1. **[ACJC 17 Promos)]**
An arithmetic progression and a geometric progression each has first term $\frac{1}{4}$. The sum of their second terms is $\frac{1}{4}$ and the sum of their third terms is $\frac{1}{8}$. Given that the geometric progression is convergent, find the sum to infinity. [6]

2. **[CJC 17 Promos]**
In a country, households spend 65% of their income on average. To boost the countrys economy, the government decides to pump in x million dollars within its country. This investment sets off a chain reaction, allowing the income to undergo growth in stages as modelled below:

   - At the start of Stage 1, household income increases by x million dollars. Households can afford to spend more, allowing the economy to grow. Based on the average expenditure of 65% of household income, the growth amount is

     $$0.65x \text{ million dollars.}$$

     At the end of Stage 1, the overall increase in income of the country is

     $$x + 0.65x \text{ million dollars.}$$

   - At the start of Stage 2, the amount of $x + 0.65x$ million dollars generates more growth of

     $$0.65^2x \text{ million dollars.}$$

     At the end of Stage 2, the overall increase in income of the country is

     $$x + 0.65x + 0.65^2x \text{ million dollars.}$$

   - The process continues indefinitely and creates a phenomenon called the multiplier effect in the country.

   In this context, the multiplier is the ratio of the total change in the countrys income to the initial increase of x million dollars.

   (a) i. Show that the growth amount in stage 5 due to the investment is $0.116x$ million dollars, correct to 3 decimal places. [1]

   ii. By which stage will the growth amount due to the investment fall below 1% of the initial investment? [2]

   iii. By the end of which stage will the overall increase in income due to the investment exceed 2.5 times of the initial investment? [3]

   iv. By considering an infinite geometric series, show that the size of the multiplier is $\dfrac{20}{7}$. [1]

   (b) It is proposed that to increase the multiplier effect, changes have to be made to the spending habits of households through tax rebates. Determine the percentage of household income that should be spent on average in order to achieve an overall increase of $3x$ million dollars by the end of stage 3. Leave your answer to 1 decimal place. [3]

3. **[DHS 17 Promos]**
The sum of the first n terms of a sequence, S_n, is a quadratic polynomial in n. It is given that the first term and second term of the sequence is 50 and 45 respectively, and that the sum of the next 10 terms is 175. Find S_n and hence show that this sequence is an arithmetic progression. [5]

4. **[DHS 17 Promos]**
At the end of 2015, the farmers of a chocolate factory had 40 000 square metres of fertile land for planting cocoa. At the end of 2016, the farmers realised that the area of fertile land had decreased by 800 square metres due to nutrient loss from the soil. In each subsequent year, the decrease in the area of fertile land is $\frac{7}{8}$ of the decrease in the previous year.

   (a) Show that the remaining area of the fertile land at the end of the n^{th} year starting from 2016 (where 2016 is the 1st year, 2017 is the 2nd year, and so on) can be expressed as $a + b\left(\frac{7}{8}\right)^n$, where a and b are constants to be determined. [3]

   (b) After how many complete years, starting from 2016, will the area of fertile land first fall below 35 000 square metres? Show your working clearly. [3]

   (c) Find the area of fertile land available to the farmer in the long run. [1]

5. **[JJC 17 Promos]**

   (a) The first term of an arithmetic progression is a and its common difference is $2a$. Given that the $(N-2)$th term of the arithmetic progression is 663 and the Nth term is 731, find a and N and also the sum of the first N terms. [5]

   (b) A ball is dropped from a height of 2 metres. Each time it falls vertically on to a horizontal floor, it rebounds to $\frac{4}{5}$ of the height from which it fell. Show that the total distance covered by the ball when it touched the floor for the nth time is given by

$$18 - 16\left(\frac{4}{5}\right)^{n-1} \text{ metres.}$$

[3]

   i. If l is the total distance the ball travels before coming to rest, write down the value of l. [1]

   ii. Hence find the least value of n if the total distance exceeds 90% of l. [2]

6. **[NJC 17 Promos]**
The tenth, third and first term of an arithmetic series are the fourth, fifth and sixth terms of a geometric series respectively. It is also given that the terms of the arithmetic series are increasing.

   (a) Prove that $d = \frac{5a}{4}$, where a and d are the first term and the common difference of the arithmetic series respectively. Deduce why the geometric series converges. [4]

   (b) Show that the first term of the geometric series if $\frac{16807}{32}a$. Hence, find the sum to infinity in terms of a. [3]

7. **[SAJC 17 Promos]**
Sam plans to save \$900 on 1 January 2018. On the first day of each subsequent month he will save \$3 more than in the previous month, so that he will save \$903 on 1 February 2018, \$906 on 1 March 2018, and so on.

(a) On which date will he first have saved over \$80 000 in total? [4]

Sally will put \$900 on 1 January 2018 into a stock portfolio with a monthly growth rate of $r\%$, so that on the last day of each month, the amount in the portfolio on that day is increased by $r\%$. She will put a further \$900 into the portfolio on the first day of each subsequent month.

(b) Find an expression, in terms of r and n, for the value of the portfolio on the first day of the nth month (where January 2018 is the 1st month, February 2018 is the 2nd month, and so on). Hence, find the minimum monthly growth rate for Sallys portfolio such that the value of the portfolio on 2 January 2023 will exceed the amount Sam will have saved on 2 January 2023. [4]

Answers

1. $\frac{\sqrt{2}}{4}$.
2. (a) ii. Stage 11.
   iii. Stage 4.
   (b) 81.1%.
3. $S_n = -\frac{5}{2}n^2 + \frac{105}{2}n$.
4. (a) $a = 33\,600, b = 6400$.
   (b) 12.
   (c) 33 600 square metres.
5. (a) $a = 17, N = 22, 8228$.
   (b) $l = 18$.
   Least $n = 11$.
6. (a) Since $-1 < r = \frac{2}{7} < 1$, the geometric series converges.
   (b) $S_\infty = \frac{117649}{160}a$.
7. (a) 79 months. 1 July 2024.
   (b) $\frac{90000}{r}\left((1 + \frac{r}{100})^n - 1\right)$.
   0.314%.