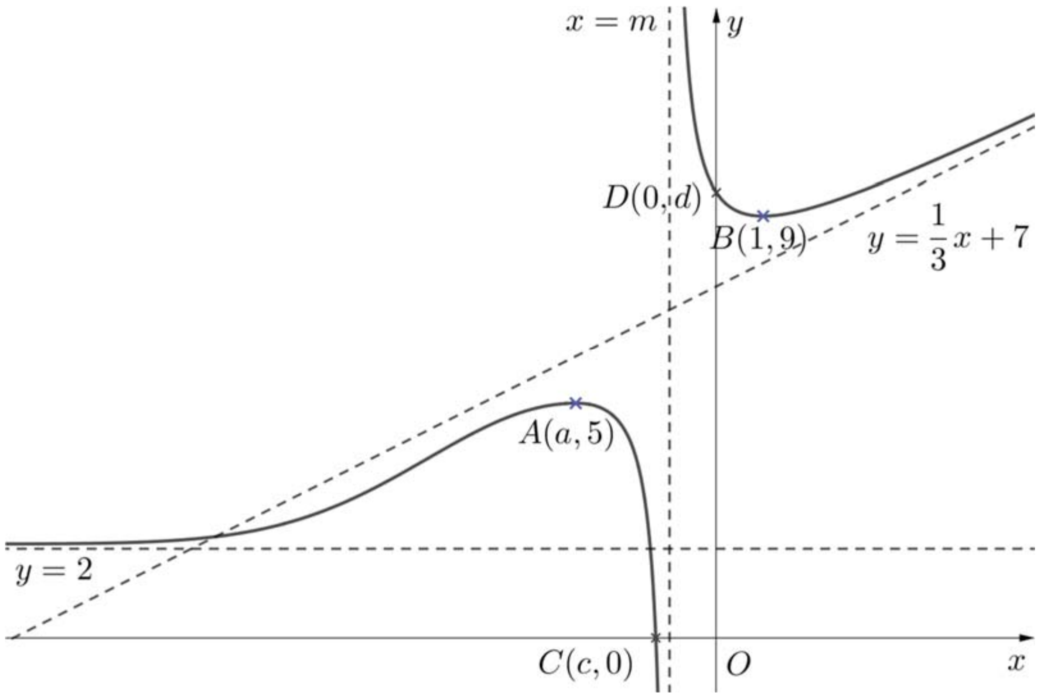
1 Given that $\mathrm{f}(x)=\tan\left(\frac{\pi}{4}-x\right)$, find $\mathrm{f}(0)$, $\mathrm{f}'(0)$ and $\mathrm{f}''(0)$. Hence write down the first three non-zero terms in the Maclaurin series for $\mathrm{f}(x)$. [4]

2 The function $\mathrm{f}(x)$ is a quadratic polynomial such that $\mathrm{f}(4)=3$ and $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x=\frac{20}{3}$. If the curve with equation $y=\mathrm{f}(x)$ is transformed by a translation of 1 unit in the negative x-direction, the new curve has a y-intercept at $\frac{3}{2}$. Find $\mathrm{f}(x)$. [4]

3 The diagram below shows the curve with equation $y=\mathrm{f}(x)$. It has turning points $A(a,5)$ and $B(1,9)$ and asymptotes with equations $y=2$, $x=m$ and $y=\frac{1}{3}x+7$. The curve also crosses the axes at the points $C(c,0)$ and $D(0,d)$. The gradient of the curve at D is -3.



Sketch, on separate diagrams, the following curves.

(a) $y=\frac{1}{\mathrm{f}(x)}$ [3]

(b) $y=\mathrm{f}'(x)$ [3]

Label the coordinates of the points corresponding to *A*, *B*, *C*, and *D* (where applicable), the points where the curve crosses the axes, and the equations of any asymptotes.

4 A curve C has equation $y = e^x$.

(a) Find the exact volume of revolution when the region bounded by C, the line $y = 3$ and the y-axis is rotated completely about the y-axis. [5]

(b) Describe a pair of transformations which transforms the curve with equation $y = e^{1+2x}$ on to the curve C. [2]

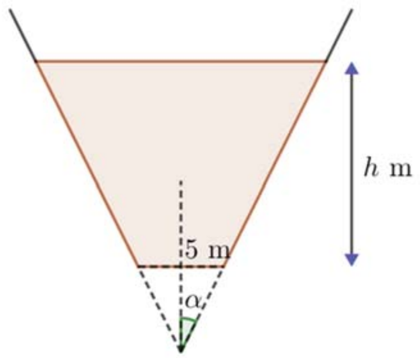
5 A curve C has parametric equations

$$x = \sin\theta, \qquad y = \sin 2\theta, \qquad \text{for } 0 \le \theta \le \pi.$$

(i) Sketch C, labelling the coordinates of any axial intercepts. [2]

(ii) Find the exact area of the region bounded by C. [5]

6 A vessel is formed by removing a smaller cone of radius 5 m from a bigger cone whose semi-vertical angle is α, where $\tan\alpha = 0.5$. Water flows out of the vessel at a rate of $k\sqrt{h}$ m^3 per minute, where k is a positive constant. At time t minutes, the height of the water surface from the hole is h m (see diagram).



(i) Show that the volume of the water V, in m^3, is given by $\frac{1}{12}\pi\left[(h+10)^3 - 1000\right]$. [4]

(ii) Find the rate of change of h, in terms of k, when $V = 120\pi$. [4]

7 Determine the constants A and B such that

$$\frac{1}{(3x^2+1)(x^2+3)} = \frac{A}{3x^2+1} + \frac{B}{x^2+3}.$$

Hence, find the exact value of p, where $p > -1$, such that

$$\int_0^1 \frac{8}{(3x^2+1)(x^2+3)}\,dx = \int_{-1}^{p} \left|\frac{\sqrt{3}}{9}x\right| dx,$$

giving your answer in terms of π. [8]

8 A curve C has equation $\ln(y+1) = 1 + \tan^{-1} x$, where $y > -1$.

(i) Explain why C has no tangent parallel to the y-axis. [2]

(ii) Without using a calculator, find the equation of the normal to C at the point where it crosses the y-axis. [3]

(iii) Determine the range of values of x for which C is concave downwards. [3]

 [Turn over

9 Non-zero and non-parallel vectors **a**, **b** and **c** are such that $\mathbf{b}\times 3\mathbf{c}=\mathbf{c}\times\mathbf{a}$.

(i) Determine the relationship between **c** and $\mathbf{a}+3\mathbf{b}$, justifying your answer. [2]

It is given that **a** and $3\mathbf{b}$ are unit vectors and that the angle between **a** and **b** is $60°$.

(ii) Evaluate $|\mathbf{a}+3\mathbf{b}|$. [3]

(iii) Given further that $\mathbf{a}+3\mathbf{b}$ makes an angle of $60°$, $120°$ and $135°$ with the positive x-, y- and z-axes respectively, show that **c** is parallel to $\mathbf{i}-\mathbf{j}-\sqrt{2}\mathbf{k}$. [3]

10 Functions f and g are defined by

$$\mathrm{f}: x \mapsto 2-x+\frac{8}{x+2}, \quad x\in\mathbb{R}, x\neq -2, x>k,$$
$$\mathrm{g}: x \mapsto x^2-6x+a, \quad x\in\mathbb{R}, x>0,$$

where a is a constant.

(i) State the least value of k for which the function f^{-1} exists. [1]

Using this value of k,

(ii) Without finding f^{-1}, sketch, on the same diagram, the graphs of $y=\mathrm{f}(x)$, $y=\mathrm{f}^{-1}(x)$ and $y=\mathrm{f}^{-1}\mathrm{f}(x)$, showing clearly their geometrical relationship. State the equations of any asymptotes. [4]

(iii) Find the smallest integer value of a for which the composite function fg exists and use this value to state the range of fg. [4]

(iv) Given instead that $a=10$, solve the inequality $\mathrm{fg}(x)+\mathrm{g}(x)\leq 4$ algebraically. [5]

11 The plane p passes through the points A, B and C with coordinates $(-3,-4,11)$, $(1,-2,0)$ and $(-5,2,-1)$ respectively. The point M has position vector given by $\mathbf{m}=3\mathbf{i}+6\mathbf{j}+15\mathbf{k}$.

(i) Show that $\overrightarrow{AM}$ is perpendicular to p. [3]

(ii) Find the coordinates of N which is the mirror image of M in p. [2]

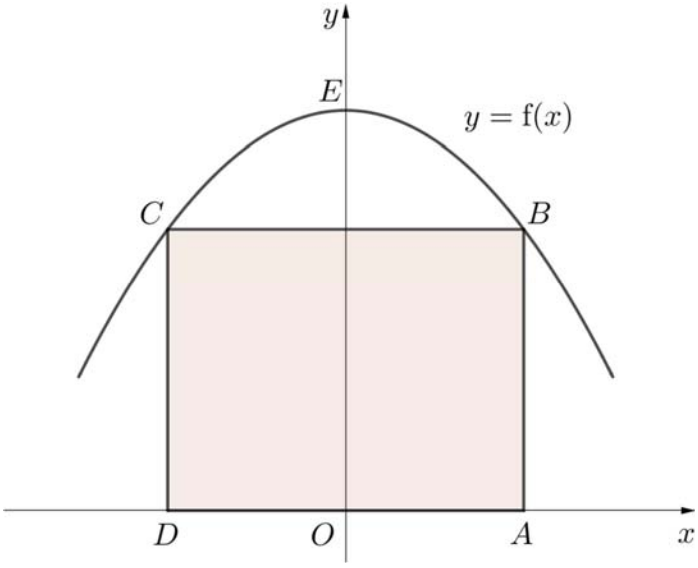
(iii) Find a vector equation of the line which is a reflection of the line MB in p. [2]

The plane q has equation $11x+6y-z-4+k(-x-3y+z-4)=0$ for some constant k.

(iv) Given that q contains M, find a cartesian equation of q. [2]

(v) By finding the line of intersection between p and q, or otherwise, find a cartesian equation of the plane which is a reflection of q in p. [5]

12 It is given that $\mathrm{f}(x) = 9 - \frac{x^2}{6}$. The diagram below shows a vertical cross section of a building. The cross section of the roof of the building can be modelled by the curve $y = \mathrm{f}(x)$, where $-6 \le x \le 6$ and x is in metres. The ground level is represented by the x-axis.



The cross section of the living space under the roof can be modelled by a rectangle $ABCD$ with points $D(-a, 0)$ and $A(a, 0)$, where $0 < a \le 6$.

The P/A ratio is a measure of the thermal insulation of a space. The lower the P/A ratio, the smaller the amount of insulation is required, thus saving costs. The P/A ratio can be defined by the function $I(a) = \frac{P(a)}{A(a)}$ where $P(a)$ is the perimeter and $A(a)$ is the area of the rectangle $ABCD$.

(i) Show that $I(a) = \frac{12}{54 - a^2} + \frac{1}{a}$. [3]

(ii) Given that $a = a_1$ is the value of a which gives the minimum value of I, show that a_1 satisfies the equation $a^2 + \sqrt{24}a^{\frac{3}{2}} - 54 = 0$. [3]

(iii) Find a_1, correct to 3 decimal places, and show that it minimises I. [2]

The developer would prefer the living space $ABCD$ to be at least 80% of the area $ABECD$, which is currently not satisfied. It is suggested to replace the curve $y = \mathrm{f}(x)$ to model the roof with part of the curve in part **(iv)**.

(iv) Sketch the curve with equation $(2y - 19)^2 - 2x^2 = 1$ for $y \le 9$ for **all** values of x, stating the equations of any asymptotes. [2]

(v) Based on the value of a_1 found in part **(iii)**, determine if the developer would accept the suggestion. [2]