

MINISTRY OF EDUCATION, SINGAPORE in collaboration with UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE General Certificate of Education Advanced Level Higher 2

MATHEMATICS

Paper 1

9740/01

October/November 2016 3 hours

Additional Materials: Answer Paper Graph paper List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question.

Express $\frac{4x^2 + 4x - 14}{x - 4} - (x + 3)$ as a single simplified fraction. [2] 1

Hence, without using a calculator, solve the inequality

$$\frac{4x^2 + 4x - 14}{x - 4} < x + 3.$$
 [3]

- (i) Use your calculator to find the gradient of the curve $y = 2^{\cos x}$ at the points where x = 0 and $x = \frac{1}{2}\pi$. 2 [2]
 - (ii) Find the equations of the tangents to this curve at the points where x = 0 and $x = \frac{1}{2}\pi$ and find the coordinates of the point where these tangents meet. [3]
- The curve $y = x^4$ is transformed onto the curve with equation y = f(x). The turning point on $y = x^4$ 3 corresponds to the point with coordinates (a, b) on y = f(x). The curve y = f(x) also passes through the point with coordinates (0, c). Given that f(x) has the form $k(x-l)^4 + m$ and that a, b and c are positive constants with c > b, express k, l and m in terms of a, b and c. [2]

By sketching the curve y = f(x), or otherwise, sketch the curve $y = \frac{1}{f(x)}$. State, in terms of a, b and c, the coordinates of any points where $y = \frac{1}{f(x)}$ crosses the axes and of any turning points. [4]

- 4 An arithmetic series has first term a and common difference d, where a and d are non-zero. A geometric series has first term b and common ratio r, where b and r are non-zero. It is given that the 4th, 9th and 12th terms of the arithmetic series are equal to the 5th, 8th and 15th terms of the geometric series respectively.
 - (i) Show that r satisfies the equation $5r^{10} 8r^3 + 3 = 0$. Given that |r| < 1, solve this equation, giving your answer correct to 2 decimal places. [4]
 - (ii) Using this value of r, find, in terms of b and n, the sum of the terms of the geometric series after, but not including, the nth term, simplifying your answer. [3]
- 5 The vectors **u** and **v** are given by $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = a\mathbf{i} + b\mathbf{k}$, where a and b are constants.
 - (i) Find $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} \mathbf{v})$ in terms of a and b.
 - (ii) Given that the i- and k-components of the answer to part (i) are equal, express $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} \mathbf{v})$ in terms of a only. Hence find, in an exact form, the possible values of a for which $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} - \mathbf{v})$ is a unit vector. [4]
 - (iii) Given instead that $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} \mathbf{v}) = 0$, find the numerical value of $|\mathbf{v}|$.

[2]

[2]

 $\mathbf{6} \quad \boldsymbol{\textcircled{G}}(\mathbf{i})$ Prove by the method of mathematical induction that

$$\sum_{r=1}^{n} r(r^2 + 1) = \frac{1}{4}n(n+1)(n^2 + n + 2).$$
[5]

[2]

(ii) A sequence u_0, u_1, u_2, \dots is given by

$$u_0 = 2$$
 and $u_n = u_{n-1} + n^3 + n$ for $n \ge 1$.

Find u_1, u_2 and u_3 .

(iii) By considering
$$\sum_{r=1}^{n} (u_r - u_{r-1})$$
, find a formula for u_n in terms of n . [3]

7 Do not use a calculator in answering this question.

- (a) Verify that -1 + 5i is a root of the equation $w^2 + (-1 8i)w + (-17 + 7i) = 0$. Hence, or otherwise, find the second root of the equation in cartesian form, p + iq, showing your working. [5]
- (b) The equation $z^3 5z^2 + 16z + k = 0$, where k is a real constant, has a root z = 1 + ai, where a is a positive real constant. Find the values of a and k, showing your working. [5]
- 8 It is given that y = f(x), where f(x) = tan(ax + b) for constants a and b.
 - (i) Show that $f'(x) = a + ay^2$. Use this result to find f''(x) and f'''(x) in terms of a and y. [5]
 - (ii) In the case where $b = \frac{1}{4}\pi$, use your results from part (i) to find the Maclaurin series for f(x) in terms of *a*, up to and including the term in x^3 . [3]
 - (iii) Find the first two non-zero terms in the Maclaurin series for $\tan 2x$. [3]

[Questions 9, 10 & 11 are printed on the next page.]

[6]

[3]

[3]

[3]

[2]

4

- 9 A stone is held on the surface of a pond and released. The stone falls vertically through the water and the distance, x metres, that the stone has fallen in time t seconds is measured. It is given that x = 0 and $\frac{dx}{dt} = 0$ when t = 0.
 - (i) The motion of the stone is modelled by the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} = 10.$$

- (a) By substituting $y = \frac{dx}{dt}$, show that the differential equation can be written as $\frac{dy}{dt} = 10 2y$.
- (b) Find y in terms of t and hence find x in terms of t.
- (ii) A second model for the motion of the stone is suggested, given by the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = 10 - 5\sin\frac{1}{2}t.$$

Find x in terms of t for this model.

- (iii) The pond is 5 metres deep. For each of these models, find the time the stone takes to reach the bottom of the pond, giving your answers correct to 2 decimal places. [2]
- 10 (a) The function f is given by $f: x \mapsto 1 + \sqrt{x}$, for $x \in \mathbb{R}$, $x \ge 0$.
 - (i) Find $f^{-1}(x)$ and state the domain of f^{-1} .
 - (ii) Show that if ff(x) = x then $x^3 4x^2 + 4x 1 = 0$. Hence find the value of x for which ff(x) = x. Explain why this value of x satisfies the equation $f(x) = f^{-1}(x)$. [5]
 - (b) The function g, with domain the set of non-negative integers, is given by

 $g(n) = \begin{cases} 1 & \text{for } n = 0, \\ 2 + g(\frac{1}{2}n) & \text{for } n \text{ even}, \\ 1 + g(n-1) & \text{for } n \text{ odd}. \end{cases}$

- (i) Find g(4), g(7) and g(12).
 - (ii) Does g have an inverse? Justify your answer.
- 11 The plane *p* has equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} a \\ 4 \\ -2 \end{pmatrix}$, and the line *l* has equation $\mathbf{r} = \begin{pmatrix} a-1 \\ a \\ a+1 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$, where *a* is a constant and λ , μ and *t* are parameters.
 - (i) In the case where a = 0,
 - (a) show that *l* is perpendicular to *p* and find the values of λ , μ and *t* which give the coordinates of the point at which *l* and *p* intersect, [5]
 - (b) find the cartesian equations of the planes such that the perpendicular distance from each plane to p is 12. [5]
 - (ii) Find the value of a such that l and p do not meet in a unique point. [3]



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MATHEMATICS

Paper 2

9740/02

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Section A: Pure Mathematics [40 marks]



Water is poured at a rate of 0.1 m^3 per minute into a container in the form of an open cone. The semi-vertical angle of the cone is α , where $\tan \alpha = 0.5$. At time *t* minutes after the start, the radius of the water surface is *r* m (see diagram). Find the rate of increase of the depth of water when the volume of water in the container is 3 m^3 . [7]

[The volume of a cone of base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.]

- 2 (a) (i) Find $\int x^2 \cos nx \, dx$, where *n* is a positive integer. [3] (ii) Hence find $\int_{\pi}^{2\pi} x^2 \cos nx \, dx$, giving your answers in the form $a \frac{\pi}{n^2}$, where the possible values of *a* are to be determined. [2]
 - (b) The region bounded by the curve $y = \frac{x\sqrt{x}}{9-x^2}$, the x-axis and the lines x = 0 and x = 2 is rotated through 2π radians about the x-axis. Use the substitution $u = 9 x^2$ to find the exact volume of the solid obtained, simplifying your answer. [5]
- 3 A curve D has parametric equations

1

$$x = t - \cos t$$
, $y = 1 - \cos t$, for $0 \le t \le 2\pi$.

- (i) Sketch the graph of *D*. Give in exact form the coordinates of the points where *D* meets the *x*-axis, and also give in exact form the coordinates of the maximum point on the curve. [4]
- (ii) Find, in terms of a, the area under D for $0 \le t \le a$, where a is a positive constant less than 2π .

[3]

[4]

The normal to D at the point where $t = \frac{1}{2}\pi$ cuts the x-axis at E and the y-axis at F.

(iii) Find the exact area of triangle OEF, where O is the origin.

4 (a) Two loci in the Argand diagram are given by the equations

|z-3-i| = 1 and $\arg z = \alpha$, where $\tan \alpha = 0.4$.

The complex numbers z_1 and z_2 , where $|z_1| < |z_2|$, correspond to the points of intersection of these loci.

(i) Draw an Argand diagram to show both loci, and mark the points represented by z_1 and z_2 .

(ii) Find the two values of z which represent points on |z - 3 - i| = 1 such that $|z - z_1| = |z - z_2|$. [4]

(b) (i) The complex number 2 – 2i is denoted by w. By writing w in polar form $re^{i\theta}$, where r > 0and $-\pi < \theta \le \pi$, find exactly all the cube roots of w in polar form. [3]

(ii) Find the smallest positive whole number value of *n* such that $\arg(w^*w^n) = \frac{1}{2}\pi$. [3]

Section B: Statistics [60 marks]

5 In a game of chance, a player has to spin a fair spinner. The spinner has 7 sections and an arrow which has an equal chance of coming to rest over any of the 7 sections. The spinner has 1 section labelled **R**, 2 sections labelled **B** and 4 sections labelled **Y** (see diagram).



The player then has to throw one of three fair six-sided dice, coloured red, blue or yellow. If the spinner comes to rest over **R** the red die is thrown, if the spinner comes to rest over **B** the blue die is thrown and if the spinner comes to rest over **Y** the yellow die is thrown. The yellow die has one face with * on it, the blue die has two faces with * on it and the red die has three faces with * on it. The player wins the game if the die thrown comes to rest with a face showing * uppermost.

(i) Find the probability that a player wins a game.

[2]

[2]

- (ii) Given that a player wins a game, find the probability that the spinner came to rest over **B**. [1]
- (iii) Find the probability that a player wins 3 consecutive games, each time throwing a die of a different colour.

6 The number of employees of a company, classified by department and gender, is shown below.

	Production	Development	Administration	Finance		
Male	2345	1013	237	344		
Female	867	679	591	523		

- ④ (i) The directors wish to survey a sample of 100 of the employees. This sample is to be a stratified sample, based on department and gender.
 - (a) How many males should be in the sample?
 - (b) How many females from the Development department should be in the sample? [1]

[1]

The Managing Director knows that, some years ago, the mean age of employees was 37 years. He believes that the mean age of employees now is less than 37 years.

 (ii) State why the stratified sample from part (i) should not be used for a hypothesis test of the Managing Director's belief.

The Company Secretary obtains a suitable sample of 80 employees in order to carry out a hypothesis test of the Managing Director's belief that the mean age of the employees now is less than 37 years. You are given that the population variance of the ages is 140 years².

- (iii) Write down appropriate hypotheses to test the Managing Director's belief. You are given that the result of the test, using a 5% significance level, is that the Managing Director's belief should be accepted. Determine the set of possible values of the mean age of the sample of employees. [4]
- (iv) You are given instead that the mean age of the sample of employees is 35.2 years, and that the result of a test at the α % significance level is that the Managing Director's belief should not be accepted. Find the set of possible values of α . [3]

7 The management board of a company consists of 6 men and 4 women. A chairperson, a secretary and a treasurer are chosen from the 10 members of the board. Find the number of ways the chairperson, the secretary and the treasurer can be chosen so that

(i)	they are all women,	[1]
(ii)	at least one is a woman and at least one is a man.	[3]
The	10 members of the board sit at random around a table. Find the probability that	

- (iii) the chairperson, the secretary and the treasurer sit in three adjacent places, [3]
- (iv) the chairperson, the secretary and the treasurer are all separated from each other by at least one other person.

8 A website about electric motors gives information about the percentage efficiency of motors depending on their power, measured in horsepower. Xian has copied the following table for a particular type of electric motor, but he has copied one of the efficiency values wrongly.

Power, x	1	1.5	2	3	5	7.5	10	20	30	40	50
Efficiency, y%	72.5	82.5	84.0	87.4	87.5	88.5	89.5	90.2	91.0	91.7	92.4

(i) Plot a scatter diagram on graph paper for these values, labelling the axes, using a scale of 2 cm to represent 10% efficiency on the *y*-axis and an appropriate scale for the *x*-axis. On your diagram, circle the point that Xian has copied wrongly.

For parts (ii), (iii) and (iv) of this question you should **exclude** the point for which Xian has copied the efficiency value wrongly.

(ii) Explain from your scatter diagram why the relationship between x and y should not be modelled by an equation of the form y = ax + b. [1]

(iii) Suppose that the relationship between x and y is modelled by an equation of the form $y = \frac{c}{x} + d$, where c and d are constants. State with a reason whether each of c and d is positive or negative. [2]

- (iv) Find the product moment correlation coefficient and the constants c and d for the model in part (iii).
- (v) Use the model $y = \frac{c}{x} + d$, with the values of c and d found in part (iv), to estimate the efficiency value (y) that Xian has copied wrongly. Give two reasons why you would expect this estimate to be reliable. [3]
- 9 (a) The random variable X has distribution N(15, a^2) and P(10 < X < 20) = 0.5. Find the value of a. [2]
 - (b) The random variable Y has distribution B(4, p) and P(Y = 1) + P(Y = 2) = 0.5. Show that $4p^4 12p^2 + 8p = 1$ and hence find the possible values of p. [4]
 - (c) On a television quiz show contestants have to select the right answer from one of three alternatives.
 George decides to do this entirely by guesswork. Use a suitable approximation, which should be stated, to find the probability that George guesses at least 30 questions right out of 100. [4]

- ⊙ 10 Mia owns a field. Various types of weed are found in Mia's field.
 - (i) State, in this context, two conditions that must be met for the numbers of a particular type of weed in Mia's field to be well modelled by a Poisson distribution.

For the remainder of this question assume that these conditions are met.

There is an average of 1.5 dandelion plants (a type of weed) per m² in Mia's field.

- (ii) Find the probability that in 1 m^2 of Mia's field there are at least 2 dandelion plants. [2]
- (iii) Find the probability that in 4 m^2 of Mia's field there are at most 3 dandelion plants. [2]
- (iv) Use a suitable approximation, which should be stated, to find the probability that the number of dandelion plants in an 80 m² area of Mia's field is between 110 and 140 inclusive.

The distribution of daisies (another type of weed) per m² in Mia's field can be modelled by Po(λ). The probability that the number of daisies in a 1 m² area of the field is less than or equal to 2 is the same as the probability that the number of daisies in a 2 m² area of the field is more than 2.

[4]

(v) Write down an equation in λ and solve it to find λ .