5 Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\left(\sqrt{x}\right)^{x}\right]=\dfrac{1}{2}\left(\sqrt{x}\right)^{x}\left(1+\ln x\right)$. [4]

Hence by using the substitution $u=\sqrt{x}$, evaluate $\int_{1}^{2}\left(u\right)^{u^2+1}\left(1+2\ln u\right)\mathrm{d}u$ exactly. [3]

6 The function f is defined by

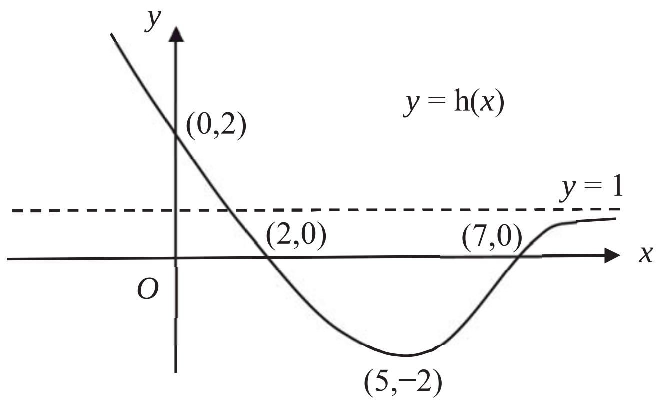
$$\mathrm{f}: x \mapsto x+1+\frac{1}{x-1},\quad x\in\mathbb{R},\ x\neq 1.$$

(i) Sketch the graph of f and explain why the inverse of f does not exist. [3]

(ii) If the domain of f is further restricted to $a \le x \le b$, state the values for a and b such that the inverse of f exists and the range of f remains unchanged. [1]

For the rest of the question, the domain of f is as originally defined.

The function h is continuous for all real values of x. The diagram below shows the graph of $y=\mathrm{h}(x)$. It has exactly one horizontal asymptote at $y=1$, one turning point at $(5,-2)$ and cuts the axes at $(0,2)$, $(2,0)$ and $(7,0)$.



(iii) Explain whether the composite functions fh or hf exists. Find the range of the composite function that exists. [4]

7 **Do not use a calculator in answering this question.**

Given that $y\sin 2x+\cos y=0$, $0<y<\pi$.

(i) Show that $2y\cos 2x+(\sin 2x-\sin y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$. [2]

(ii) By differentiating the above result, find the Maclaurin series for y, up to and including the term in x^2. [3]

(iii) Deduce an approximate value of x that satisfies the equation $\cos\left(50\pi x\right)-50\pi x\sin 2x=0$. [3]

1 The government of a particular country reported that the mean monthly water consumption of households is 17.9 m^3. This is followed by a nationwide water conservation campaign to encourage households to reduce their water consumption. Subsequently after the campaign, the water consumption $x\ \text{m}^3$ of a random sample of 50 households is recorded and their data are summarised as follows.

$$\sum(x-15)=103, \quad \sum(x-15)^2=599.$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Test at the 5% significance level whether the campaign is effective.

State, giving a reason, whether any assumption about the distribution of the monthly water consumption of households is needed in order for the test to be valid. [4]

(iii) Explain the meaning of '5% significance level' in the context of the question. [1]

2 A group of students consisting of two girls and six boys turned up for an interview.

(a) Eight colour tags are distributed randomly to this group of students, with each student receiving only one tag. There are 1 red, 3 green and 4 yellow tags. Suppose that each coloured tag is identical, find the probability that the two girls get yellow tags. [2]

(b) While waiting for their turn for the interview, the students randomly sat on the eight chairs lined up in a row outside the interview room. Find the probability that

(i) the two girls sit adjacent to each other, [2]

(ii) one of the boys, Peter, does not sit next to any girl, [3]

(iii) the two girls sit adjacent to each other, given that Peter does not sit next to any girl. [2]

7 A sequence $u_1, u_2, u_3, \ldots$ is defined such that $u_n = \dfrac{1}{(2n-1)^2}$ for all $n \geq 1$.

(i) Show that $u_{n+1} - u_n = -\dfrac{8n}{(2n-1)^2(2n+1)^2}$ for $n \geq 1$. [2]

(ii) Hence, find $\displaystyle\sum_{n=1}^{N} \frac{n}{(2n-1)^2(2n+1)^2}$. [2]

(iii) Give a reason why the series in part (ii) is convergent and state the sum of infinity. [2]

(iv) Use your answers to parts (ii) and (iii) to find $\dfrac{2}{15^2} + \dfrac{3}{35^2} + \dfrac{4}{63^2} + \ldots$. [2]

8 Suppose that a is a non-zero constant and the sequence $\{T_n\}$ is defined by $T_n = 1 - \dfrac{a}{2^n}$, $n \geq 1$. Given further that T_1, T_3, T_4 form the first three consecutive terms of a geometric progression, find

(i) the value of a, [3]

(ii) the sum to infinity of the above geometric series, [3]

(iii) $\displaystyle\sum_{n=1}^{N} \ln(1 - T_n)$. [3]

9 (i) Sketch the graph of $y = x - 1 + \dfrac{1}{x-1}$, showing all essential features clearly. [3]

(ii) The function f is given by $\mathrm{f} : x \mapsto x - 1 + \dfrac{1}{x-1}$ for $x \in (-\infty, 0] \cup (1, a]$. Find the largest possible value of a such that f^{-1} exists. [1

For the rest of the question, the domain of f is the one found in (ii).

(iii) Sketch the graph of f and f^{-1} on the same axes, showing the relationship between them clearl [

(iv) The function g is given by $\mathrm{g} : x \mapsto \sqrt{3 - x}, x < 3$. Show that function gf^{-1} exists and find i range in exact form. [

Statistics (45 marks)

5 A pack of 10 cards contains 2 red and 8 black cards. In a game, Ben draws cards at random, without replacement, from the pack, one at a time, until he has drawn out 2 black cards. The total number of cards Ben draws from the pack is denoted by X.

(i) Find $P(X = x)$ for all possible values of x. [3]

(ii) Find $E(X)$ and $Var(X)$. [2]

6 The Hand Foot Mouth Disease is a disease that is present all year round in Singapore with seasonal outbreaks every year. The mean duration of a child's infectious period is 150 hours. The Health Promotion Institution is concerned over the recent spike of Hand Foot Mouth Disease cases in Singapore and launched an investigation to check whether the mean duration of a child's infectious period has changed. A random sample of 100 infected children is selected and the duration of each child's infectious period is recorded. The results are summarized as follows.

$$\sum(x-150)=450 \quad \text{and} \quad \sum(x-150)^2=42005$$

Test at the 5% level of significance whether the mean duration of a child's infectious period has changed. State, giving a reason, whether it is necessary to assume that the duration of a child's infectious period follows a normal distribution for the test to be valid. [6]

7 A company sells bicycles and conducted a market survey among the users to understand more about consumers' behaviour. The results showed that among all bicycles, 60% belongs to males and the rest to females. 90% of the bicycles belonging to the males and 70% of those belonging to the females are racers (a type of bicycle). A bicycle is chosen at random. Find the probability that it

(i) is a racer or that it belongs to a female, [2]

(ii) belongs to a female given that it is not a racer. [3]

Two bicycles are chosen at random. Find the probability that one is a racer and one is not. [2]

8 A committee consisting of six persons is to be selected from five women and six men. Find the number of possible selections such that

(i) the chosen committee will contain exactly two men, [1]

(ii) the chosen committee will have more men than women in the committee. [2]

The chosen committee consists of a married couple together with three other men and one other woman. They are seated round a table for six. Find the number of possible arrangements such that

(iii) the husband is sitting next to his wife, [2]

(iv) the husband is separated from his wife by at least two other people on each of his left and right sides. [2]