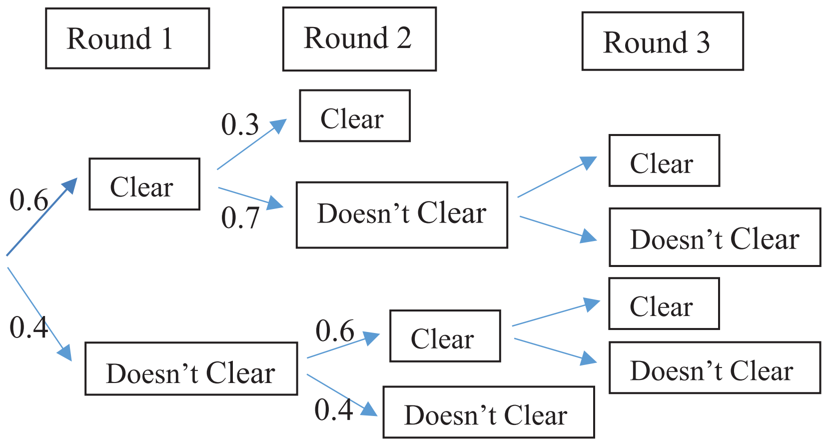
Qn	Solution	Mark Scheme
5	**Discrete Random Variables**	**[6]**
(i)	$u+v+u+v=1 \Rightarrow v=0.5-u$ $\mathrm{E}(X)=2u+3v+4u+5v$ $=6u+8v$ $=6u+8(0.5-u)$ $=4-2u$	
(ii)	$\mathrm{E}(X^2)=2^2u+3^2v+4^2u+5^2v$ $=20u+34v$ $=20u+34(0.5-u)$ $=20u+17-34u$ $=17-14u$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$ $=(17-14u)-(4-2u)^2$ $=1.16$ Using GC, $u=0.1$ or $u=0.4$. Since $u>v$, $u=0.4$ and $v=0.1$.	

Qn	**Solution**	**Mark Scheme**
6	**P&C and Probability**	**[9]**
(i)	 $\mathrm{P}(\text{a player plays 3 rounds})$ $= \mathrm{P}(\text{clears round 1 but does not clear round 2})$ $+\mathrm{P}(\text{does not clear round 1 but clears round 2})$ $=(0.6)(0.7)+(0.4)(0.6)=0.66$	
(ii)	$\mathrm{P}(\text{clears round 1} \mid \text{does not clear round 2})$ $=\dfrac{\mathrm{P}(\text{clears round 1 and does not clear round 2})}{\mathrm{P}(\text{does not clear round 2})}$ $=\dfrac{(0.6)(0.7)}{(0.6)(0.7)+(0.4)(0.4)}$ $=\dfrac{21}{29}=0.724$ (3 s.f)	
(iii)	$\mathrm{P}(\text{a player clears exactly 2 rounds})$ $=\mathrm{P}(\text{clears round 1 and round 2})$ $+\mathrm{P}(\text{clears round 1, does not clear round 2, clears round 3})$ $+\mathrm{P}(\text{does not clear round 1, clears round 2 and round 3})$ $=(0.6)(0.3)+0.2$ $=0.38$	
(iv)	Number of ways for last digit = 5 Number of ways required = 9 × 8 × 7 × 6 × 5 × 5 = 75600	
(v)	Number of ways for 3 odd digits = $^5C_3$ = 10 Number of ways for 3 even digits = $^5C_3$ = 10 Number of ways required = 10 × 10 × 6! = 72000	

Qn	Solution	Mark Scheme
7	**Hypothesis Testing**	**[10]**
	Let X be the weight of a randomly chosen mini bread (in grams). Let μ denote the population mean weight of mini breads (in grams) Unbiased estimate of population mean, $\bar{x} = \frac{3571}{80}$ $= 44.6375$ Unbiased estimate of population variance, $s^2 = \frac{1}{79}\left(159701 - \frac{(3571)^2}{80}\right)$ $= 3.8036$ $= 3.80$ (3 s.f.)	
	$\text{H}_0: \mu = 45$ $\text{H}_1: \mu < 45$ Under H_0, since $n = 80$ is large, by Central Limit Theorem, $\bar{X} \sim \text{N}\left(45, \frac{3.8036}{80}\right)$ approximately Test Statistic: $Z = \frac{\bar{X} - 45}{\sqrt{\frac{3.8036}{80}}}$ Level of significance : 4 % Reject H_0 if p - value < 0.04 Using GC, p-value = 0.0482 Since p-value = $0.0482 > 0.04$, we do not reject H_0 and conclude that there is insufficient evidence, at the 4% level of significance, that the population mean weight is less than 45 grams. Thus, the customer's claim is not supported at the 4% significance level.	
	Sample mean based on combined sample $= \frac{\sum x + 20k}{80 + 20} = \frac{3571 + 20k}{100}$ $\text{H}_0: \mu = 45$ $\text{H}_1: \mu < 45$ Under H_0, $X \sim \text{N}(45, 1.5^2) \Rightarrow \bar{X} \sim \text{N}\left(45, \frac{1.5^2}{100}\right)$ Test statistic: $Z = \frac{\bar{X} - 45}{\sqrt{\frac{1.5^2}{100}}}$ Level of significance : 4 % Reject H_0 if $z - \text{value} < -1.7507$ $z - \text{value} = \frac{\bar{x} - 45}{\sqrt{\frac{1.5^2}{100}}}$	

Since there is sufficient evidence that the customer's claim is valid at 4% level of significance, H0 is rejected

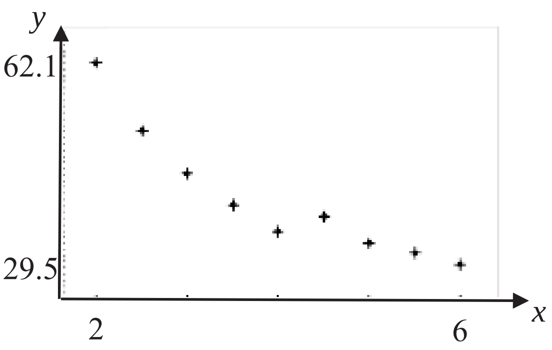
$$\frac{\overline{x}-45}{\sqrt{\frac{1.5^2}{100}}} < -1.7507$$

$$\overline{x} < 44.737395$$

$$\frac{3571+20k}{100} < 44.737395$$

$$k < 45.137$$

$$k < 45.1 \quad \text{(3 s.f)}$$

Qn	Solution	Mark Scheme
8	**Correlation and Regression**	**[10]**
(i)	$\overline{x}=\frac{2.0+2.5+3.0+3.5+4.0+4.5+5.0+5.5+6.0}{9}=4$ $\overline{y}=68.8067-7.12667(4)=40.30002$ $\frac{62.1+51.2+44.1+39.1+35+k+33+31.4+29.5}{9}=40.30002$ $k=37.30018$ $=37.3$ (to 1 d.p) (shown)	
(ii)		
(iii)	Model **(A)**: $r=-0.922$ Model **(B)**: $r=-0.866$ Model **(C)**: $r=0.990$ Since the value of $\lvert r\rvert=0.990$ for Model (C) is closest to 1, Model (C) is the best model. Using GC, equation of suitable regression line: $y=13.685+\frac{94.313}{x}$ $y=13.7+\frac{94.3}{x}$ (3 s.f)	
(iv)	When $x=4.2$, $y=13.685+\frac{94.313}{4.2}=36.1\ (3\text{ s.f})$ Since $x=4.2$ is within the data range of x **and** $\lvert r\rvert=0.990$ is close to 1, the estimated reaction time is reliable.	

Qn	Solution	Mark Scheme
9	**Normal and Sampling Distributions**	**[12]**
(i)	Let A be the mass of a randomly chosen Alpha apple in grams. Given $A \sim \mathrm{N}\left(\mu, \sigma^2\right)$ $\bar{A} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{8}\right)$ and $A_1 + A_2 + A_3 + \ldots + A_8 \sim \mathrm{N}\left(8\mu, 8\sigma^2\right)$ $\mathrm{P}\left(A_1 + A_2 + \ldots + A_8 > 3000\right) = 0.5 \Rightarrow 8\mu = 3000 \Rightarrow \mu = 375$ $\mathrm{P}\left(\bar{A} < 370\right) = 0.25$ $\mathrm{P}\left(Z < \frac{370-\mu}{\sqrt{\frac{\sigma^2}{8}}}\right) = 0.25$ $\frac{370-375}{\sqrt{\frac{\sigma^2}{8}}} = -0.67449$ $\sigma = 21.0$ (3s.f)	
(ii)	$A \sim \mathrm{N}\left(380, 20^2\right)$ Let $S = A_1 + A_2 + \ldots + A_8 \sim \mathrm{N}\left(8 \times 380, 8 \times 20^2\right) = \mathrm{N}(3040, 3200)$ $\mathrm{P}(2900 < S < 3100) = 0.849$ (3s.f.)	
(iii)	Let B be the mass of a randomly chosen Beta apple in grams. $B \sim \mathrm{N}\left(250, 18^2\right)$ Let $T = B_1 + B_2 + \ldots + B_{12} \sim \mathrm{N}\left(12 \times 250, 12 \times 18^2\right) = \mathrm{N}(3000, 3888)$. Let $C = 0.7S + 0.8T$. $\mathrm{E}(C) = 0.7 \times 3040 + 0.8 \times 3000 = 4528$ $\mathrm{Var}(C) = 0.7^2 \times 3200 + 0.8^2 \times 3888 = 4056.32$ $\therefore C \sim \mathrm{N}(4528, 4056.32)$ $\mathrm{P}(C > 4500) = 0.670$ (3 s.f)	
(iv)	Assume that the distributions of the masses of all apples are independent of one another.	
	$p = \mathrm{P}(0.9C > 4100) = \mathrm{P}(C > 4555.6) < \mathrm{P}(C > 4500)$ Thus p is lower than the answer in part **(iii)**.	

Qn	Solution	Mark Scheme
10	**Binomial Distribution and Probability**	**[13]**
(a)	The **conditional** probability, p, that a randomly chosen sweet is red is not the same for the 1st to the 5th sweets. For example, for the first sweet, $p=\frac{6}{10}$. For the second sweet, $p=\frac{5}{9}$ if the first sweet is red and $p=\frac{6}{9}$ if the first sweet is not red. Hence, whether a randomly chosen sweet is red or not is **not independent** of other sweets.	
(b)(i)	Let X be the no of sweets, out of 10, that are red. $X \sim \mathrm{B}(10,p)$ Given that $\mathrm{P}(X=5)=0.21253$, $\binom{10}{5}p^5(1-p)^5=0.21253$ $p^5(1-p)^5=0.00084337$ $p(1-p)=0.24277$ $k=0.24277=0.243 \quad (3\text{ s.f.})$ Using GC, $p=0.415 \quad \text{or} \quad 0.585 \quad (3\text{ s.f.})$	
(ii)	$X \sim \mathrm{B}(10,0.6)$ $\mathrm{P}(X \le 8 \mid X>2)$ $=\frac{\mathrm{P}(2<X\le 8)}{\mathrm{P}(X>2)}$ $=\frac{\mathrm{P}(X\le 8)-\mathrm{P}(X\le 2)}{1-\mathrm{P}(X\le 2)}$ $=\frac{0.94135}{0.98771}$ $=0.953$	
(iii)	Required Probability $=\mathrm{P}(X\le 5)\times \mathrm{P}(X\ge 5)\times 2-\mathrm{P}(X=5)^2$ $=\mathrm{P}(X\le 5)\times(1-\mathrm{P}(X\le 4))\times 2-\mathrm{P}(X=5)^2$ $=0.572$ **OR** Required Probability $=\mathrm{P}(X\le 5)\times \mathrm{P}(X>5)\times 2+\mathrm{P}(X=5)\times \mathrm{P}(X<5)\times 2+\mathrm{P}(X=5)^2$ $=0.572$	

(iv)	$\mathrm{P}\left(\lvert X_1 - X_2 \rvert \geq 8\right)$ $= \left[\underbrace{\mathrm{P}\left(X_1 = 10\right) \times \mathrm{P}\left(X_2 \leq 2\right)}_{\boxed{\text{Consider 3 cases}}} + \underbrace{\mathrm{P}\left(X_1 = 9\right) \times \mathrm{P}\left(X_2 \leq 1\right)}_{\boxed{\text{Consider 2 cases}}} + \mathrm{P}\left(X_1 = 8\right) \times \mathrm{P}\left(X_2 = 0\right) \right] \times 2$ $= 0.000309$	