

1. [ACJC MYE 18]

Without using a calculator, find the roots of the equation

$$z^2(5 - 5i) - 2z + (1 + i) = 0,$$

giving your answers in cartesian form  $a + bi$ .

[3]

2. [ACJC MYE 18]

The complex number  $w$  has modulus  $r$  and argument  $\theta$ , where  $-\pi < \theta \leq \pi$  and  $w^*$  denotes the conjugate of  $w$ .

(a) State the modulus and argument of  $\frac{w^2}{w^*}$ .

[2]

(b) Given that  $w^2 = rw^*$ , find the possible exact values of  $\theta$ .

[2]

3. [ACJC CA2 18]

One root of the equation  $w^3 - 9w^2 + 20w - 37 - b^2 = 0$ , where  $b$  is real and positive, is  $w = 1 + bi$ .

By expressing  $w^3 - 9w^2 + 20w - 37 - b^2$  as a product of a linear and a quadratic factor, find the value of  $b$  and the other roots.

[4]

4. [ACJC CA2 18]

Given that  $\left(\frac{(2 - 2i)^2}{-\sqrt{3} + ki}\right)^* = 4e^{i\theta}$ , where  $k$  is a positive real number and  $-\pi < \theta \leq \pi$ ,

(a) show that  $k = 1$ .

[2]

(b) Hence find the exact value of  $\theta$ .

[2]

5. [AJC 18 MYE]

**Do not use a calculator in answering this question.**

The complex numbers  $p$  and  $q$  satisfy the following simultaneous equations

$$\begin{aligned} p + (2 + i)q &= 5 + i \\ ip - 2q &= 4. \end{aligned}$$

Find  $p$  and  $q$  in the form  $x + yi$ , where  $x$  and  $y$  are real.

[4]

6. [RVHS MYE 18 (modified)]

Find the three smallest positive real number  $n$  such that  $(1 + i\sqrt{3})^n$  is purely imaginary.

[3]

7. [CJC MYE 18]

It is given that

$$z^3 + 2z^2 + (k - 8\sqrt{2}i)z + 8 - 4\sqrt{2}ki = 0,$$

where  $k$  is a real constant, has a real root.

(a) show that  $-2$  is the real root and find the value of  $k$ .

[4]

(b) Hence find the other roots, giving your answers in exact cartesian form  $a + bi$ .

[6]

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8. [NYJC MYE 18]

The complex numbers  $3 + 2i$ ,  $z$  and  $4 - 6i$  are the first three terms of a geometric progression.

Find, in cartesian form  $a + bi$ , where  $a, b \in \mathbb{R}$ ,

- (a) two possible complex numbers  $z$ , [2]
- (b) two possible values for the common ratio  $r$ , [1]
- (c) the 13th term of the progression for which the real part of the common ratio,  $r$ , found in (b) is negative. [2]

## Answers

- 1.  $-0.2 + 0.4i$  or  $0.4 - 0.2i$ .
- 2. (a)  $3\theta$ .  
(b)  $-\frac{2\pi}{3}, 0, \frac{2\pi}{3}$ .
- 3.  $b = \sqrt{5}$ .  
 $w = 7, 1 + i\sqrt{5}$  or  $1 - i\sqrt{5}$ .
- 4.  $-\frac{2\pi}{3}$ .
- 5.  $p = 6 - 6i, q = 1 + 3i$ .
- 6. 1.5, 4.5, 7.5.
- 7. (a)  $k = 4$ .  
(b)  $-2 - 2\sqrt{2}i$  and  $-2 + 2\sqrt{2}i$ .
- 8. (a)  $z = 5 - i$  or  $-5 + i$ .  
(b)  $r = 1 - i$  or  $-1 + i$ .  
(c)  $-192 - 128i$ .

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Question:	1	2	3	4	5	6	7	8	Total
Marks	3	4	4	4	4	3	10	5	37
Score:									

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