	Since $\lim_{n\to\infty} \sqrt[n]{a_n} = \frac{2}{3} < 1$, by the Cauchy Test, the series converges for all real values of x
(b)(ii)	$\sum_{r=0}^{\infty} \frac{2^r r}{3^r} = 0 + 1\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right)^3 + 4\left(\frac{2}{3}\right)^4 + \ldots$ $= \frac{2}{3}\left[1 + 2\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)^3 + \ldots\right]$ $= \frac{2}{3}\left(1 - \frac{2}{3}\right)^{-2}$ since $1 + 2\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)^3 + \ldots = \left(1 - \frac{2}{3}\right)^{-2}$ with $y = \frac{2}{3}$ $= 6$
5	Let X be the mass of a standard packet of sugar in grams. Let Y be the mass of a large packet of sugar in grams. $X \sim \mathrm{N}\left(520, 8^2\right)$ $Y \sim \mathrm{N}\left(1030, 11^2\right)$ $X_1 + X_2 - Y \sim \mathrm{N}(10, 249)$ $\mathrm{P}\left(X_1 + X_2 > Y\right) = \mathrm{P}\left(X_1 + X_2 - Y > 0\right)$ $= 0.73687$ $= 0.737$ (3 s.f.)
	$\frac{X_1 + X_2 + Y}{3} \sim \mathrm{N}\left(690, \frac{83}{3}\right)$ $\mathrm{P}\left(680 < \frac{X_1 + X_2 + Y}{3} < 700\right) = 0.94272$ $= 0.943$ (3 s.f.)
6(i)	Total number of ways to select 5 members $= {}^{13}C_5$ Number of ways to select 5 members with 3 Biology students $= {}^{10}C_2$ Number of ways to select 5 members with at most 2 Biology students $= {}^{13}C_5 - {}^{10}C_2 = 1242$

(ii)	Let the number of Biology, History, and Literature students be B, H, L respectively. $\mathrm{P}(\mathrm{H}>\mathrm{L} \mid \mathrm{B}\le 2)=\dfrac{\mathrm{P}((\mathrm{H}>\mathrm{L})\cap(\mathrm{B}\le 2))}{\mathrm{P}(\mathrm{B}\le 2)}$ $=\dfrac{\mathrm{n}((\mathrm{H}>\mathrm{L})\cap(\mathrm{H}+\mathrm{L}\ge 3))}{\mathrm{n}(\mathrm{B}\le 2)}$ Number of ways to select cast members with H > L when there are 3 humanities students = ${}^3C_2\times({}^4C_2\times{}^6C_1+{}^4C_3\times{}^6C_0)=120$ Number of ways to select cast members with H > L when there are 4 humanities students = ${}^3C_1\times({}^4C_3\times{}^6C_1+{}^4C_4\times{}^6C_0)=75$ Number of ways to select cast members with H > L when there are 5 humanities students = ${}^3C_0\times({}^4C_3\times{}^6C_2+{}^4C_4\times{}^6C_1)=66$ Required Probability $=\dfrac{\mathrm{n}((\mathrm{H}>\mathrm{L})\cap(\mathrm{H}+\mathrm{L}\ge 3))}{\mathrm{n}(\mathrm{B}\le 2)}$ $=\dfrac{120+75+66}{1242}$ $=\dfrac{261}{1242}=0.210\ \ (3\text{ s.f.})$
7(i)	$\mathrm{P}(X=1)=\mathrm{P}(X=2)=....=\mathrm{P}(X=n)=\dfrac{1}{n}$ $\mathrm{E}(X)=\sum_{x=1}^{n}x\mathrm{P}(X=x)=\sum_{x=1}^{n}x\cdot\left(\dfrac{1}{n}\right)$ $=\dfrac{1}{n}\left(\dfrac{n(n+1)}{2}\right)$ $=\dfrac{n+1}{2}$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$

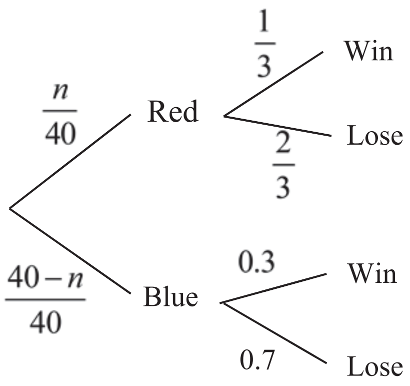
	$\mathrm{E}(X^2)=\sum_{x=1}^{n} x^2 \cdot \left(\frac{1}{n}\right)$ $=\frac{1}{n}\left(\frac{n}{6}(n+1)(2n+1)\right)$ $=\frac{(n+1)(2n+1)}{6}$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-\left(\mathrm{E}(X)\right)^2$ $=\frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2$ $=\frac{(n+1)}{12}\left(2(2n+1)-3(n+1)\right)$ $=\frac{(n+1)(n-1)}{12}$ $=\frac{n^2-1}{12}$
(ii)	$\mathrm{E}(X)=\frac{7}{2}$ and $\mathrm{Var}(X)=\frac{6^2-1}{12}=\frac{35}{12}$ $\mathrm{P}\left(\lvert X-\mu\rvert>\sigma\right)=\mathrm{P}\left(\left\lvert X-\frac{7}{2}\right\rvert>\sqrt{\frac{35}{12}}\right)$ $=\mathrm{P}\left(X>\frac{7}{2}+\sqrt{\frac{35}{12}}\right)+\mathrm{P}\left(X<\frac{7}{2}-\sqrt{\frac{35}{12}}\right)$ $=\mathrm{P}(X>5.21)+\mathrm{P}(X<1.79)$ $=\mathrm{P}(X=6)+\mathrm{P}(X=1)$ $=\frac{2}{6}=\frac{1}{3}$

Let S be the random variable "no. of observations, out of 20, such that the total score is an outlier".

$S \sim \mathrm{B}(20, \frac{1}{3})$

$\mathrm{P}(S \geq 8) = 1 - \mathrm{P}(S \leq 7) = 0.339$

8(i)



P(a player wins the game)

$= \frac{15}{40} \times \frac{1}{3} + \frac{25}{40} \times \frac{3}{10}$

$= \frac{5}{16}$

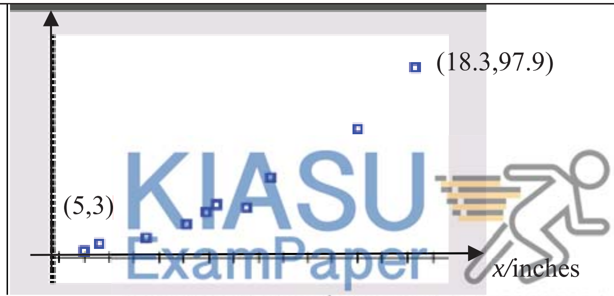
P(exactly 2 of 3 players win)

$= {}^{3}C_{2} \left(\frac{5}{16}\right)^{2} \left(\frac{11}{16}\right)$

$= \frac{825}{4096}$

Alternatively,

	Let X be the random variable "the number of people who wins the game out of 3" $X \sim \mathrm{B}(3, \frac{5}{16})$ $\mathrm{P}(X = 2) = 0.201$ (to 3 s.f.)
(ii)	P(a player wins the game) $= \frac{n}{40} \times \frac{1}{3} + \frac{40-n}{40} \times \frac{3}{10}$ $= \frac{10n}{1200} + \frac{360-9n}{1200}$ $= \frac{360+n}{1200}$ $\mathrm{f}(n)$ = P(player draws blue | player wins) $= \frac{\mathrm{P}(\text{player draws blue and wins})}{\mathrm{P}(\text{player wins})}$ $= \frac{\frac{40-n}{40} \times \frac{3}{10}}{\frac{360+n}{1200}}$

	$=\dfrac{120-3n}{400}\times\dfrac{1200}{360+n}$ $=\dfrac{3(120-3n)}{360+n}$ $=\dfrac{360-9n}{360+n}$ $=\dfrac{-9(360+n)+3600}{360+n}$ $=-9+\dfrac{3600}{360+n}$ As n increases, $\dfrac{3600}{360+n}$ decreases, hence f(n) decreases. Hence f is decreasing for all n, $0\le n\le 40$. This means that as the number of red counters increase, the probability that a winning player drew a blue counter decreases.
9(i)	
(ii)	(a) product moment correlation coefficient, $r=0.96346$ (b) product moment correlation coefficient, $r=0.98710$

(iii)	The second model $\ln y = c + d\ln x$ is the better model because its product moment correlation coefficient is closer to one as compared to the product moment correlation coefficient of the first model. From the scatter plot, it can be seen that the data seems to indicate a non-linear (curvilinear) relationship between y and x. Hence the model $y = a + bx$ is not appropriate.
(iv)	We have to use the regression line $\ln y$ against $\ln x$. From GC, the equation is $\ln y = -2.5866 + 2.4665\ln x$ When $x = 20$, $\ln y = -2.5866 + 2.4665\ln 20$ $y = e^{4.8025} = 121.82 = 121$ $x = 20$ is outside the data range and hence the relationship $\ln y = c + d\ln x$ may not hold. Hence the estimate may not be reliable.
10(i)	Assumptions 1. Every eraser is equally likely to be blue. 2. The colour of a randomly selected eraser is independent of the colour of other erasers.
(ii)	Let Y be the number of blue erasers, out of 36. $Y \sim B(36, 0.20)$ $P(Y \le 6) = 0.40069 \approx 0.401$
(iii)	Let W be the number of boxes that contain at most six blue erasers, out of 200. $W \sim B(200, 0.40069)$ $P(W \ge 40\% \text{ of } 200) = P(W \ge 80) = 1 - P(W \le 79)$ $= 0.53477 \approx 0.535$
(iv)	Let T denote the number of cartons where each carton contains at least 40% of the boxes that contains at most six blue erasers per box. $T \sim B(150, 0.53477)$ $E(T) = 150 \times 0.53477 = 80.216$ $\text{Var}(T) = 150 \times 0.53477 \times (1 - 0.53477) = 37.319$

	Since n is large $(n=30)$, by the Central Limit Theorem, $\overline{T}=\frac{T_1+T_2+...+T_{30}}{30}\sim N(80.216,\frac{37.319}{30})$ approximately. $P(\overline{T}<80)=0.423219\approx 0.423$ (3 sig. fig.)
(v)	Let R be the number of blue erasers, out of 36. $R\sim B(36,p)$ $P(R=1)=\binom{36}{1}p^1(1-p)^{35}=36p(1-p)^{35}$
(vi)	$P(R=2)=\binom{36}{2}p^2(1-p)^{34}=630p^2(1-p)^{34}$ $P(R=1)=2P(R=2)$ $36p(1-p)^{35}=2\times 630p^2(1-p)^{34}$ $36(1-p)=1260p$ $1-p=35p$ $1=36p$ $p=\frac{1}{36}$

11(i)	Let T be the random variable " time taken in seconds for a computer to boot up", with population mean μ . Unbiased estimate of the population mean, $\bar{t}=\frac{802.5}{25}=32.1$ Unbiased estimate of the population variance, $s^2=\frac{1}{24}\left[26360.25-\frac{802.5^2}{25}\right]=25$
(ii)	A statistic is said to be an unbiased estimate of a given parameter when the mean of the sampling distribution of the statistic can be shown to be equal to the parameter being estimated. For example, $\mathrm{E}(\overline{X})=\mu$.
(iii)	Test H_0: $\mu=30$ against $H_1:\mu>30$ at the 5% level of significance. Under H_0, $\overline{T}\sim N(30,\frac{25}{25})$. Using GC, $\bar{t}=32.1$ gives rise to $z_{calc}=2.1$ and p-value = 0.0179 Since p-value = 0.0179 $\leq$ 0.05, we reject H_0 and conclude that there is sufficient evidence at the 5% significance level that the specification is not being met (<u>or</u> the computer requires more than 30 seconds to boot up). "5% significance level" is the probability of wrongly concluding that the mean boot up time for the computer is more than 30 seconds when in fact it is not more than 30 seconds.
(iv)	The critical value for the test is 31.645. For the specification to be met, H_0 is not rejected. $\bar{t}<31.6$ (3 s.f.) Since $\bar{t}>0$, Answer is $0<\bar{t}<31.6$.
(v)	Under H_0, $\overline{Y}\sim N\left(30,\frac{\sigma^2}{25}\right)$

	$Z = \dfrac{\overline{Y} - (30)}{\frac{\sigma}{\sqrt{25}}} \sim \mathrm{N}(0,1)$ Using a 1 – tailed z test, $z_{\text{calc}} = \dfrac{32.4 - 30}{\frac{\sigma}{5}} = \dfrac{12}{\sigma}$, $z_{\text{crit}} = 1.64485$ In order not to reject H0, $z_{\text{calc}} < 1.64485$ $\dfrac{12}{\sigma} < 1.64485$ $\sigma > 7.2955$ $\sigma > 7.30$