Topic 13 Permutation and Combinations

1 Find the number of ways in which the letters of the word ELEVATED can be arranged if

(i) there are no restrictions, [1]

(ii) T and D must not be next to one another, [2]

(iii) consonants (L, V, T, D) and vowels (E, A) must alternate, [3]

(iv) between any two Es there must be at least 2 other letters. [3]

(2009/P2/8)

---

2 A team in a particular sport consists of 1 goalkeeper, 4 defenders, 2 midfielders and 4 attackers. A certain club has 3 goalkeepers, 8 defenders, 5 midfielders and 6 attackers.

(i) How many different teams can be formed by the club? [2]

One of the midfielders in the club is the brother of one of the attackers in the club.

(ii) How many different teams can be formed which include exactly one of the two brothers? [3]

The two brothers leave the club. The club manager decides that one of the remaining midfielders can play as either a midfielder or as a defender.

(iii) How many different teams can now be formed by the club? [3]

(2014/P2/6)

---

3 This question is about arrangements of all eight letters in the word CABBAGES.

(i) Find the number of different arrangements of the eight letters that can be made. [2]

(ii) Write down the number of these arrangements in which the letters are **not** in alphabetical order. [1]

(iii) Find the number of different arrangements that can be made with both the A's together and both the B's together. [2]

(iv) Find the number of different arrangements that can be made with no two adjacent letters the same. [4]

(2015/P2/11)

---

4 A children's game is played with 20 cards, consisting of 5 sets of 4 cards. Each set consists of a father, mother, daughter and son from the same family. The family names are Red, Blue, Green, Yellow and Orange. So, for example, the Red family cards are father Red, mother Red, daughter Red and son Red.

The 20 cards are arranged in a row.

(i) In how many different ways can the 20 cards be arranged so that the 4 cards in each family set are next to each other? [2]

(ii) In how many different ways can the cards be arranged so that all five father cards are next to each other, all four Red family cards are next to each other and all four Blue family cards are next to each other? [3]

The cards are now arranged at random in a circle.

(iii) Find the probability that no two father cards are next to each other. [4]

(2017/P2/6)

Topic 14 Probability

1 A company buys $p\%$ of its electronic components from supplier A and the remaining $(100 - p)\%$ from supplier B. The probability that a randomly chosen component supplied by A is faulty is 0.05. The probability that a randomly chosen component supplied by B is faulty is 0.03.

(i) Given that $p = 25$, find the probability that a randomly chosen component is faulty. [2]

(ii) For a general value of p, the probability that a randomly chosen component that is faulty was supplied by A is denoted by $\mathrm{f}(p)$. Show that $\mathrm{f}(p) = \dfrac{0.05p}{0.02p + 3}$. Prove by differentiation that f is an increasing function for $0 \leqslant p \leqslant 100$, and explain what this statement means in the context of the question. [6]

(2009/P2/7)

---

2 For events A and B it is given that $\mathrm{P}(A) = 0.7$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \mid B') = 0.8$. Find

(i) $\mathrm{P}(A \cap B')$, [2]

(ii) $\mathrm{P}(A \cup B)$, [2]

(iii) $\mathrm{P}(B' \mid A)$. [2]

For a third event C, it is given that $\mathrm{P}(C) = 0.5$ and that A and C are independent.

(iv) Find $\mathrm{P}(A' \cap C)$. [2]

(v) Hence state an inequality satisfied by $\mathrm{P}(A' \cap B \cap C)$. [1]

(2010/P2/7)

---

3 The digits 1, 2, 3, 4 and 5 are arranged randomly to form a five-digit number. No digit is repeated. Find the probability that

(i) the number is greater than 30 000, [1]

(ii) the last two digits are both even, [2]

(iii) the number is greater than 30 000 and odd. [4]

(2010/P2/8)

---

4 Camera lenses are made by two companies, A and B. 60% of all lenses are made by A and the remaining 40% by B. 5% of the lenses made by A are faulty. 7% of the lenses made by B are faulty.

(i) One lens is selected at random. Find the probability that

(a) it is faulty, [2]

(b) it was made by A, given that it is faulty. [1]

(ii) Two lenses are selected at random. Find the probability that

(a) exactly one of them is faulty, [2]

(b) both were made by A, given that exactly one is faulty. [3]

(2011/P2/9)

---

5 A committee of 10 people is chosen at random from a group consisting of 18 women and 12 men. The number of women on the committee is denoted by R.

(i) Find the probability that $R = 4$. [3]

(ii) The most probable number of women on the committee is denoted by r. By using the fact that $P(R = r) > P(R = r + 1)$, show that r satisfies the inequality

$$(r+1)!(17-r)!(9-r)!(r+3)! > r!(18-r)!(10-r)!(r+2)!$$

and use this inequality to find the value of r. [5]

(2011/P2/11)

---

6 The probability that a hospital patient has a particular disease is 0.001. A test for the disease has probability p of giving a positive result when the patient has the disease, and equal probability p of giving a negative result when the patient does not have the disease. A patient is given the test.

(i) Given that $p = 0.995$, find the probability that

(a) the result of the test is positive, [2]

(b) the patient has the disease given that the result of the test is positive. [2]

(ii) It is given instead that there is a probability of 0.75 that the patient has the disease given that the result of the test is positive. Find the value of p, giving your answer correct to 6 decimal places. [3]

(2012/P2/5)

---

7 A group of fifteen people consists of one pair of sisters, one set of three brothers and ten other people. The fifteen people are arranged randomly in a line.

(i) Find the probability that the sisters are next to each other. [2]

(ii) Find the probability that the brothers are *not* all next to one another. [2]

(iii) Find the probability that the sisters are next to each other and the brothers *are* all next to one another. [2]

(iv) Find the probability that *either* the sisters are next to each other *or* the brothers are all next to one another *or* both. [2]

Instead the fifteen people are arranged in a circle.

(v) Find the probability that the sisters are next to each other. [1]

(2012/P2/7)

---

8 For events A and B it is given that $P(A) = 0.7$, $P(B \mid A') = 0.8$ and $P(A \mid B') = 0.88$. Find

(i) $P(B \cap A')$, [1]

(ii) $P(A' \cap B')$, [2]

(iii) $P(A \cap B)$. [3]

(2013/P2/8)

---

9 A machine is used to generate codes consisting of three letters followed by two digits. Each of the three letters generated is equally likely to be any of the twenty-six letters of the alphabet A–Z. Each of the two digits generated is equally likely to be any of the nine digits 1–9. The digit 0 is not used. Find the probability that a randomly chosen code has

(i) three different letters and two different digits, [2]

(ii) the second digit higher than the first digit, [2]

(iii) exactly two letters the same or two digits the same, but not both, [4]

(iv) exactly one vowel (A, E, I, O or U) and exactly one even digit. [4]

(2013/P2/11)

---

10 A game has three sets of ten symbols, and one symbol from each set is randomly chosen to be displayed on each turn. The symbols are as follows.

Set 1	+	+	+	+	×	×	×	○	○	★
Set 2	+	+	+	×	○	○	○	○	★	★
Set 3	+	+	×	×	×	×	○	○	○	★

For example, if a + symbol is chosen from set 1, a ○ symbol is chosen from set 2 and a ★ symbol is chosen from set 3, the display would be + ○ ★.

(i) Find the probability that, on one turn,

(a) ★ ★ ★ is displayed, [1]

(b) at least one ★ symbol is displayed, [2]

(c) two × symbols and one + symbol are displayed, in any order. [3]

(ii) Given that exactly one of the symbols displayed is ★, find the probability that the other two symbols are + and ○. [4]

(2014/P2/10)

---

11 For events A, B and C it is given that $\mathrm{P}(A) = 0.45$, $\mathrm{P}(B) = 0.4$, $\mathrm{P}(C) = 0.3$ and $\mathrm{P}(A \cap B \cap C) = 0.1$. It is also given that events A and B are independent, and that events A and C are independent.

(i) Find $\mathrm{P}(B \mid A)$. [1]

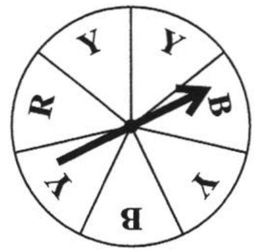
(ii) Given also that events B and C are independent, find $\mathrm{P}(A' \cap B' \cap C')$. [3]

(iii) Given instead that events B and C are **not** independent, find the greatest and least possible values of $\mathrm{P}(A' \cap B' \cap C')$. [4]

(2015/P2/9)

---

12 In a game of chance, a player has to spin a fair spinner. The spinner has 7 sections and an arrow which has an equal chance of coming to rest over any of the 7 sections. The spinner has 1 section labelled **R**, 2 sections labelled **B** and 4 sections labelled **Y** (see diagram).



The player then has to throw one of three fair six-sided dice, coloured red, blue or yellow. If the spinner comes to rest over **R** the red die is thrown, if the spinner comes to rest over **B** the blue die is thrown and if the spinner comes to rest over **Y** the yellow die is thrown. The yellow die has one face with $*$ on it, the blue die has two faces with $*$ on it and the red die has three faces with $*$ on it. The player wins the game if the die thrown comes to rest with a face showing $*$ uppermost.

(i) Find the probability that a player wins a game. [2]

(ii) Given that a player wins a game, find the probability that the spinner came to rest over **B**. [1]

(iii) Find the probability that a player wins 3 consecutive games, each time throwing a die of a different colour. [2]

(2016/P2/5)

---

13 The management board of a company consists of 6 men and 4 women. A chairperson, a secretary and a treasurer are chosen from the 10 members of the board. Find the number of ways the chairperson, the secretary and the treasurer can be chosen so that

(i) they are all women, [1]

(ii) at least one is a woman and at least one is a man. [3]

The 10 members of the board sit at random around a table. Find the probability that

(iii) the chairperson, the secretary and the treasurer sit in three adjacent places, [3]

(iv) the chairperson, the secretary and the treasurer are all separated from each other by at least one other person. [3]

(2016/P2/7)

Topic 15 Discrete Random Variables

1 A bag contains 6 red counters and 3 yellow counters. In a game, Lee removes counters at random from the bag, one at a time, until he has taken out 2 red counters. The total number of counters Lee removes from the bag is denoted by T.

(i) Find $P(T = t)$ for all possible values of t. [3]

(ii) Find $E(T)$ and $Var(T)$. [2]

Lee plays this game 15 times.

(iii) Find the probability that Lee has to take at least 4 counters out of the bag in at least 5 of his 15 games. [2]

(2017/P2/5)

Topic 16 Binomial and Normal Distributions

1 The thickness in cm of a mechanics textbook is a random variable with the distribution $N(2.5, 0.1^2)$.

(i) The mean thickness of n randomly chosen mechanics textbooks is denoted by $\overline{M}$ cm. Given that $P(\overline{M} > 2.53) = 0.0668$, find the value of n. [3]

The thickness in cm of a statistics textbook is a random variable with the distribution $N(2.0, 0.08^2)$.

(ii) Calculate the probability that 21 mechanics textbooks and 24 statistics textbooks will fit onto a bookshelf of length 1 m. State clearly the mean and variance of any normal distribution you use in your calculation. [3]

(iii) Calculate the probability that the total thickness of 4 statistics textbooks is less than three times the thickness of 1 mechanics textbook. State clearly the mean and variance of any normal distribution you use in your calculation. [3]

(iv) State an assumption needed for your calculations in parts **(ii)** and **(iii)**. [1]

(2009/P2/9)

---

2 A fixed number, n, of cars is observed and the number of those cars that are red is denoted by R.

(i) State, in context, two assumptions needed for R to be well modelled by a binomial distribution. [2]

Assume now that R has the distribution $B(n, p)$.

(ii) Given that $n = 20$ and $p = 0.15$, find $P(4 \leqslant R < 8)$. [2]

(iii) Given that $n = 20$ and $P(R = 0 \text{ or } 1) = 0.2$, write down an equation for the value of p, and find this value numerically. [2]

(2009/P2/11i,ii,v)

---

3 In this question you should state clearly the values of the parameters of any normal distribution you use.

Over a three-month period Ken makes X minutes of peak-rate telephone calls and Y minutes of cheap-rate calls. X and Y are independent random variables with the distributions $N(180, 30^2)$ and $N(400, 60^2)$ respectively.

(i) Find the probability that, over a three-month period, the number of minutes of cheap-rate calls made by Ken is more than twice the number of minutes of peak-rate calls. [4]

Peak-rate calls cost \$0.12 per minute and cheap-rate calls cost \$0.05 per minute.

(ii) Find the probability that, over a three-month period, the total cost of Ken's calls is greater than \$45. [3]

(iii) Find the probability that the total cost of Ken's peak-rate calls over two independent three-month periods is greater than \$45. [3]

(2010/P2/9)

---

4 The continuous random variable X has the distribution $N(\mu, \sigma^2)$. It is known that $P(X < 40.0) = 0.05$ and $P(X < 70.0) = 0.975$. Calculate the values of μ and σ. [4]

(2011/P2/5)

---

5 When I try to contact (by telephone) any of my friends in the evening, I know that on average the probability that I succeed is 0.7. On one evening I attempt to contact a fixed number, n, of different friends. If I do not succeed with a particular friend, I do not attempt to contact that friend again that evening. The number of friends whom I succeed in contacting is the random variable R.

(i) State, in the context of this question, two assumptions needed to model R by a binomial distribution. [2]

(ii) Explain why one of the assumptions stated in part **(i)** may not hold in this context. [1]

Assume now that these assumptions do in fact hold.

(iii) Given that $n = 8$, find the probability that R is at least 6. [1]

(2011/P2/7i-iii)

---

6 In an opinion poll before an election, a sample of 30 voters is obtained.

(i) The number of voters in the sample who support the Alliance Party is denoted by A. State, in context, what must be assumed for A to be well modelled by a binomial distribution. [2]

Assume now that A has the distribution $B(30, p)$.

(ii) Given that $p = 0.15$, find $P(A = 3 \text{ or } 4)$. [2]

(iii) For an unknown value of p it is given that $P(A = 15) = 0.06864$ correct to 5 decimal places. Show that p satisfies an equation of the form $p(1-p) = k$, where k is a constant to be determined. Hence find the value of p to a suitable degree of accuracy, given that $p < 0.5$. [5]

(2012/P2/9i,ii,iv)

---

7 The continuous random variable Y has the distribution $N(\mu, \sigma^2)$. It is known that $P(Y < 2a) = 0.95$ and $P(Y < a) = 0.25$. Express μ in the form ka, where k is a constant to be determined. [4]

(2013/P2/6)

---

8 On average one in 20 packets of a breakfast cereal contains a free gift. Jack buys n packets from a supermarket. The number of these packets containing a free gift is the random variable F.

(i) State, in context, two assumptions needed for F to be well modelled by a binomial distribution. [2]

Assume now that F has a binomial distribution.

(ii) Given that $n = 20$, find $P(F = 1)$. [1]

(2013/P2/7i,ii)

---

9 Yan is carrying out an experiment with a fair 6-sided die and a biased 6-sided die, each numbered from 1 to 6.

Yan rolls the fair die 10 times. Find the probability that it shows a 6 exactly 3 times. [1]

(2014/P2/7i)

---

10 'Droppers' are small sweets that are made in a variety of colours. Droppers are sold in packets and the colours of the sweets in a packet are independent of each other. On average, 25% of Droppers are red.

(i) A small packet of Droppers contains 10 sweets. Find the probability that there are at least 4 red sweets in a small packet. [2]

A large packet of Droppers contains 100 sweets.

(ii) Yip buys 15 large packets of Droppers. Find the probability that no more than 3 of these packets contain at least 30 red sweets. [2]

(2015/P2/6i,iii)

---

11 In this question you should state clearly the values of the parameters of any normal distribution you use.

The masses in grams of apples have the distribution $N(300, 20^2)$ and the masses in grams of pears have the distribution $N(200, 15^2)$. A certain recipe requires 5 apples and 8 pears.

(i) Find the probability that the total mass of 5 randomly chosen apples is more than 1600 grams. [2]

(ii) Find the probability that the total mass of 5 randomly chosen apples is more than the total mass of 8 randomly chosen pears. [3]

The recipe requires the apples and pears to be prepared by peeling them and removing the cores. This process reduces the mass of each apple by 15% and the mass of each pear by 10%.

(iii) Find the probability that the total mass, after preparation, of 5 randomly chosen apples and 8 randomly chosen pears is less than 2750 grams. [4]

(2015/P2/12)

---

12 **(a)** The random variable X has distribution $N(15, a^2)$ and $P(10 < X < 20) = 0.5$. Find the value of a. [2]

(b) The random variable Y has distribution $B(4, p)$ and $P(Y = 1) + P(Y = 2) = 0.5$. Show that $4p^4 - 12p^2 + 8p = 1$ and hence find the possible values of p. [4]

(2016/P2/9a,b)

---

13 On average 8% of a certain brand of kitchen lights are faulty. The lights are sold in boxes of 12.

(i) State, in context, two assumptions needed for the number of faulty lights in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty lights in a box has a binomial distribution.

(ii) Find the probability that a box of 12 of these kitchen lights contains at least 1 faulty light. [1]

The boxes are packed into cartons. Each carton contains 20 boxes.

(iii) Find the probability that each box in one randomly selected carton contains at least one faulty light. [1]

(iv) Find the probability that there are at least 20 faulty lights in a randomly selected carton. [2]

(v) Explain why the answer to part **(iv)** is greater than the answer to part **(iii)**. [1]

The manufacturer introduces a quick test to check if lights are faulty. Lights identified as faulty are discarded. If a light is faulty there is a 95% chance that the quick test will correctly identify the light as faulty. If the light is not faulty, there is a 6% chance that the quick test will incorrectly identify the light as faulty.

(vi) Find the probability that a light identified as faulty by the quick test is **not** faulty. [3]

(vii) Find the probability that the quick test correctly identifies lights as faulty or not faulty. [1]

(viii) Discuss briefly whether the quick test is worthwhile. [1]

(2017/P2/9)

---

14 A small component for a machine is made from two metal spheres joined by a short metal bar. The masses in grams of the spheres have the distribution $N(20, 0.5^2)$.

(i) Find the probability that the mass of a randomly selected sphere is more than 20.2 grams. [1]

In order to protect them from rusting, the spheres are given a coating which increases the mass of each sphere by 10%.

(ii) Find the probability that the mass of a coated sphere is between 21.5 and 22.45 grams. State the distribution you use and its parameters. [3]

(iii) The masses of the metal bars are normally distributed such that 60% of them have a mass greater than 12.2 grams and 25% of them have a mass less than 12 grams. Find the mean and standard deviation of the masses of metal bars. [4]

(iv) The probability that the total mass of a component, consisting of two randomly chosen coated spheres and one randomly chosen bar, is more than k grams is 0.75. Find k, stating the parameters of any distribution you use. [4]

(2017/P2/10)

Topic 17 Hypothesis Testing

1 A company supplies sugar in small packets. The mass of sugar in one packet is denoted by X grams. The masses of a random sample of 9 packets are summarised by

$$\Sigma x = 86.4, \quad \Sigma x^2 = 835.92.$$

(i) Calculate unbiased estimates of the mean and variance of X. [2]

The mean mass of sugar in a packet is claimed to be 10 grams. The company directors want to know whether the sample indicates that this claim is incorrect.

(ii) Suppose now that the population variance of X is known, and that the assumption made in part **(ii)** is still valid. What change would there be in carrying out the test? [1]

(2009/P2/10i,iii)

---

2 The time required by an employee to complete a task is a normally distributed random variable. Over a long period it is known that the mean time required is 42.0 minutes. Background music is introduced in the workplace, and afterwards the time required, t minutes, is measured for a random sample of 11 employees. The results are summarised as follows.

$$n = 11 \qquad \Sigma t = 454.3 \qquad \Sigma t^2 = 18\,778.43$$

Find unbiased estimates of the population mean and variance. [7]

(2010/P2/6)

---

3 In a factory, the time in minutes for an employee to install an electronic component is a normally distributed continuous random variable T. The standard deviation of T is 5.0 and under ordinary conditions the expected value of T is 38.0. After background music is introduced into the factory, a sample of n components is taken and the mean time taken for randomly chosen employees to install them is found to be $\bar{t}$ minutes. A test is carried out, at the 5% significance level, to determine whether the mean time taken to install a component has been reduced.

(i) State appropriate hypotheses for the test, defining any symbols you use. [2]

(ii) Given that $n = 50$, state the set of values of $\bar{t}$ for which the result of the test would be to reject the null hypothesis. [3]

(iii) It is given instead that $\bar{t} = 37.1$ and the result of the test is that the null hypothesis is not rejected. Obtain an inequality involving n, and hence find the set of values that n can take. [4]

(2011/P2/10)

---

4 On a remote island a zoologist measures the tail lengths of a random sample of 20 squirrels. In a species of squirrel known to her, the tail lengths have mean 14.0 cm. She carries out a test, at the 5% significance level, of whether squirrels on the island have the same mean tail length as the species known to her. She assumes that the tail lengths of squirrels on the island are normally distributed with standard deviation 3.8 cm.

(i) State appropriate hypotheses for the test. [1]

The sample mean tail length is denoted by $\bar{x}$ cm.

(ii) Use an algebraic method to calculate the set of values of $\bar{x}$ for which the null hypothesis would not be rejected. (Answers obtained by trial and improvement from a calculator will obtain no marks.) [3]

(iii) State the conclusion of the test in the case where $\bar{x} = 15.8$. [2]

(2012/P2/6)

---

5 A motoring magazine editor believes that the figures quoted by car manufacturers for distances travelled per litre of fuel are too high. He carries out a survey into this by asking for information from readers. For a certain model of car, 8 readers reply with the following data, measured in km per litre.

14.0 12.5 11.0 11.0 12.5 12.6 15.6 13.2

Calculate unbiased estimates of the population mean and variance. [2]

(2013/P2/9i)

---

6 A market stall sells pineapples which have masses that are normally distributed. The stall owner claims that the mean mass of the pineapples is at least 0.9 kg. Nur buys a random selection of 8 pineapples from the stall. The 8 pineapples have masses, in kg, as follows.

0.80 1.00 0.82 0.85 0.93 0.96 0.81 0.89

Find unbiased estimates of the population mean and variance of the mass of pineapples. [7]

(2015/P2/8)

---

7 The number of employees of a company, classified by department and gender, is shown below.

	Production	Development	Administration	Finance
Male	2345	1013	237	344
Female	867	679	591	523

The Company Secretary obtains a suitable sample of 80 employees in order to carry out a hypothesis test of the Managing Director's belief that the mean age of the employees now is less than 37 years. You are given that the population variance of the ages is 140 years^2.

(i) Write down appropriate hypotheses to test the Managing Director's belief. You are given that the result of the test, using a 5% significance level, is that the Managing Director's belief should be accepted. Determine the set of possible values of the mean age of the sample of employees. [4]

(ii) You are given instead that the mean age of the sample of employees is 35.2 years, and that the result of a test at the α% significance level is that the Managing Director's belief should not be accepted. Find the set of possible values of α. [3]

(2016/P2/6iii,iv)

8 The production manager of a food manufacturing company wishes to take a random sample of a certain type of biscuit bar from the thousands produced one day at his factory, for quality control purposes. He wishes to check that the mean mass of the bars is 32 grams, as stated on the packets.

(i) State what it means for a sample to be random in this context. [1]

The masses, x grams, of a random sample of 40 biscuit bars are summarised as follows.

$$n = 40 \qquad \Sigma(x - 32) = -7.7 \qquad \Sigma(x - 32)^2 = 11.05$$

(ii) Calculate unbiased estimates of the population mean and variance of the mass of biscuit bars. [2]

(iii) Test, at the 1% level of significance, the claim that the mean mass of biscuit bars is 32 grams. You should state your hypotheses and define any symbols you use. [5]

(iv) Explain why there is no need for the production manager to know anything about the population distribution of the masses of the biscuit bars. [2]

(2017/P2/7)