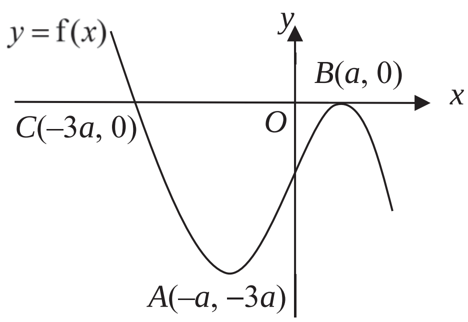
1 **(i)** Sketch the curve with equation $y=\left|\dfrac{\alpha x}{x+1}\right|$, where α is a positive constant, stating the equations of the asymptotes. On the same diagram, sketch the line with equation $y = \alpha x - 2$. [3]

(ii) Solve the inequality $\left|\dfrac{\alpha x}{x+1}\right| \geq \alpha x - 2$, giving your answers in term of α. [3]

2 Interpret geometrically the vector equation $\mathbf{r} = \mathbf{a} + \mu\mathbf{m}$ where $\mathbf{a}$ and $\mathbf{m}$ are constant vectors and μ is a parameter. [2]

Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, such that $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors. The point C lies on the line AB such that the area of the triangle OBC is 6 units^2. Given that $\mathbf{a}$ is a unit vector, $|\mathbf{b}| = 4$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is 30°, find the possible position vectors of C in terms of $\mathbf{a}$ and $\mathbf{b}$. [6]

3 **(a)** The diagram shows the curve $y = \mathrm{f}(x)$, where a is a positive constant. The curve has a minimum point at $A(-a, -3a)$, a maximum point at $B(a, 0)$ and cuts the x-axis at the point $C(-3a, 0)$.



Sketch, labelling each graph clearly and showing the coordinates of the points corresponding to A, B and C whenever possible, the graphs of

(i) $y = 3\mathrm{f}(x-a)$, [2]

(ii) $y = \mathrm{f}\left(\left|\dfrac{x}{2}\right|\right)$, [2]

(iii) $y = \dfrac{1}{\mathrm{f}(x)}$. [2]

(b) The curve with equation $y = 2 + \mathrm{e}^{-x}$ is reflected in the line $y = 5$. Find the equation of the reflected curve. [2]

4 It is given that

$$f(x)=\begin{cases} 4x+2 & \text{for } 0\le x\le 1,\\ \dfrac{12}{x+\sqrt{x}} & \text{for } 1<x\le 4,\end{cases}$$

and that $f(x+4)=f(x)$ for all real values of x.

(i) Sketch the graph of $y=f(x)$ for $-2<x\le 6$. [3]

(ii) Use the substitution $u=\sqrt{x}$ to find the exact value of $\int_0^4 f(x)\,dx$. [5]

5 **(a)** Find $\int \sin x\cos 3x\,dx$. [2]

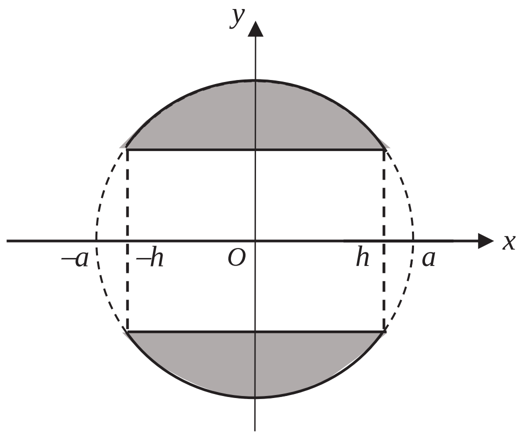
(b) Find $\int \frac{x-1}{\sqrt{1+2x-x^2}}\,dx$. Find the greatest integer value of b such that $\int_0^b \frac{x-1}{\sqrt{1+2x-x^2}}\,dx$ is defined. [5]

(c) Find $\int x\cos x\,dx$. Hence find the exact value of $\int_0^{2\pi} x|\cos x|\,dx$. [5]

6 **(a)** In an arithmetic progression, the 8th term is 20 and the 27th term is greater than the 15th term by 24. It is given that the sum of the first n terms is greater than the sum of the 8th to the 40th term by more than 1218. Find the smallest value of n. [6]

(b) An infinite geometric progression is such that the sum of all the terms after the nth term is equal to twice the nth term. Show that the sum to infinity of the progression is three times the first term. [3]

7 A napkin-holder is formed by boring a cylindrical hole, of length $2h$, through a wooden sphere of radius a, where a is a fixed constant. The axis of the hole passes through the centre O of the sphere. The diagram shows a cross-section through O, with x- and y-axes taken parallel and perpendicular to the axis of the hole respectively.



(i) Let S denote the napkin holder's total surface area, which is made up of its internal (cylindrical) area and its external (spherical) area. It is given that the external surface area is $4\pi ah$.

(a) Show that $S = 4\pi h\left(a + \sqrt{a^2 - h^2}\right)$. [2]

(b) Use differentiation to find, in terms of a, the exact maximum value of S as h varies. [You do not need to verify that this value of S is the maximum.] [4]

(ii) Let V denote the volume of the wood forming the napkin-holder. By considering the napkin-holder as a solid of revolution about the x-axis, find V in terms of h, verifying that it is independent of a. [4]

8 Functions f and g are defined by

$$\mathrm{f} : x \mapsto \frac{x^2 - 6x - 6}{x+1}, \quad x \in \mathbb{R}, \quad x \neq -1,$$
$$\mathrm{g} : x \mapsto \frac{ax+1}{x+b}, \qquad x \in \mathbb{R}, \qquad x \neq -b,$$

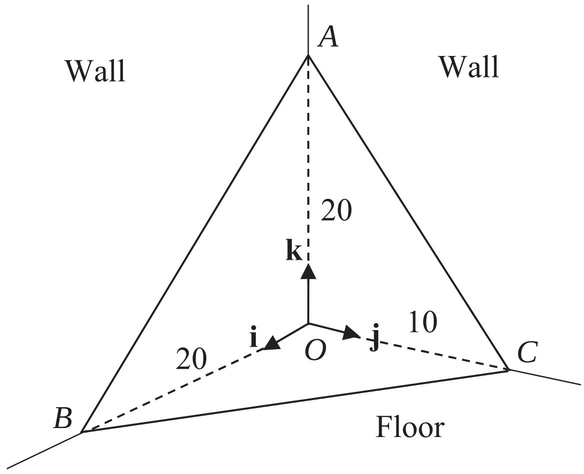
where a and b are constants.

(i) Sketch the graph of $y = \mathrm{f}(x)$, giving the coordinates of the turning points and the equation of the asymptotes. Write down the range of f. [3]

(ii) Find the value of a and the range of values of b such that both composite functions fg and gf exist. [4]

(iii) Find $\mathrm{g}^{-1}(x)$. Given that $\mathrm{g}^{-1}(x) = \mathrm{g}(x)$ for all real x, $x \neq -b$, find b in terms of a. [3]

9



A shoe store owner plans to install a triangular mirror *ABC* with negligible thickness at one of the floor corner of his shop to allow his customers to view the fitting of their selected shoes. Points (x, y, z) are defined relative to the corner point at *O* where the two vertical walls, which are perpendicular to each other, and the horizontal floor meet. The z-axis points vertically upwards. The x-axis and y-axis are the intersections of the floor with the two walls. A, B and C lie on the z-axis, x-axis and y-axis and are 20 units, 20 units and 10 units from *O* respectively. The units of length are measured in inches.

(i) Find the cartesian equation of the face of the mirror *ABC*. Hence find the exact shortest distance from *O* to the face of the mirror *ABC*. [4]

(ii) Find the coordinates of the point *N* on the face of the mirror *ABC* which is nearest to *O*. [2]

(iii) Find the acute angle between the face of the mirror *ABC* and the floor. [2]

As a safety measure, a triangular plank *OBR* is installed to support the mirror, where *R* is a point between *A* and *C* such that $AR : RC = \mu : 1-\mu$. The face of the mirror *ABC* meets the plank *OBR* on *l*.

(iv) Given that *N* in **(iii)** lies on *l*, find the coordinates of *R*. [5]

10 A curve *C* has parametric equations

$$x = 2t + \sin 2t, \quad y = \cos 2t, \quad \text{for } 0 \le t \le \pi.$$

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\tan t$. What can be said about the tangent to *C* at the point where $t = \frac{\pi}{2}$. [4]

(ii) Find the exact x-coordinates, x_1 and x_2 where $x_1 < x_2$, of the two points where *C* cuts the x-axis. [2]

(iii) Sketch *C*, indicating the exact coordinates of the end-points. [2]

(iv) Find the exact area of the region bounded by *C* and the x-axis. [5]

(v) Find the value of t at which the tangent to *C* at the point $x = x_1$ intersects *C* again. [3]

End of Paper