1. **[AJC 18 J2 MYE]**
Two non-zero vectors, $\mathbf{a} + k\mathbf{b}$ and $\mathbf{a} - k\mathbf{b}$ are perpendicular to each other, where k is a positive constant and $\mathbf{a}$ is a unit vector.

   Find the magnitude of $\mathbf{b}$ in terms of k.

2. **[AJC 18 J2 MYE]**
The plane Π contains the origin O and is parallel to vectors $-\mathbf{i} + \mathbf{k}$ and $\mathbf{i} + 2\mathbf{j}$.

   (a) Find an equation of the plane Π in scalar product form. [2]

   The point P has coordinates $(3, 1, 2)$.

   (b) By finding the foot of perpendicular, N, of point P to the plane Π, show that the position vector of the mirror image of P in Π is $-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. [3]

   The point Q has coordinates $(4, 0, 1)$.

   (c) Find the exact length of projection of PQ on to the plane Π. [3]

   (d) Hence, or otherwise, find the exact area of the triangle PNQ. [2]

3. **[DHS J2 18 MYE]**
Relative to the origin O, two points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors. The points C and D lie on the midpoints of OA and AB respectively. The line segments OD and BC intersect at the point P such that $OP : PD = \lambda : 1 - \lambda$.

   (a) Find $\overrightarrow{OP}$ in terms of $\lambda, \mathbf{a}$ and $\mathbf{b}$. [1]

   (b) Show that the ratio of $BP : PC$ is $2 : 1$. [4]

4. **[MI J2 18 MYE]**
Referred to the origin O, a laser beam l_1 is fired from the point A with coordinates $(1, -2, 4)$ and passes through the point B with coordinates $(2, -1, s)$. Another laser beam can be modelled as the line l_2 with equation $\dfrac{x-1}{-3} = \dfrac{y+2}{4}, z = 4$.

   (a) Find the cosine of the acute angle between the two laser beams, leaving your answer in terms of s. [3]

   It is given that $s = 3$.

   (b) Find the coordinates of the point corresponding to the shortest distance from the point B to the laser beam l_2. Hence find the shortest distance. [4]

   The laser beams are fired at a glass block. One of the surfaces of the galss block can be modelled as a plane with equation $\mathbf{r} \cdot \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} = 1$.

   (c) Find the position vector of the point where the laser beam l_1 meets the glass block. [3]

   (d) Another surface of the glass block has equation $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 6$. Find the exact thickness of the glass block. [2]

Answers

1. $\frac{1}{k}$.

2. (a) $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = 0$.

   (c) $\frac{\sqrt{26}}{3}$.

   (d) $\frac{\sqrt{26}}{2}$.

3. $\frac{\lambda}{2}(\mathbf{a} + \mathbf{b})$.

4. (a) $\frac{1}{5\sqrt{(s-4)^2+2}}$.

   (b) $(\frac{22}{25}, -\frac{46}{25}, 4)$, $\frac{\sqrt{74}}{5}$.

   (c) $\frac{1}{2}\begin{pmatrix} 5 \\ -1 \\ 5 \end{pmatrix}$.

   (d) $\frac{7\sqrt{6}}{6}$.