| Qn |  |
| :---: | :---: |
| 5(i) | $\text { Number of ways }=\frac{9!}{2!2!3!}=15120$ |
| 5(ii) | Number of ways $=\frac{7!}{3!}=840$ |
| 5(iii) | $\begin{aligned} \text { Number of ways } & =\frac{5!}{2!} \cdot{ }^{6} C_{3} \\ & =1200 \end{aligned}$ |
| 5(iv) | Let the event D be such that the D's are together, the event E be such that the E's are together and $S$ be such that the S 's are together. $\begin{aligned} n(D \cup E \cup S)= & n(D)+n(E)+n(S)-n(D \cap E) \\ & -n(E \cap S)-n(D \cap S)+n(D \cap E \cap S) \\ = & \frac{8!}{2!3!}+\frac{7!}{2!2!}+\frac{8!}{2!3!}-\frac{6!}{2!}-\frac{6!}{2!}-\frac{7!}{3!}+5! \\ = & 6540 \\ \text { Number of ways }= & n\left(D^{\prime} \cap E^{\prime} \cap S^{\prime}\right) \\ = & n(S)-n(D \cup E \cup S) \\ = & 15120-6540 \\ = & 8580 \end{aligned}$ |

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| Qn |  |
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| 6(i) | Let $X$ denotes the number of 1 -year old flares that fail to fire successfully, out of the $100, X \sim \mathrm{~B}(100,0.005)$ $\mathrm{P}(X \leq 2)=0.985897 \approx 0.986$ |
| 6(ii) | Let $Y$ denotes the number of boxes with a hundred 1-year old flares with at most 2 that fail to fire, out of 50 boxes, ie $Y \sim \mathrm{~B}(50,0.985897)$ $\mathrm{P}(Y \leq 48)=0.156856 \approx 0.157$ |
| 6(iii) | Let $T$ denotes the number of 10 -year old flares that fire successfully, out of the $6, T \sim \mathrm{~B}(6,0.75)$ <br> (a) Required prob $=(1-0.970) \times \mathrm{P}(T \geq 4)$ $\begin{aligned} & =0.03 \times(1-P(T \leq 3)) \\ & =0.0249 \end{aligned}$ <br> (b) $\quad \mathrm{P}$ (at least 4 of the 7 flares fire successfully) $\begin{aligned} & =0.024917+0.970 \times \mathrm{P}(T \geq 3) \\ & =0.024917+0.970 \times(1-\mathrm{P}(T \leq 2)) \\ & =0.958 \end{aligned}$ |


| Qn |  |
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| 7(i) | Let $X$ be the rv denoting the amount of time taken by a cashier to deal with a randomly chosen customer, ie $X \sim \mathrm{~N}\left(150,45^{2}\right)$. $\mathrm{P}(X>180)=0.25249 \approx 0.252$ |
| 7(ii) | Assume that the time taken to deal with each customer is independent of the other, ie $X_{1}+X_{2} \sim \mathrm{~N}\left(2 \times 150,2 \times 45^{2}\right)$ $\mathrm{P}\left(X_{1}+X_{2}<200\right)=0.058051 \approx 0.0581$ |
| 7(iii) | Let $Y$ be the rv denoting the amount of time taken by a the second cashier to deal with a randomly chosen customer, ie $Y \sim \mathrm{~N}\left(150,45^{2}\right)$. $\begin{aligned} & \quad X_{1}+X_{2}+X_{3}+X_{4} \sim \mathrm{~N}\left(4 \times 150,4 \times 45^{2}\right) \\ & \text { and } Y_{1}+Y_{2}+Y_{3} \sim \mathrm{~N}\left(3 \times 150,3 \times 45^{2}\right) \end{aligned}$ $\mathrm{P}\left(X_{1}+X_{2}+X_{3}+X_{4}<Y_{1}+Y_{2}+Y_{3}\right)=\mathrm{P}\left(X_{1}+X_{2}+X_{3}+X_{4}-\left(Y_{1}+Y_{2}+Y_{3}\right)<0\right)$ <br> Using $X_{1}+X_{2}+X_{3}+X_{4}-\left(Y_{1}+Y_{2}+Y_{3}\right) \sim N\left(150,7 \times 45^{2}\right)$ $\mathrm{P}\left(X_{1}+X_{2}+X_{3}+X_{4}-\left(Y_{1}+Y_{2}+Y_{3}\right)<0\right)=0.10386 \approx 0.104$ |


| Qn |  |
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| 8(i) |  |
| 8(ii) | Using GC, $r=0.884$ for the model $u=a x+b$ $u=a \mathrm{e}^{b x} \Rightarrow \ln u=b x+\ln a$ <br> Using GC, $r=0.906$ for the model $u=a \mathrm{e}^{b x}$ <br> Since the value of $r$ is closer to 1 for the $2^{\text {nd }}$ model, $u=a e^{b x}$ is a better model. $\ln u=0.013633 x+0.94964$ $\begin{aligned} & u=\mathrm{e}^{0.013633 x+0.94964} \\ & u=2.58 \mathrm{e}^{0.0136 x}=2.6 \mathrm{e}^{0.014 x} \end{aligned}$ |
| 8(iii) | $7=2.58 \mathrm{e}^{0.0136 x} \Rightarrow x=\frac{\ln \left(\frac{7}{2.58}\right)}{0.0136}=73.391 \approx 73$ <br> A patient with urea serum is 7 mmgl per litre is approximately 73 years old. <br> ExamPaper <br> Since $r=0.906$ is close to $\mathrm{f}^{\circ}$ and $\mathrm{m}^{\circ}$ is within the data range of urea serum, estimate is reliable. |
| 8(iv) <br> (a) | The product moment correlation coefficient in part (ii) will not be changed if the units for the urea serum is given in mol per decilitre. |
| 8(iv) <br> (b) | $u=0.258 \mathrm{e}^{0.0136 x}$ |


| Qn |  |
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| 9(i) | $\begin{aligned} & \begin{aligned} \mathrm{P}(X=2) & =\frac{18}{18} \frac{2}{17} \frac{15}{16} \frac{3!}{2!} \\ & =\frac{45}{136} \\ \mathrm{P}(X=0) & =\frac{18}{18} \frac{15}{17} \frac{12}{16} \\ & =\frac{45}{68} \end{aligned} \\ & \begin{aligned} \mathrm{P}(X=3) & =\frac{18}{18} \frac{2}{17} \frac{1}{16} \\ & =\frac{1}{136} \end{aligned} \end{aligned}$ |
| 9(ii) | $\begin{aligned} & \mathrm{E}(X)=\frac{93}{136} \\ & \mathrm{E}\left(X^{2}\right)=0 \times \frac{45}{68}+2^{2} \times \frac{45}{136}+3^{2} \times \frac{1}{136}=\frac{189}{136} \\ & \operatorname{Var}(X)=\frac{189}{136}-\left(\frac{93}{136}\right)^{2} \\ & \approx 0.922 \end{aligned}$ |
| 9(iii) | Since $n=40$ is łarge, by Central <br>  $P(\bar{X}>1)=0.0186$ |


| Qn |  |
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| 9(iv) | $\begin{aligned} & \text { Expected winnings }=-\frac{45}{68} a+\frac{45}{136}(a+10)+\frac{1}{136}(a+10) \\ & -\frac{11}{34} a+\frac{115}{34}>0 \\ & a<\frac{115}{11} \\ & a<10 . \ddot{4} \dot{5} \end{aligned}$ <br> The possible amounts will be $1 \leq a \leq 10$ and $a \in \mathbb{Z}$. |


| Qn |  |
| :---: | :---: |
| 10(i) | Let $X$ be the thickness of the coating on a randomly chosen computer device. Let $\mu$ be the mean thickness of the coating of a computer device. <br> Assume that the standard deviation of the coating of a computer device remains unchanged. <br> To test : $\begin{aligned} & H_{0}: \mu=100 \\ & H_{1}: \mu \neq 100\end{aligned}$ <br> Level of Significance: 5\% <br> Under $H_{0}$, since sample size $n=50$ is large, by Central Limit Theorem, $Z=\frac{\bar{X}-100}{10 / \sqrt{50}} \sim N(0,1)$ approx. <br> Reject $H_{0}$ if $p-$ value $\leq 0.05$. <br> Calculations: $\bar{x}=103.4$ $p-\text { value }=0.0162$ <br> Conclusion: Since $p$-value $<0.05$, we reject $H_{0}$ and conclude that there is significant evidence at $5 \%$ level of significance that the process is not in control. |
| 10(ii) | Reject $H_{0}$ is $\left\|z_{\text {calc }}\right\| \geq 1.960$ <br> For $H_{0}$ to be rejected $\begin{aligned} & \left\|\frac{\bar{x}-100}{10 / \sqrt{50}}\right\| \geq 1995996 \\ \Rightarrow & \bar{x} \leq 100-1.95996\left(\frac{10}{\sqrt{50}}\right) \text { or } \bar{x} \geq 100+1.95996\left(\frac{10}{\sqrt{50}}\right) \\ \Rightarrow & \bar{x} \leq 97.228 \text { or } \bar{x} \geq 102.772 \end{aligned}$ <br> Thus the required range of values of $\bar{x}$ is $0<\bar{X} \leq 97.2$ or $\bar{x} \geq 102.8$. |

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| Qn |  |
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| 10(iii) | $\begin{aligned} & \bar{y}=\frac{4164}{40}=104.1 \\ & \Sigma(y-100)=4164-4000=164 \\ & s^{2}=\frac{1}{39}\left[\Sigma(y-100)^{2}-\frac{\left(\Sigma(y-100)^{2}\right.}{40}\right] \\ &=\frac{1}{39}\left[9447-\frac{164^{2}}{40}\right] \\ &=\frac{43873}{195}=224.9897 \end{aligned}$ |
| 10(iv) | The standard deviation may have changed due to the wear out of mechanical parts as well. |
| 10(v) | To test : $\begin{aligned} & H_{0}: \mu=100 \\ & H_{1}: \mu \neq 100 \end{aligned}$ <br> Level of Significance: 4\% <br> Under $H_{0}$, since sample size $n=40$ is large, by Central Limit Theorem, $Z=\frac{\bar{Y}-100}{S / \sqrt{40}} \sim N(0,1) \text { approx. }$ <br> Reject $H_{0}$ if $\text { f } p-\text { value } \leq 0.04 \text {. }$ <br> Calculations: $\bar{E} \times=1044.4, s^{2}=224.2997$ $p-\text { value }=0.0839$ <br> Conclusion: Since $p$-value $>0.04$, we do not reject $H_{0}$ and conclude that there is insignificant evidence at $4 \%$ level of significance that the process is not in control. |

