is positive, [2]

(b) the patient has the disease given that the result of the test is positive. [2]

(ii) It is given instead that there is a probability of 0.75 that the patient has the disease given that the result of the test is positive. Find the value of p, giving your answer correct to 6 decimal places. [3]

6 On a remote island a zoologist measures the tail lengths of a random sample of 20 squirrels. In a species of squirrel known to her, the tail lengths have mean 14.0 cm. She carries out a test, at the 5% significance level, of whether squirrels on the island have the same mean tail length as the species known to her. She assumes that the tail lengths of squirrels on the island are normally distributed with standard deviation 3.8 cm.

(i) State appropriate hypotheses for the test. [1]

The sample mean tail length is denoted by $\bar{x}$ cm.

(ii) Use an algebraic method to calculate the set of values of $\bar{x}$ for which the null hypothesis would not be rejected. (Answers obtained by trial and improvement from a calculator will obtain no marks.) [3]

(iii) State the conclusion of the test in the case where $\bar{x} = 15.8$. [2]

7 A group of fifteen people consists of one pair of sisters, one set of three brothers and ten other people. The fifteen people are arranged randomly in a line.

(i) Find the probability that the sisters are next to each other. [2]

(ii) Find the probability that the brothers are *not* all next to one another. [2]

(iii) Find the probability that the sisters are next to each other and the brothers *are* all next to one another. [2]

(iv) Find the probability that *either* the sisters are next to each other *or* the brothers are all next to one another *or* both. [2]

Instead the fifteen people are arranged in a circle.

(v) Find the probability that the sisters are next to each other. [1]

8 Amy is revising for a mathematics examination and takes a different practice paper each week. Her marks, $y\%$ in week x, are as follows.

Week x	1	2	3	4	5	6
Percentage mark y	38	63	67	75	71	82

(i) Draw a scatter diagram showing these marks. [1]

(ii) Suggest a possible reason why one of the marks does not seem to follow the trend. [1]

(iii) It is desired to predict Amy's marks on future papers. Explain why, in this context, neither a linear nor a quadratic model is likely to be appropriate. [2]

It is decided to fit a model of the form $\ln(L - y) = a + bx$, where L is a suitable constant. The product moment correlation coefficient between x and $\ln(L - y)$ is denoted by r. The following table gives values of r for some possible values of L.

L	91	92	93
r		−0.929944	−0.929918

(iv) Calculate the value of r for $L = 91$, giving your answer correct to 6 decimal places. [1]

(v) Use the table and your answer to part **(iv)** to suggest with a reason which of 91, 92 or 93 is the most appropriate value for L. [1]

(vi) Using this value for L, calculate the values of a and b, and use them to predict the week in which Amy will obtain her first mark of at least 90%. [4]

(vii) Give an interpretation, in context, of the value of L. [1]

9 In an opinion poll before an election, a sample of 30 voters is obtained.

(i) The number of voters in the sample who support the Alliance Party is denoted by A. State, in context, what must be assumed for A to be well modelled by a binomial distribution. [2]

Assume now that A has the distribution $\mathrm{B}(30, p)$.

(ii) Given that $p = 0.15$, find $\mathrm{P}(A = 3 \text{ or } 4)$. [2]

(iii) Given instead that $p = 0.55$, explain whether it is possible to approximate the distribution of A with

(a) a normal distribution,

(b) a Poisson distribution. [3]

(iv) For an unknown value of p it is given that $\mathrm{P}(A = 15) = 0.06864$ correct to 5 decimal places. Show that p satisfies an equation of the form $p(1-p) = k$, where k is a constant to be determined. Hence find the value of p to a suitable degree of accuracy, given that $p < 0.5$. [5]

10 Gold coins are found scattered throughout an archaeological site.

(i) State two conditions needed for the number of gold coins found in a randomly chosen region of area 1 square metre to be well modelled by a Poisson distribution. [2]

Assume that the number of gold coins in 1 square metre has the distribution $\mathrm{Po}(0.8)$.

(ii) Find the probability that in 1 square metre there are at least 3 gold coins. [1]

(iii) It is given that the probability that 1 gold coin is found in x square metres is 0.2. Write down an equation for x, and solve it numerically given that $x < 1$. [2]

(iv) Use a suitable approximation to find the probability that in 100 square metres there are at least 90 gold coins. State the parameter(s) of the distribution that you use. [3]

Pottery shards are also found scattered throughout the site. The number of pottery shards in 1 square metre is an independent random variable with the distribution $\mathrm{Po}(3)$. Use suitable approximations, whose parameters should be stated, to find

(v) the probability that in 50 square metres the total number of gold coins and pottery shards is at least 200, [4]

(vi) the probability that in 50 square metres there are at least 3 times as many pottery shards as gold coins. [3]