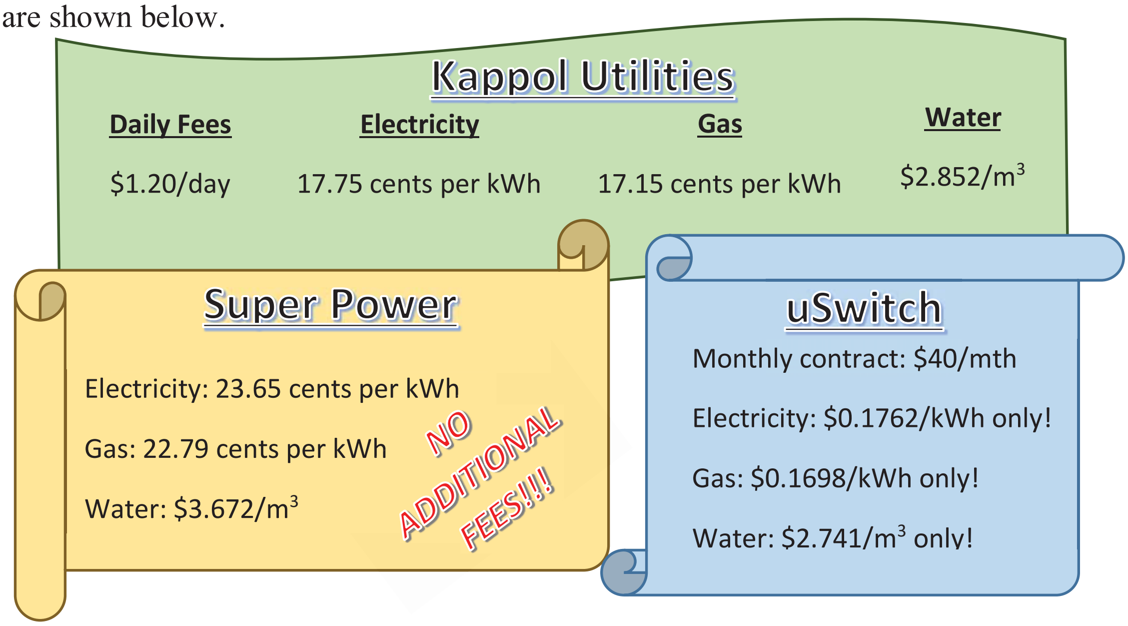
1 The rates from three different utilities retailers Kappol Utilities, Super Power and uSwitch are shown below.



Based on Mr Lim's utilities consumption for the 30 days in the month of June, Mr Lim found that if he subscribed all his electricity, gas and water services from a single utilities retailer, he would have to pay Kappol Utilities, Super Power and uSwitch a bill of $143.06, $140.78 and $144.96 respectively.
Find the amount of electricity and gas in kWh, and water in m^3 that Mr Lim used in the month of June. [3]

2 **(i)** Differentiate $\tan^{-1}\left(x^k\right)$, where k is a positive integer, leaving your answer in terms of k. [1]

(ii) Hence find $\int x^{k-1}\tan^{-1}\left(x^k\right)\mathrm{d}x$ in terms of k. [3]

3 **(i)** Show that the x-coordinate of the point(s) of intersection between a horizontal line $y=k$ where $k\in\mathbb{R}$ and the curve $y=\dfrac{ax+b}{x^2+1}$ where $a,b\in\mathbb{R}$ would satisfy the equation

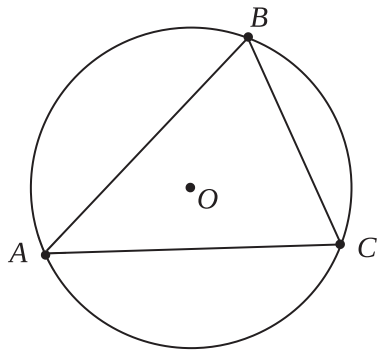
$$kx^2-ax+\left(k-b\right)=0\ .$$ [1]

(ii) Given that $-1\le k\le 4$, find the values of a and b. [4]

4 An arithmetic sequence $a_1,\ a_2,\ a_3,\ \ldots$ has common difference d, where $d\neq 0$ and $a_1\neq 0$.
Another geometric sequence $b_1,\ b_2,\ b_3,\ \ldots$ has common ratio r, where $r>0,\ r\neq 1$. It is given that $a_1=b_1$, $a_3=b_3$ and $a_7=b_5$.
If $a_n=b_m$ where $n,m\in\mathbb{Z}^+$, find n in terms of m. [6]

[Turn over

5



The diagram above shows a circle centred at O with radius 1 unit.

It is given that $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $\overrightarrow{OC}=-\frac{2}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}$.

(a) Show that the area of triangle ABC is given by $|\mathbf{a}\times\mathbf{b}|$ square units. Given that $\angle ACB=\frac{\pi}{3}$, find the exact area of triangle ABC. [5]

(b) The point M lies on CB produced such that the area of triangle AMC is three times that of the area of triangle ABC. Find $\overrightarrow{OM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. [2]

6 **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sec^n\theta\right)$. [1]

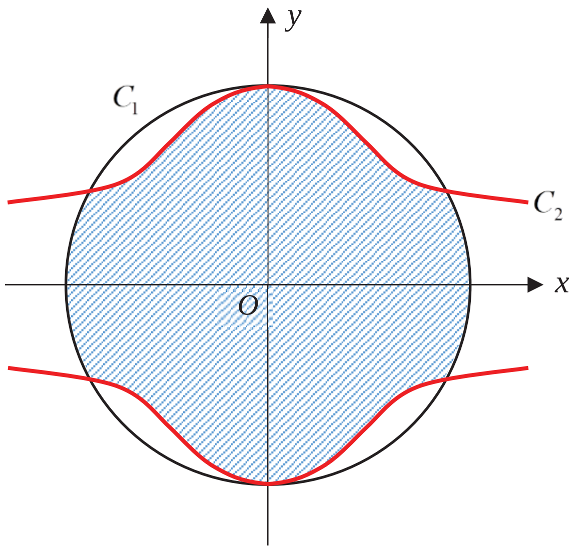
(ii) Using the substitution $x=2\tan^2\theta$, show that $\int_0^2 x\sqrt{1+\frac{x}{2}}\,\mathrm{d}x$ can be expressed as

$$k\int_a^b\left(\sec^5\theta\tan\theta-\sec^3\theta\tan\theta\right)\mathrm{d}\theta,$$

where a, b and k are constants to be determined. [4]

(iii) Hence find the exact value of $\int_0^2 x\sqrt{1+\frac{x}{2}}\,\mathrm{d}x$, expressing your answer in the form $A\left(\sqrt{2}+B\right)$, where A and B are constants to be determined. [2]

7



The curves C_1 and C_2 have equations $x^2+y^2=4$ and $y^2=\dfrac{4}{1+x^2}$ respectively.

(i) Find the exact coordinates of the points of intersection of C_1 and C_2. [2]

(ii) Find the area of the shaded region, giving your answer correct to 3 decimal places. [2]

(iii) Find the exact volume of the solid obtained when the shaded region is rotated through π radians about the x-axis. [3]

8 **(i)** Show that $\dfrac{1}{(2r+3)\sqrt{2r+1}\ +\ (2r+1)\sqrt{2r+3}}=\dfrac{1}{2}\left(\dfrac{1}{\sqrt{2r+1}}-\dfrac{1}{\sqrt{2r+3}}\right)$. [2]

(ii) Given that $S_n=\displaystyle\sum_{r=1}^{n}\frac{1}{(2r+3)\sqrt{2r+1}+(2r+1)\sqrt{2r+3}}$, find S_n in terms of n. [3]

(iii) Find the smallest value of n for which the difference between S_n and S_∞ is less than 0.05. [3]

9 The parametric equations of a curve are given by

$$x=\sin t\cos t,\quad y=\cos\left(t+\frac{\pi}{4}\right),\text{ where } 0\le t\le\frac{\pi}{2}.$$

(i) Show that $\dfrac{dy}{dx}=\dfrac{1}{\sqrt{2}\left(\sin t-\cos t\right)}$. [2]

(ii) Hence find the equation of the tangent parallel to the y-axis. [2]

The curve cuts the y-axis at the points P and Q.

(iii) Find the exact coordinates of P and Q. [2]

(iv) The point R on the curve has coordinates $\left(\sin\theta\cos\theta,\ \cos\left(\theta+\dfrac{\pi}{4}\right)\right)$.

Show that the area of triangle PQR is given by $\dfrac{\sqrt{2}}{4}\sin 2\theta$. Hence find the value of θ for which the area of triangle PQR is a maximum. (You need not prove that it is a maximum.) [2]

[Turn over

10 **(i)** Describe a sequence of transformations which transforms the graph of $y=\frac{1}{x}$ to the graph of $y=\frac{3x+5}{x+2}$. [3]

(ii) Sketch the graphs of $y=\frac{3x+5}{x+2}$ and $16(x+3)^2+9(y-4)^2=144$ on a single diagram, indicating clearly any axial intercepts, points of intersection of the two graphs and the equations of asymptotes. [5]

(iii) Hence find the set of values of x that satisfies the inequality

$$\frac{3x+5}{x+2} > 4-\sqrt{\frac{144-16(x+3)^2}{9}}.$$ [2]

11 The function f is defined by

$$\mathrm{f}: x \mapsto 2\left|4-(x-\mathrm{e})^2\right|, \text{ where } x \in \mathbb{R}.$$

(i) Sketch the graph of $y=\mathrm{f}(x)$, showing clearly the exact coordinates of any axial intercepts and turning points. [2]

(ii) Show that f^{-1} does not exist. [1]

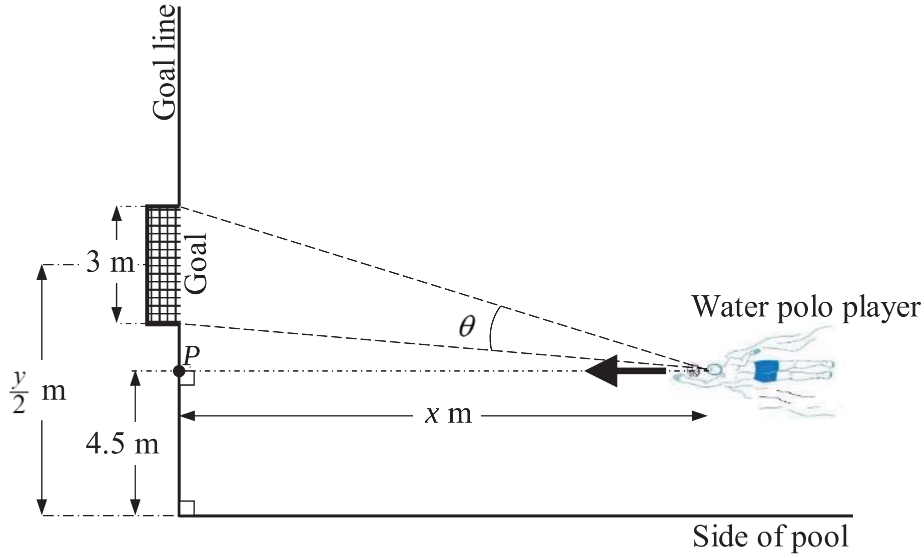
(iii) The domain of f is restricted to $a \le x < \mathrm{e}$, where $a \in \mathbb{R}$.
State the smallest exact value of a for which f^{-1} exists. Hence find f^{-1} in similar form. [5]

The function g is defined by

$$\mathrm{g}: x \mapsto \frac{x}{x+1}, \text{ where } x \in \mathbb{R}, x \neq -1.$$

(iv) Using the domain in part **(iii)**, show that gf^{-1} exists and find the exact range of gf^{-1}. [3]

12



The diagram (not drawn to scale) shows part of a rectangular water polo pool with a fixed width of y m. A goal 3 m wide is placed on the goal line at one end of the pool with the centre of the goal $\frac{y}{2}$ m from the side of the pool. A water polo player at a distance of x m perpendicular to the goal line and a distance of 4.5 m away from the side of the pool swims directly towards a point P on the goal line. A visual angle θ of the goal is the angle subtended at the eye of the water polo player by the goal.

(i) Show that $\tan\theta = \dfrac{12x}{4x^2 + A}$ where $A = (y-6)(y-12)$. [3]

(ii) The desired visual angle of the goal is obtained when θ is a maximum. Find by differentiation, the value of x such that the desired visual angle of the goal is obtained. Leave your answer in exact form in terms of A. [4]

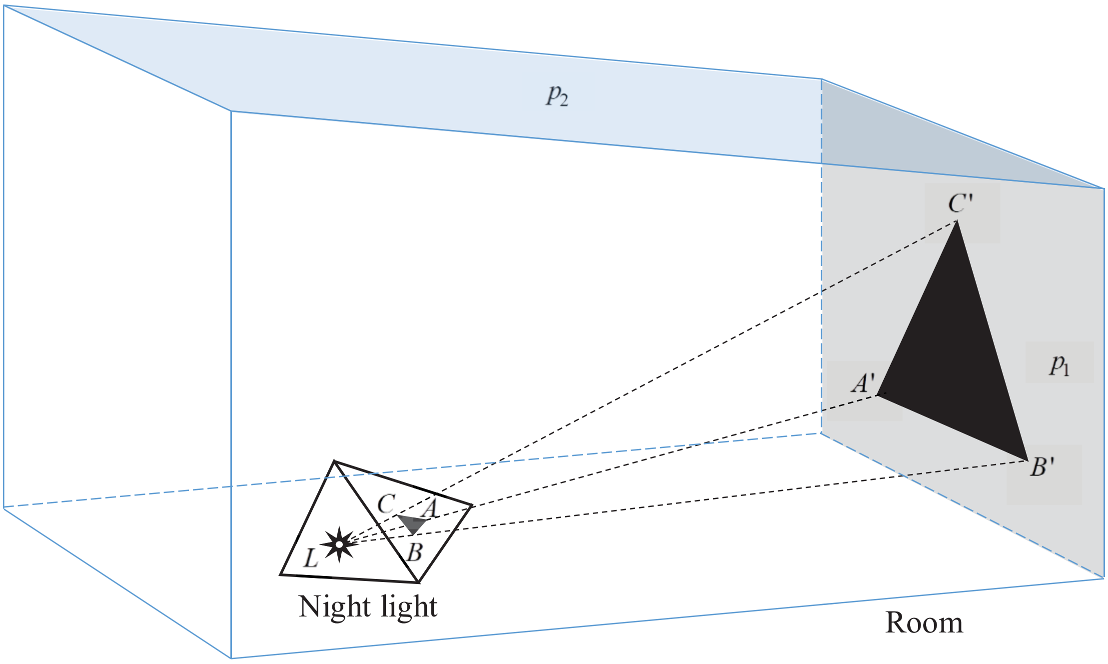
For the rest of the question, let $y = 20$.

(iii) Show that the water polo player needs to be $2\sqrt{7}$ m away from the goal line in order to obtain the desired visual angle. [2]

(iv) The water polo player swims at a constant speed of 50 m per minute. Find the rate of change of θ at the instant when the water polo player is 15 m away from the goal line. [3]

[Turn over

13



In the diagram (not drawn to scale), a night light with a light emitting diode L housed in a transparent acrylic case is placed in a room. A triangular sticker is stuck onto one face of the acrylic case. Referred to the origin O, the three vertices of the triangular sticker have coordinates $A(2,\ 3,\ 3)$, $B(4,\ 3,\ 4)$ and $C(2,\ 2,\ 5)$, where the units of measurement are in centimetres. When the night light is switched on, a triangular shadow $A'B'C'$ of the triangular sticker is cast on a vertical plane wall p_1 on one side of the room.

It is given that $\overrightarrow{OA'} = \begin{pmatrix} -3 \\ 13 \\ 8 \end{pmatrix}$ and $\overrightarrow{OB'} = \begin{pmatrix} 9 \\ 13 \\ 14 \end{pmatrix}$.

(i) Show that $\overrightarrow{OL} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$. [4]

(ii) Given that p_1 is parallel to the x-z plane, find $\overrightarrow{OC'}$. [4]

The slanted ceiling p_2 is a plane with equation $y + 4z = 1053$.

(iii) Find the shortest distance between L and p_2. [2]

(iv) Find the line l when p_2 meets p_1. [2]