1 The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by $S_n = n^2(An + B)$, where A and B are non-zero constants.

(i) Find an expression for u_n in terms of A, B and n, simplifying your answer. [3]

(ii) It is also given that the second term is 6 and the sixth term is 218. Find the values of A and B. [2]

(iii) Determine if the sum to infinity exists. [1]

2 **Do not use a calculator in answering this question.**

The curve C has equation $x^2 + 3xy - y^2 + 4 = 0$.

(i) Find the exact x-coordinates of the stationary points of C. [4]

(ii) For the stationary point with $x < 0$, determine whether it is a maximum or minimum. [3]

3 **(i)** Show that $\dfrac{1}{2r-1} - \dfrac{1}{2r+3} = \dfrac{k}{(2r-1)(2r+3)}$, where k is a constant whose value is to be determined. [1]

(ii) Hence find the sum of the series

$$\frac{1}{1\times5} + \frac{1}{3\times7} + \frac{1}{5\times9} + \cdots + \frac{1}{(2n-1)(2n+3)}.$$

5 21 45

(There is no need to express your answer as a single algebraic fraction.) [3]

(iii) Find the exact value of the infinite series

$$\frac{1}{11\times15} + \frac{1}{13\times17} + \frac{1}{15\times19} + \cdots. \quad [3]$$

Section A: Pure Mathematics [55 marks]

1 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point M is on AB produced such that $AB : BM = 3:2$ and the point N is on OB such that $ON : NB = 1:2$. It is given that $|\mathbf{a}| = 3$, $\mathbf{b}$ is a unit vector and $\angle AOB = 60°$.

(i) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) Show that the area of triangle OMN can be written as $k|\mathbf{a} \times \mathbf{b}|$, where k is a constant to be found. Hence evaluate the exact area of triangle OMN. [4]

(iii) Find the length of projection OM onto ON. [3]

2 (i) Show that $\frac{1}{r} - \frac{2}{(r-1)} + \frac{1}{(r-2)} = \frac{2}{r(r-1)(r-2)}$. [2]

(ii) Hence find $\sum_{r=3}^{n} \frac{1}{r(r-1)(r-2)}$, giving your answer in the form $k - \mathrm{f}(n)$, where k is a constant. [4]

(iii) Explain why $\sum_{r=3}^{\infty} \frac{1}{r(r-1)(r-2)}$ is a convergent series and state the value of the sum to infinity. [2]

(iv) Use your answer in (ii) to find $\sum_{r=1}^{n-1} \frac{1}{(r+1)(r+2)(r+3)}$.

[2]

3 Show that

$$\int \frac{8x+1}{4+3x^2}\,\mathrm{d}x = a\ln\left(4+3x^2\right) + b\tan^{-1}\frac{\sqrt{3}}{2}x + c,$$

where a and b are constants to be determined. [4]

A region is bounded by the curve $y = \frac{8x+1}{4+3x^2}$ and lines $y = \frac{57}{32}x$, $y = 0$ and $x = 2$. The curve and line $y = \frac{57}{32}x$ intersect at the point $\left(\frac{2}{3}, \frac{19}{16}\right)$.

(i) Find the exact value of the area of this region. [4]

(ii) Find the volume generated by this region when it is rotated 2π radians about the x-axis, giving your answer correct to 3 decimal places. [3]

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4 A function f is said to be self-inverse if $f(x) = f^{-1}(x)$ for all x in the domain of f.

The function g is defined by

$$g : x \mapsto \frac{ax+b}{cx+d}\ ,\ x \in \mathbb{R},\ x \neq -\frac{d}{c}\ ,$$

where a, b, c and d are constants.

(i) Find $g^{-1}(x)$. Given that g is self-inverse, state the relationship between a and d. [3]

The functions p and q are defined by

$$p : x \mapsto \frac{2x+3}{x-2}\ ,\ x \in \mathbb{R},\ x \neq 2\ ,$$

$$q : x \mapsto 3-x\ ,\ x \in \mathbb{R}\,.$$

(ii) Find $p^2(x)$, and hence find $p^{101}(x)$. [3]

(iii) Explain why the composite function pq does not exist. [2]

(iv) State a sequence of transformations which transform the graph of $y = p(x)$ to the graph of $y = qp(x)$. [2]

5 A curve C is defined by the parametric equations $x = \tan\theta$, $y = \sec\theta$ for $0 < \theta < \frac{1}{2}\pi$.

State the ranges of values of x and y. [2]

(i) Find the Cartesian equation of C. Sketch C, giving the equation(s) of any asymptote(s) and labelling the axial intercepts. [3]

The tangent and normal at $P(\tan\theta, \sec\theta)$ meet the x-axis at Q and R respectively.

(ii) Show that the area, A, of the circle passing through P, Q and R, can be expressed as $A = \pi\left(\tan\theta + \dfrac{1}{2\tan\theta}\right)^2$. [7]

(iii) Show algebraically that $\left(t + \dfrac{1}{2t}\right)^2 - 2 \geq 0$ for all $t \in \mathbb{R}, t \neq 0$. Deduce the minimum value of A. [3]

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7 The function f is defined by

$$\mathrm{f}: x \mapsto -\sqrt{1+x},\ x \in \mathbb{R},\ x \geq -1.$$

(i) Show that f^{-1} exists. [1]

(ii) On a single diagram, sketch the graph of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ together with the line $y = x$, indicating clearly the coordinates of the points where the curves cross the x- and y-axes. [2]

(iii) Solve the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$, giving your answer in exact form. [3]

(iv) State the value of x such that $\mathrm{f}^{-1}(x) = 4$. [1]

(v) The graph of $y = \mathrm{f}(x)$ undergoes, in succession, the following transformations:

Step 1: a reflection in the y-axis;

Step 2: a translation of 1 unit in the negative x-direction.

Find the equation of the resultant curve in the form $y = \mathrm{g}(x)$, stating its domain clearly. [2]

Section B: Statistics [40 marks]

9 **(i)** A card set containing 100 cards has one number from the set $\{1, 2, 3, \ldots, 100\}$ printed on each card, and every card has a different number. Two boys Alex and Bobby play a game using the card set. Each of them picks a card at random without replacement, with Alex picking his card first. Find the probability that the number on Bobby's card is bigger than that on Alex's card. [2]

(ii) Chris joins Alex and Bobby in the game, with Alex picking this card first, followed by Bobby and Chris being the last to pick. Find the probability that the number on Chris's card is biggest. [2]

(iii) From the results in **(i)** and **(ii)**, does the order of picking the card affects the chance of getting the largest number? Justify your answer. [1]

10 The random variable X has the distribution $X \sim \mathrm{N}(\mu, \sigma^2)$. It is given that X_1 and X_2 are 2 independent observations of X with $\mathrm{P}(4 < X < 6) = 0.5$ and $\mathrm{P}(X_1 + X_2 > 10) = 0.5$.

(i) Show that $\mu = 5$ and state, with a reason, the value of $\mathrm{P}(2X > 10)$. [3]

(ii) Find the value of σ, giving your answer to 5 significant figures. [3]

(iii) A random sample of n observations of X is obtained. Given that there is a probability of more than 95% such that the sample mean is within 0.1 units of μ, show that $\mathrm{P}\left(Z < \dfrac{a\sqrt{n}}{b}\right) < c$, where a, b, c are real values to be determined. Hence find the least value of n. [4]

1 The government of a particular country reported that the mean monthly water consumption of households is 17.9 m^3. This is followed by a nationwide water conservation campaign to encourage households to reduce their water consumption. Subsequently after the campaign, the water consumption x m^3 of a random sample of 50 households is recorded and their data are summarised as follows.

$$\sum(x-15)=103, \quad \sum(x-15)^2=599.$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Test at the 5% significance level whether the campaign is effective.

State, giving a reason, whether any assumption about the distribution of the monthly water consumption of households is needed in order for the test to be valid. [4]

(iii) Explain the meaning of '5% significance level' in the context of the question. [1]

2 A group of students consisting of two girls and six boys turned up for an interview.

(a) Eight colour tags are distributed randomly to this group of students, with each student receiving only one tag. There are 1 red, 3 green and 4 yellow tags. Suppose that each coloured tag is identical, find the probability that the two girls get yellow tags. [2]

(b) While waiting for their turn for the interview, the students randomly sat on the eight chairs lined up in a row outside the interview room. Find the probability that

(i) the two girls sit adjacent to each other, [2]

(ii) one of the boys, Peter, does not sit next to any girl, [3]

(iii) the two girls sit adjacent to each other, given that Peter does not sit next to any girl. [2]

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Section B: Probability and Statistics [50 marks]

3 There are 6 boys and 8 girls in a class. Two of the boys are twins and three of the girls are triplets.

(i) Find the number of ways for the 6 boys to be seated in a row of 8 seats so that there will be two empty seats. [1]

(ii) Find the number of ways for the class to be seated in two rows of 7 chairs each, with the triplets seated together in the front row and the twins together in the back row. [2]

(iii) All the 14 children are to be seated in a circle. Find the probability that no two of the triplets are seated together. [3]

4 In a game, three fair tetrahedral dice are thrown onto a horizontal surface. Each die has four faces, numbered 1, 2, 3 and 4. The score of each die is the number on the face that the die lands on. The score of the game, denoted by X, is the highest of the three scores on the dice.

(i) Show that $\mathrm{P}(X=4)=\dfrac{37}{64}$. [3]

(ii) Find $\mathrm{P}(X=x)$ for all the remaining possible values of x. [3]

(iii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [2]

Joshua plays the game 10 times.

(iv) Find the probability that Joshua scores less than 4 in at least 6 of his 10 games. [2]

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