1. **[AJC MYE 18]**

   (a) Show that $[(k+2)!]k - (k!)(k-2) = (k!)(k^3 + 3k^2 + k + 2)$. [1]

   (b) Hence find $\displaystyle\sum_{k=3}^{n} \left[(k!)(k^3 + 3k^2 + k + 2)\right]$ in terms of n. [3]

   (c) Explain if the series in (b) converges. [1]

2. **[AJC MYE 18]**

   A school is commemorating her 35th Anniversary with a 35 000 km Resilience Run. Two groups of students pledge to accumulate a total distance of 35 000 km, but they intend to clock the distance with their own methods as shown below:

   - Group A: covers collectively 20 km on Day 1, and the distance to be covered on each subsequent day is increased by $\frac{1}{11}$ times the distance covered on the previous day.
   - Group B: covers collectively 20 km on Day 1, and the distance to be covered on each subsequent day is increased by d km.

   (a) Find the total distance, to the nearest kilometer, Group A would have covered on days 1 to 30 inclusive. [2]

   (b) Find the least value of N for which the distance covered by Group A on Day N exceeds 3% of the total pledged distance. [2]

   (c) Find the range of values of d if Group B wishes to cover a total distance not less than that covered by Group A by the end of the 30th day. [2]

   Assume now that $d = 6$ km.

   (d) The school wishes to hold a Grand Finale on Founders Day and wants both groups of students to have completed a combined total of 35 000 km from Day 1 until this day inclusive. How many days before Founders Day must the students begin to clock their distances? [2]

3. **[MI MYE 18]**

   (a) Find c such that $\dfrac{1}{r} - \dfrac{2}{r+1} + \dfrac{1}{r+2} = \dfrac{c}{r(r+1)(r+2)}$. [1]

   (b) Hence find $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$.
   There is no need to express your answer as a single algebraic fraction. [2]

   (c) Determine, with the aid of a sketch of $y = \dfrac{1}{x(x+1)(x+2)}$, which of the two quantities $\displaystyle\int_1^{n+1} \frac{1}{x(x+1)(x+2)}\,\mathrm{d}x$ or $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$ is larger for $n \geq 1$. [2]

   (d) Use your answer to (b) to find $\displaystyle\sum_{r=2}^{n} \frac{1}{r^3 - r}$. [2]

4. **[MI MYE 18]**

The nth term, T_n, of a sequence is given by $\lg 3x^{n-1}$, where x is a positive constant.

(a) Show that the sequence is an arithmetic progression for all positive integers n. [2]

When $\lg 3$ is subtracted from each of the fifth, third and second terms of the above arithmetic progression, these terms become the first three consecutive terms of a geometric progression.

(b) Find the range of values of x for which the sum of the first 10 terms of the arithmetic progression differs from the sum to infinity of the geometric progression by more than 2. [5]

5. **[NJC MYE 18 (modified)]**

(a) Prove that $\dfrac{1}{(n-2)!n} = \dfrac{1}{(n-1)!} - \dfrac{1}{n!}$ for $n \geq 2$. [2]

(b) Find $\displaystyle\sum_{n=2}^{N} \frac{1}{(n-2)!n}$. [3]

(c) Explain why $\displaystyle\sum_{n=2}^{N} \frac{1}{(n-2)!n}$ in convergent and find the sum to infinity. [2]

6. **[AJC MYE 18]**

Given that $\displaystyle\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$, show that

$$\sum_{r=n}^{2n} \left(2r^3 + 3r\right) = an(n+1)(5n^2+n+3),$$

where a is a constant to be determined. [4]

7. **[TPJC MYE 18 (modified)]**

There are two national savings plans available, Plan A and Plan B, both requiring monthly deposits of a fixed amount of money on the first day of each month. In Plan A, an interest rate of 0.45% per month is given on the last day of each month.

(a) Saver A chooses Plan A and deposits \$200 on the first day of each month. Show that the total savings for saver A at the end of n months is

$$\$\frac{401800}{9}(1.0045^n - 1).$$

[2]

(b) Hence calculate the total interest saver A receives after 12 months. [2]

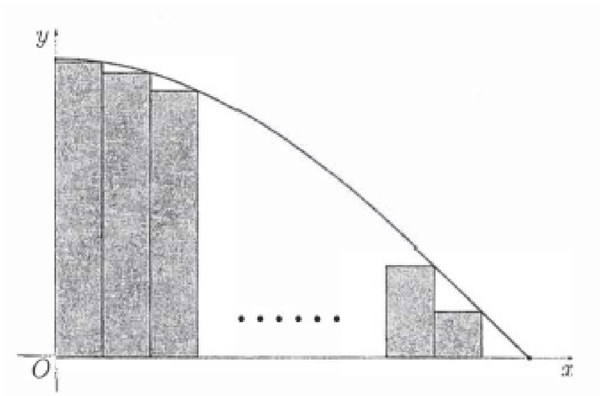
(c) Saver A decides to withdraw the money once the total savings reaches \$4000. How many months must saver A wait in order to withdraw the money? [3]

Plan B gives an interest that starts at \$2 on the last day of the first month, and rises by \$1 on the last day of each subsequent month. This means that the interest will be \$3 on the last day of the second month, and \$4 on the last day of the third month. Saver B chooes Plan B and deposits \$$y$ on the first day of each month.

(d) Find the least value of y (to nearest dollar) for saver B to deposit each month in order to save more than saver A at the end of 12 months, assuming both saver A and saver B start their saving plan at the same time. [5]

8. **[NJC MYE 18 (modified)]**

The diagram shows the curve with equation $y = \cos x$, $0 \leq x \leq \frac{\pi}{2}$. The interval $\left[0, \frac{\pi}{2}\right]$ on the x-axis is divided into n equal parts. The area under the curve in this interval may be approximated by the total area A of $n - 1$ rectangles, each of width $\frac{\pi}{2n}$.



(a) Given that $\cos\frac{\pi}{12} + \cos\frac{5\pi}{12} = \frac{\sqrt{6}}{2}$ and by considering the value of A when $n = 6$, explain clearly why

$$1 + \sqrt{2} + \sqrt{3} + \sqrt{6} < \frac{24}{\pi}.$$

[5]

(b) By considering A, find the exact value of

$$\lim_{n\to\infty}\left\{\cos\left(\frac{\pi}{2n}\right) + \cos\left(\frac{2\pi}{2n}\right) + \cdots + \cos\left(\frac{(n-1)\pi}{2n}\right)\right\}.$$

[2]

Answers

1. (b) $(n+1)!(n^2+3n-1)-54$.
   (c) Does not converge as $(n+1)!(n^2+3n-1)-54 \to \infty$ as $n \to \infty$.
2. (a) 2773 km.
   (b) 47.
   (c) $d \geq 4.99$.
   (d) $n = 55$. 54 days before the date.
3. (a) $T_n - T_{n-1} = \lg x$ which is a constant.
   (b) $0 < x < 0.656$ or $x > 0.842$.
4. (a) $c = 2$.
   (b) $\frac{1}{2}\left(\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}\right)$.
   (c) $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$ is larger.
   (d) $\frac{1}{2}\left(\frac{1}{2} - \frac{1}{n} + \frac{1}{n+1}\right)$.
5. $a = \frac{3}{2}$.
6. (b) \$71.37.
   (c) 20 months.
   (d) \$203.
7. (b) $1 - \frac{1}{N!}$.
   (c) 1.
8. $\frac{1}{\pi}$.

Question:	1	2	3	4	5	6	7	8	Total
Marks	5	8	7	7	7	4	12	7	57
Score:									