2017 MATHEMATICS HIGHER 2 SPECIMEN PAPER (SYLLABUS 9758) Paper 2

Section A: Pure Mathematics [40 marks]

1 **(i)** The function f is defined as follows:

$$f : x \mapsto 3\cos x - 2\sin x,\ x \in \mathbb{R}, -\pi \le x < \pi.$$

Write f(x) as $R\cos(x+\alpha)$, where R and α are constants to be found. Hence, or otherwise, find the range of f and sketch the curve. [4]

(ii) The function g is defined as follows:

$$g : x \mapsto 3\cos x - 2\sin x,\ x \in \mathbb{R}, -\alpha \le x \le b.$$

Given that the function g^{-1} exists, write down the largest value of b. Find $g^{-1}(x)$. [3]

2 The first four terms of a sequence of numbers are 3, 1, 1 and 3. S_n is the sum of the first n terms of this sequence.

(i) Explain why S_n cannot be a quadratic polynomial in n. [2]

It is given that S_n is a cubic polynomial.

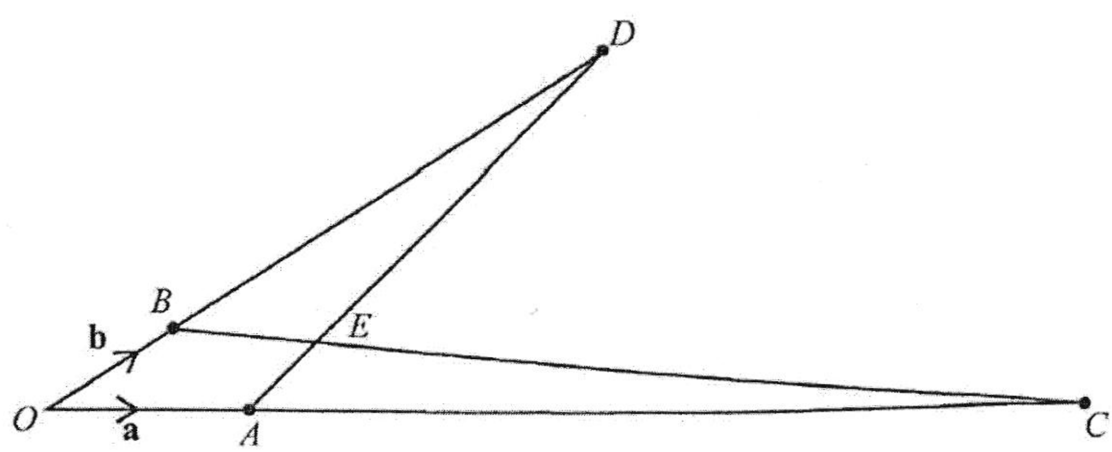
(ii) Find S_n in terms of n. [4]

(iii) Find an expression in terms of n for the nth term of the sequence. [3]

3 **(a)** The angle between the vectors $3\mathbf{i} - 2\mathbf{j}$ and $6\mathbf{i} + d\mathbf{j} - \sqrt{7}\mathbf{k}$ is $\cos^{-1}\left(\frac{6}{13}\right)$.

Show that $2d^2 - 117d + 333 = 0$. [3]

(b)



With reference to the origin O, the points A, B, C and D are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AC} = 5\mathbf{a}$ and $\overrightarrow{BD} = 3\mathbf{b}$. The lines AD and BC cross at E (see diagram).

(i) Find $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [6]

(ii) The point F divides the line CD in the ratio 5 : 3. Show that O, E and F are collinear, and find OE: OF. [4]

4 (i) Given that $y = \tan(e^{2x} - 1)$, show that $\frac{dy}{dx} = ke^{2x}(1 + y^2)$, where k is to be found. Hence find the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ when $x = 0$. [6]

(ii) Write down the first three non-zero terms in the Maclaurin series for $\tan(e^{2x} - 1)$. [1]

(iii) The first two non-zero terms in the Maclaurin series for $\tan(e^{2x} - 1)$ are equal to the first two non-zero terms in the series expansion of $e^{ax}\ln(1 + nx)$. By using appropriate expansions from the List of Formulae **(MF 26)**, find the constants a and n. Hence find the third non-zero term of the series expansion of $e^{ax}\ln(1 + nx)$. for these values of a and n.

[4]

Section B: Probability and Statistics [60 marks]

5 This question is about six couples. Each couple consists of a husband and wife.

The 12 people visit a theatre, and sit in a row of 12 seats.

(i) In how many different ways can the 12 people sit so that each husband and wife in a couple sit next to each other? [2]

(ii) In how many different ways can the 12 people sit so that the 6 wives all sit next to each other, and none of the wives sits next to her own husband? [3]

The group decides to form a committee to arrange future outings. The committee will consist of 3 of the 12 people. At least 1 of the wives will be on the committee but no husband and wife couple will be included.

(iii) In how many ways can the committee be formed? [3]

6 Giant pumpkins are often irregular in shape. In order to account for the different shapes of pumpkins, growers of giant pumpkins measure the size of a pumpkin by a combination of three measurements, called the 'over the top' length. Pumpkin growers keep records so that they can estimate the mass of giant pumpkins while they are still growing. The over the top lengths (d m) and the masses (m kg) of a random sample of 7 giant pumpkins are as follows.

d	2.31	2.9	4.05	5.5	6.7	7.92	9.17
m	11	14	47	104	170	282	449

(i) Draw a scatter diagram of these data, and explain how you know from your diagram that the relationship between m and d should not be modelled by an equation of the form $y = ax+b$. [2]

(ii) Which of the formulae $m = ed^2 + f$ and $m = gd^3 + h$, where e, f, g *and* h are constants, is the better model for the relationship between m and d? Explain fully how you decided, and find the constants for the better formula. [5]

(iii) Use the formula you chose from part **(ii)** to estimate the mass of a giant pumpkin with
(a) over the top length 6 m,
(b) over the top length 12m.

Explain which of your two estimates is more reliable. [3]

7 'Bings' are sweets that are sold in packets of 6. Each packets is made up of randomly chosen coloured sweets. On average 10% of Bings are yellow.

(i) Explain why a binomial distribution is appropriate for modelling the number of yellow sweets in a packet. Find the probability that a randomly chosen packet of Bings contains no more than one yellow sweet. [3]

(ii) Kev buys 90 randomly chosen packets of Bings. Find the probability that at least 80 of these packets contain no more than one yellow sweet. [2]

On average the proportion of Bings that are red is p. It is known that the modal number of red sweets in a packet is 2.

(iii) Use this information to find exactly the range of values that p can take. [4]

8 A bag contains 3 blue counters, 1 red counter and y yellow counters. Darvina chooses 3 counters at random from the bag, without replacement. The random variable S is the sum of the number of blue counters chosen and **twice** the number of red counters chosen.

(i) Show that $\mathrm{P}(S=3)=\dfrac{6(3y+1)}{(y+4)(y+3)(y+2)}$. [2]

(ii) Given that $\mathrm{P}(S=3)=\dfrac{7}{20}$, calculate y. Hence find the probability distribution of S. [6]

9 A type of metal bolt is manufactured with a nominal radius of 0.8cm. In fact, the radii of the bolts, is measured in cm, have the distribution $\mathrm{N}(0.8, 0.01^2)$.

(i) Find the percentage of bolts that have a radius between 0.79cm and 0.82cm. [1]

Metal washers are manufactured to fit on the bolts. The inside radii of the washers, measured in cm, have the distribution $\mathrm{N}(0.81, 0.012^2)$.

(ii) Write down the distribution of the inside circumference of the washers, in cm, and find the circumference that is exceeded by 5% of the washers. [4]

A bolt and a washer are ' a good fit' if

- the inside radius of the washer is greater than the radius of the bolt and
- the inside radius of the washer is not more than 0.04 cm greater than the radius of the bolt.

(iii) A washer is chosen at random, and a bolt is chosen at random. Find the probability that the washer and the bolt is a good fit. [3]

The outside radii of the washers, measured in cm, have the distribution $\mathrm{N}(\mu,\sigma^2)$. It is known that 15% of the washers have an outside radii greater than 1.25cm and 25% have an outside radius of less than 1.15cm.

(iv) Find the values of μ and σ . [4]