1. **[YJC 17 Promos]**
   (a) Using an algebraic method, solve the inequality
   $$4+\frac{1}{x^2-1}\leq\frac{-12}{x-1}.$$
   [5]

   (b) Hence, solve the inequality
   $$4+\frac{1}{x-1}\leq\frac{-12}{\sqrt{x}-1}.$$
   [2]

2. **[YJC 17 Promos]**
   (a) As an engineer in FreshWater Company, Mr Lim monitors the volume of water in the tank for public usage. The tank is initially filled with 5000 litres of water. At the start of each day, Mr Lim will add in 500 litres of water into the tank. In other words, the volume of water in the tank at the beginning of Day 1 is 5500 litres. At the end of each day, 15% of the volume of water in the tank at the start of that day will be used.
      i. Show that the volume of water in the tank at the end of nth day is given by
      $$0.85^n(5500)+\frac{8500(1-0.85^{n-1})}{3}.$$
      Hence, find the least number of days for the water in the tank to be less than 3000 litres. [3]
      ii. Explain why the tank will never dry up and state the volume of water that the tank will have in the long run. [2]

   (b) Mrs Tan who is also working in FreshWater Company, is a quality control engineer. The company gives her a project which requires 10 000 litres of water to be tested on the first day. On each subsequent day, the water tested is 18 litres less than on the previous day. The testing will continue daily up to and including the day when the volume of water tested is less than 12 litres.
      i. Find the total volume of water that she has tested after 20 days. [2]
      ii. Find the duration of her project. [2]

3. **[YJC 17 Promos]**
   A cake shop bakes 3 types of Swiss rolls, chocolate, vanilla and strawberry. Each chocolate, vanilla and strawberry Swiss roll is sold at \$3.60, \$3.20 and \$4.50 respectively. On a particular day, the shop baked 55 Swiss rolls. Two hours before closing, $\frac{2}{3}$ of the chocolate Swiss rolls, $\frac{7}{8}$ of the vanilla Swiss rolls and $\frac{3}{4}$ of the strawberry Swiss rolls were sold. The total amount collected at that time was \$150.10. In order to sell the remaining Swiss rolls, the cake shop offered a discount. All the chocolate and strawberry Swiss rolls were sold at 3 for the price of 2 and all the vanilla Swiss rolls were sold at 2 for the price of 1. All the Swiss rolls were sold and the total earnings for the day was \$183.90.

   Formulate the equations required and determine the number of each type of Swiss rolls baked on that day. [4]

4. **[YJC 17 Promos (modified)]**

   (a) Find $\sum_{r=1}^{2k}(r+1-2^{-r})$ in terms of k, simplifying your answer. [4]

   (b) i. Show that $\dfrac{1}{1+2^{n-1}} - \dfrac{1}{1+2^n} = \dfrac{2^{n-1}}{(1+2^{n-1})(1+2^n)}$. [1]

   ii. Hence show that $\sum_{n=1}^{N} \dfrac{2^{n-1}}{(1+2^{n-1})(1+2^n)} = \dfrac{1}{2} - \dfrac{1}{2^N}$. [3]

   iii. Give a reason why the series $\sum_{n=1}^{\infty} \dfrac{2^{n-1}}{(1+2^{n-1})(1+2^n)}$ converges and write down its value. [2]

5. **[RVHS 17 Promos (modified)]**

   (a) Without using a calculator, solve the inequality $\dfrac{x^2+4x+5}{x^2-2x-3} \leq 0$. [3]

   (b) Hence deduce the solution to the inequality $\dfrac{x^2+4|x|+5}{x^2-2|x|-3} \leq 0$. [2]

6. **[RVHS 17 Promos (modified)]**
   A sequence $U_1, U_2, U_3, \ldots$ is defined by $U_n = \dfrac{n}{\mathrm{e}^n}$.

   (a) Show that $U_{r+1} - U_r = \dfrac{r(1-e)+1}{\mathrm{e}^{r+1}}$. [1]

   (b) Hence find $\sum_{r=1}^{n} \left(\dfrac{r(1-e)+1}{\mathrm{e}^{r+1}}\right)$ in terms of n. [3]

   (c) Using the result in part (b), $\sum_{r=5}^{2} 0 \left(\dfrac{(r-1)(1-e)+1}{\mathrm{e}^r}\right)$, expressing your answer as a single fraction in terms of e. [3]

7. **[RVHS 17 Promos]**
   An arithmetic progression, A, has first term a and common difference d. The first, third and thirteenth terms of A are equal to the forth, third and second terms of a geometric progression, G, respectively.

   (a) Show that the common ratio of G is $\frac{1}{5}$. [3]

   (b) Explain if the sum to infinity to G exists. [1]

   (c) Calculate the difference of the sum of the first 20 terms of A and the sum of all the terms of G, giving your answers in terms of a.

8. **[SAJC 17 Promos]**
Sam plans to save \$900 on 1 January 2018. On the first day of each subsequent month he will save \$3 more than in the previous month, so that he will save \$903 on 1 February 2018, \$906 on 1 March 2018, and so on.

   (a) On what date will he first have saved over \$80 000 in total? [4]

   Sally will put \$900 on 1 January 2018 into a stock portfolio with a monthly growth rate of $r\%$, so that on the last day of each month, the amount in the portfolio on that day is increased by $r\%$. She will put a further \$900 into the portfolio on the first day of each subsequent month

   (b) Find an expression, in terms of r and n, for the value of the portfolio on the first day of the nth month (where January 2018 is the 1st month, February 2018 is the 2nd month, and so on). Hence, find the minimum monthly growth rate for Sallys portfolio such that the value of the portfolio on 2 January 2023 will exceed the amount Sam will have saved on 2 January 2023. [4]

9. **[PJC 17 Promos]**
A cubic curve passes through the points $\left(1, \frac{9}{2}\right)$ and $(-2, -9)$. Find the equation of the curve if it has a stationary point at $\left(-1, -\frac{3}{2}\right)$. [4]

Answers

1. (a) $x = -\frac{3}{2}$ or $-1 < x < 1$.
   (b) $0 \leq x < 1$.
2. (a) i. Least $n = 16$.
   ii. Since $r = 0.85, -1 < r < 1$. Hence the series converges.
   $\frac{8500}{3}$ liters.
   (b) i. 196 580.
   ii. 556 days.
3. $x = 27, y = 16, z = 12$.
4. (a) $(k+1)(2k+1) - 2 + \frac{1}{2^{2k}}$.
   (b) iii. $\frac{1}{2}$.
5. $-1 < x < 3$.
   $-3 < x < 3$.
6. (b) $\frac{n+1-\mathrm{e}^n}{\mathrm{e}^{n+1}}$.
   (c) $\frac{20-4\mathrm{e}^{16}}{\mathrm{e}^{20}}$.
7. (b) Since $|r| = \frac{1}{5} < 1$.
   (c) $243.75|a|$.
8. (a) $n = 79$. 1 July 2024.
   (b) $\frac{90000}{r}\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$.
   0.314%.
9. $y = 3x^2 + \frac{9}{2}x^2 - 3$.