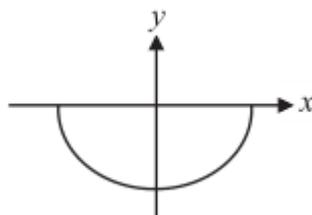


1. [ACJC Promos 19]

(a) Show that  $\int_0^{\frac{\pi}{2}} e^{-2x} \cos 2x \, dx = \frac{1}{4} (1 + e^{-\pi})$ . [4]

(b) The region bounded by the curve  $y = e^{-x} \cos x$ ,  $y = 1$  and  $x = \frac{\pi}{2}$  is rotated through  $2\pi$  radians about the  $x$ -axis. Using the result in (a), find the exact volume of the solid formed. [5]

2. [ACJC Promos 19 (modified)]



The diagram shows a curve  $C$  with parametric equations

$$x = h \cos \theta, \quad y = k \sin \theta,$$

where  $h$  and  $k$  are positive constants and  $-\pi \leq \theta \leq 0$ .

(a) Using the parametric equations of curve  $C$ , find the coordinates of the points where  $C$  crosses the axes. [2]

(b) The region enclosed by  $C$  and the  $x$ -axis is denoted by  $A$ . Find the area of  $A$  in terms of  $h$ ,  $k$  and  $\pi$ . [1]

(c) Find the cartesian equation of  $C$ , expressing  $y$  in terms of  $x$ . [2]

3. [ASRJC Promos 19]

(a) Find  $\frac{d}{dx} (\sin 2x - 2x \cos 2x)$ . Hence or otherwise find  $\int x^2 \cos 2x \, dx$ . [4]

(b) If  $0 < a < 1$ , find  $\int_0^1 (a + x) |a - x| \, dx$ , leaving your answer in terms of  $a$ . [5]

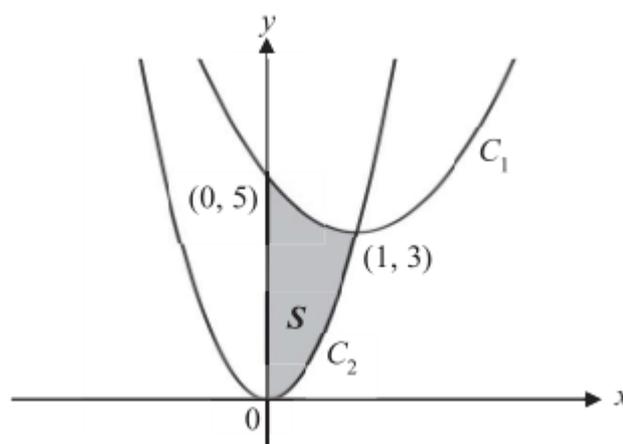
4. [RVHS Promos 19]

(a) By means of the substitution  $x = \sqrt{12} \sin \theta$ , show that

$$\int_0^{\sqrt{3}} \sqrt{12 - x^2} \, dx = \pi + \frac{3}{2}\sqrt{3}. \quad [4]$$

(b) The region  $R$  is bounded by the curve  $y = \sqrt[4]{12 - x^2}$ , the lines  $x = \sqrt{3}$  and  $x = -\sqrt{3}$ , and the  $x$ -axis. Using your result in part (a), find the exact volume of the solid formed by rotating  $R$  through  $2\pi$  radians about the  $x$ -axis. [3]

5. [ASRJC Promos 19]



The diagram above shows the curves  $C_1$  and  $C_2$  with equations  $y = 2(x - 1)^2 + 3$  and  $y = 3x^2$  respectively. The region in the first quadrant enclosed by the curves and the  $y$ -axis is denoted by  $S$ . Find the volume of the solid generated when the region  $S$  is rotated through  $2\pi$  radians about the  $y$ -axis, giving your answer correct to 4 decimal places.

[3]

6. [RVHS Promos 19]

Express  $x + 3 = A(2x + 2) + B$ , where  $A$  and  $B$  are constants to be determined.

Hence, find  $\int \frac{x + 3}{x^2 + 2x + 2} dx$ .

[3]

7. [TMJC Promos 19]

The curve  $C$  has equation  $y = \frac{\sqrt{x}}{(16 - x^2)^{\frac{1}{4}}}$ ,  $0 \leq x < 4$ .

(a) Sketch  $C$ .

[2]

(b) Find the exact volume of revolution when the region bounded by  $C$ , the line  $x = 2\sqrt{3}$  and the  $x$ -axis, is rotated  $2\pi$  radians about the  $x$ -axis.

[4]

(c) A horizontal line  $l$  intersects  $C$  at  $x = 2\sqrt{3}$ . Find the exact volume of revolution when the region bounded by  $C$ , the horizontal line  $l$  and the line  $x = 0$ , is rotated  $2\pi$  radians about the  $x$ -axis.

[2]

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8. **[DHS MYE 18 (modified)]**

A group of students consisting of two girls and six boys turned up for an interview.

- (a) Eight colour tags are distributed randomly to this group of students, with each student receiving only one tag. There are 1 red, 3 green and 4 yellow tags and each coloured tag is identical.
- i. Find the total number of ways to distribute the tags. [1]
  - ii. Find the number of ways to distribute the tags such that the two girls get yellow tags. [1]
- (b) While waiting for their turn for the interview, the students randomly sat on the eight chairs lined up in a row outside the interview room.
- i. Find the number of ways for the two girls to sit adjacent to each other. [2]
  - ii. Find the number of ways for the two girls to sit adjacent to each other and on of the boys, Peter, does not sit next to any girl. [2]
  - iii. Find the number of ways for Peter to not sit next to any girl. [1]

9. **[EJC MYE 18]**

A committee consisting of six persons is to be selected from five women and six men. Find the number of possible selections such that

- (a) the chosen committee will contain exactly two men, [1]
- (b) the chosen committee will have more men than women in the committee. [2]

The chosen committee consists of a married couple together with three other men and one other woman. They are seated round a table for six. Find the number of possible arrangements such that

- (c) The husband is sitting next to his wife, [2]
- (d) the husband is separated from his wife by at least two other people on each of his left and right sides. [2]

10. **[MI MYE 18 (modified)]**

Each of the digits 1, 2, 3, 4, 5, 6 is written on a separate card. The cards are then laid out in a row to form a number.

- (a) How many distinct 6-digit numbers can be formed? [1]
- (b) How many distinct 3-digit numbers can be formed? [1]

An additional card with digit 1 is added to the set of cards to be arranged.

- (c) How many ways are there to form a 7-digit number? [1]
- (d) How many ways are there to form a 7-digit number that starts and ends with the same digit? [1]

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11. [MI MYE 18]

Three boys, two girls and two teachers sit in a line on stools in a coffee bar.

- (a) In how many ways can they arrange themselves so that the two teachers are next to each other and not all the girls are next to each other? [3]

The eight people decide to sit at a round table with ten chairs instead.

- (b) Find the number of different possible arrangements. [2]

12. [RI MYE 18]

The eleven letters in the word INSTITUTION are individually printed on eleven identical cards.

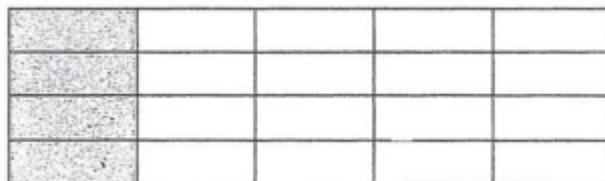
- (a) The eleven cards are arranged in a line.  
i. Find the number of different arrangements of the eleven cards that can be made. [1]

- ii. Find the number of different arrangements that can be made if S,U and O are separated from one another. [2]

- (b) Three cards are to be chosen from the eleven cards; the order in which they are chosen does not matter. Find the number of different possible selections of three cards. [3]

13. [TJC MYE 18]

As part of a military exercise, a rectangular piece of land is divided into 4 equal rows and 5 equal columns, forming 20 smaller plots as shown in the diagram below. The shaded plots are designated to be “No Man’s Land”.



3 identical simulated land mines are placed in 3 distinct plots of land. Find the number of ways this can be done if

- (a) at least one mine is placed in “No Man’s Land”, [2]

- (b) all the mines are placed in the same column or in the same row, [3]

- (c) no two mines are placed in the same column and no two mines are placed in the same row. [2]

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## Answers

1.  $\frac{\pi}{8}(4\pi + e^{-\pi} - 3)$  units<sup>3</sup>.
2. (a)  $(h, 0), (-h, 0), (0, -k)$ .  
(b)  $\frac{kh\pi}{2}$  units<sup>2</sup>.  
(c)  $y = -k\sqrt{1 - \frac{x^2}{h^2}}$ .
3. (a)  $\frac{x^2 \sin 2x}{2} - \frac{1}{4} \sin 2x + \frac{1}{2}x \cos 2x + C$ .  
(b)  $\frac{4a^3}{3} - a^2 + \frac{1}{3}$ .
4.  $\pi(2\pi + 3\sqrt{3})$  units<sup>3</sup>.
5. 5.7596 units<sup>3</sup>.
6.  $\frac{1}{2} \ln |x^2 + 2x + 2| + 2 \tan^{-1}(x + 1) + C$ .
7. (b)  $2\pi$ .  
(c)  $4\pi$ .
8. (a) i. 280.  
ii. 60.  
(b) i. 10,080.  
ii. 7200.  
iii. 14,400.
9. (a) 75.  
(b) 181.  
(c) 48.  
(d) 24.
10. (a) 720.  
(b) 120.  
(c) 2,520.  
(d) 120.
11. (a) 8,640.  
(b) 101,440.
12. (a) 554,400.  
(b) 282,240.  
(c) 37.
13. (a) 580.  
(b) 60.  
(c) 240.