YISHUN INNOVA JUNIOR COLLEGE
JC 1 MID YEAR EXAMINATION
Higher 2

CANDIDATE NAME	

CG		INDEX NO	

MATHEMATICS

9758/01

Paper 1

1 JULY 2019

2 hours 15 minutes

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9
Marks									

Total marks	

This paper consists of 16 printed pages.

1 At a particular carnival event, visitors are required to purchase vouchers in \$2, \$5 and \$10 denominations in order to participate in activities at various game booths. At the end of the event, the vouchers collected by a particular game booth called "Startgame" are sorted and counted. A total of \$400 worth of vouchers is collected. The number of \$2 vouchers collected is ten more than the number of \$10 vouchers collected. It is also found that the number of \$5 vouchers is equal to the total number of \$2 and \$10 vouchers.
Find the number of \$2, \$5 and \$10 vouchers collected by "Startgame". [4]

2 Without using a calculator, solve the inequality

$$\frac{x^2-2}{x+3} \geq 1-x .$$

[5]

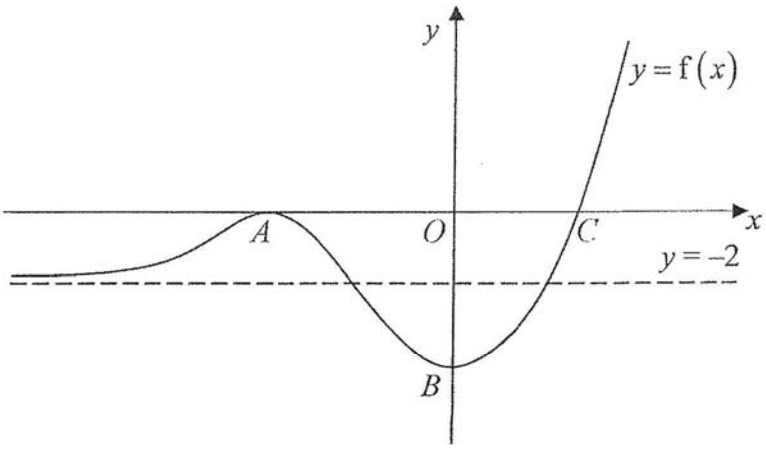
3 A curve C has equation $y = \dfrac{2x-7}{x-3}$.

(i) Express the equation of C in the form $y = A + \dfrac{B}{x-3}$, where A and B are constants to be found. [1]

(ii) Sketch the graph of C, giving the equations of any asymptotes and the coordinates of any points of intersection with x- and y-axes. [2]

(iii) Describe a sequence of transformations which transforms the graph of C on to the graph of $y = -\dfrac{1}{x}$. [2]

4 The diagram shows the curve $y = \mathrm{f}(x)$ with an asymptote $y = -2$. The curve has turning points at A and B and crosses the x-axis at the point C. The coordinates of A, B and C are $(-3, 0)$, $(0, -4)$ and $(2, 0)$ respectively.



Sketch, on separate diagrams, the graphs of

(i) $y = \mathrm{f}(-x) + 3$, [2]

(ii) $y = \mathrm{f}(|x| - 1)$, [2]

(iii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

stating the equations of any asymptotes and the coordinates of the points corresponding to A, B and C where appropriate.

(i)

(ii)

(iii)

5 A curve C has parametric equations

$$x = t^3, \ \ y = 2t^2 - 1; \ \text{ for } t \in \mathbb{R}.$$

(i) Sketch C, stating clearly the exact coordinates of the points of intersection with the axes. [2]

(ii) The line $y = -x - 1$ cuts C at points A and B. Show that the length of $AB = k\sqrt{2}$, where k is a constant to be found. [3]

(iii) Points D and E on C have parameters -1 and p respectively. The point F is the midpoint of DE. Find a cartesian equation of the curve traced by F as p varies. [3]

6 **(a)** Sketch, on the same diagram, the graphs of $y=|1-3x|$ and $y=-x^2+x+2$. [2]

Hence find the set of values of x that satisfies the inequality

$$x^2 \le x + 2 - |1 - 3x|,$$

giving your answer in exact form. [4]

(b) Sketch the curve C with equation $9x^2 - 4y^2 - 36x + 32y - 64 = 0$. [4]

(i) Deduce the range of values of k such that the line $y - 4 = k(x-2)$ does not intersect C. [1]

(ii) Deduce the value of m such that the curve $\dfrac{(x-2)^2}{m^2} + (y-4)^2 = 1$ intersects C at exactly two distinct points. [1]

7 **(a)** Given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find a formula for $\sum_{r=0}^{n} (6r+1)(2r-1)$, fully factorising your answer. [4]

(b) **(i)** Show that $\frac{1}{2n-1} - \frac{1}{2n+1} = \frac{k}{4n^2-1}$, where k is a constant to be determined. [1]

(ii) Hence find $\sum_{n=2}^{N} \frac{1}{4n^2-1}$ in terms of N. [3]

(iii) Using the result in part **(ii)**, find

$$\frac{1}{99}+\frac{1}{143}+\frac{1}{195}+\cdots+\frac{1}{16N^2-1}$$

in terms of N. [3]

8 A table top, A, is made up of wooden bars of decreasing lengths.

(i) The first bar of the table top A has length 150 cm and the lengths of the bars form a geometric progression. The 30th bar has length 10 cm.

(a) Find the total length of the first 10 bars, giving your answer correct to the nearest cm. [4]

(b) Show that the total length of all the bars must be less than 1683 cm, no matter how many bars there are. [2]

Another table top, B, consists of 25 wooden bars. The lengths of the first 13 bars are in arithmetic progression with common difference d cm, where $d > 0$, and the lengths of the last 13 bars are in arithmetic progression with common difference $-d$ cm. The 13th bar, with length 120 cm, is the longest bar of table top B and the total length of all 25 bars is 1752 cm.

(ii) Find the value of d and the total length of the first bar and the 25th bar of table top B. [5]

9 The function f is defined by

$$f : x \mapsto \frac{1}{2x+1}, \ x \in \mathbb{R}, \ x \geq 0.$$

(i) Sketch the graph of f and explain why the function f^{-1} exists. [2]

(ii) Find $f^{-1}(x)$ and write down the domain and range of f^{-1}. [4]

(iii) Sketch on the same axes in part **(i)**, the graphs of $y = f^{-1}(x)$ and $y = ff^{-1}(x)$. [2]

(i) & (iii)

(ii)

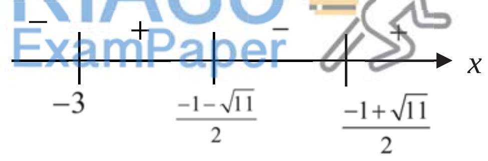
The function g is defined by

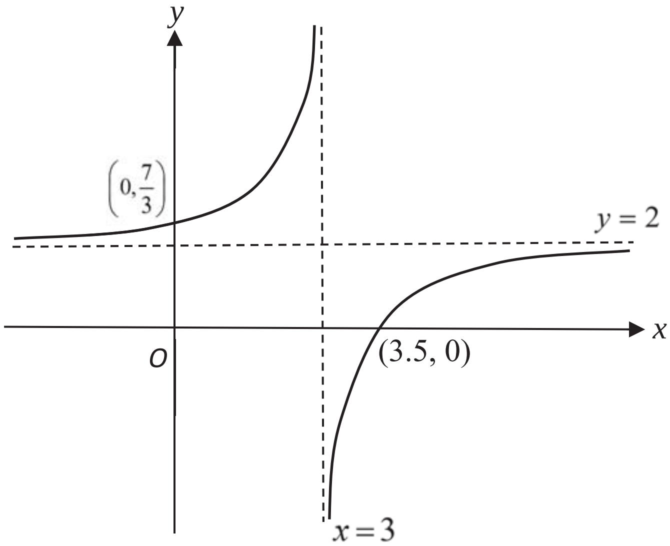
$$g : x \mapsto \ln\sqrt{x}, \quad x \in \mathbb{R}, \quad x > 0.$$

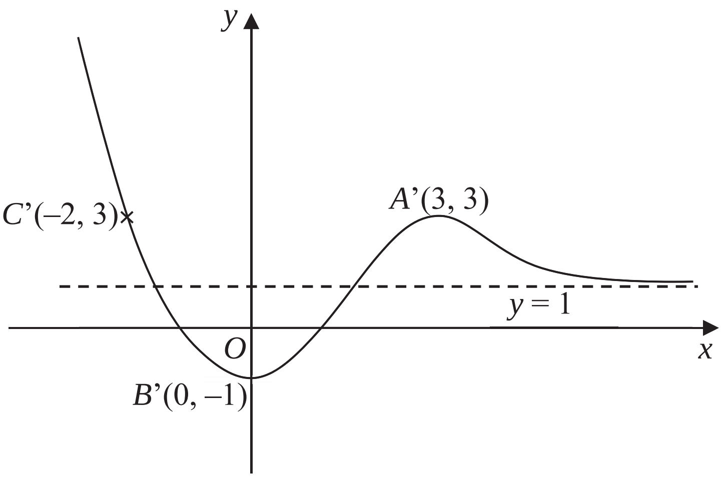
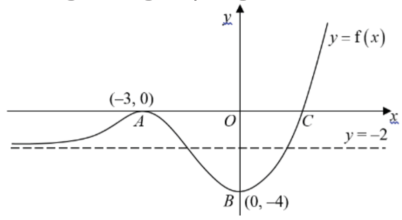
(iv) Show that the composite function gf exists. [1]

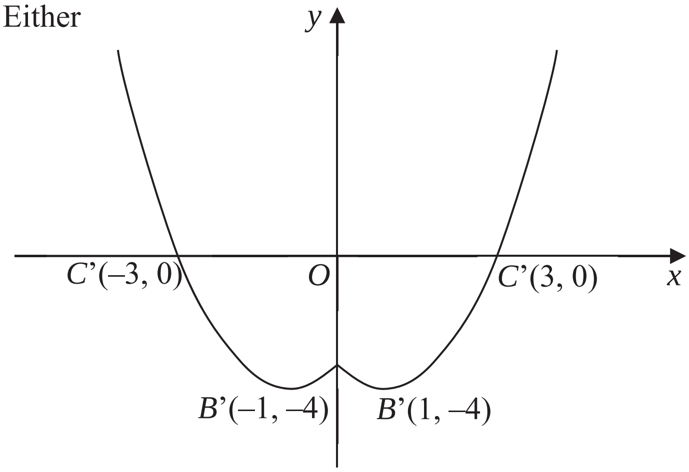
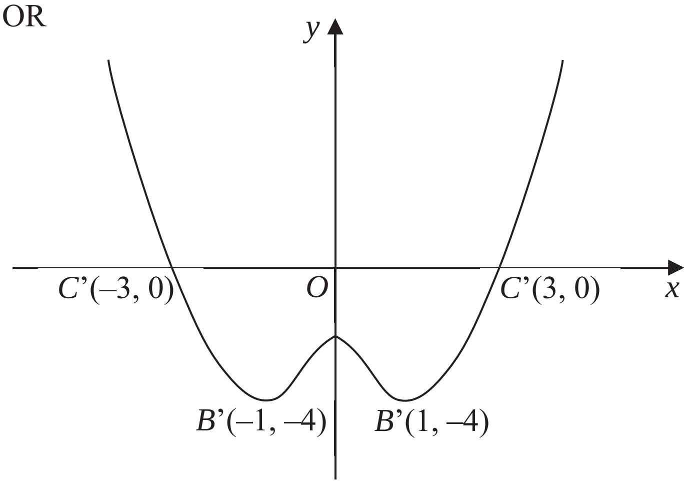
(v) Find $\mathrm{gf}(x)$ and state the domain and range of gf. [3]

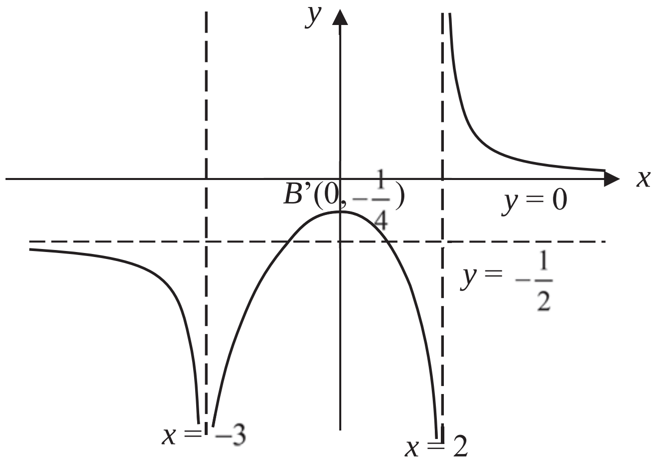
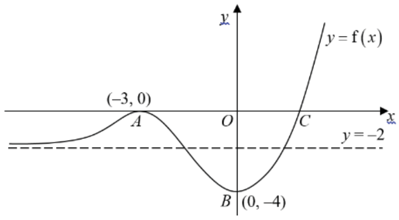
– **End Of Paper** –

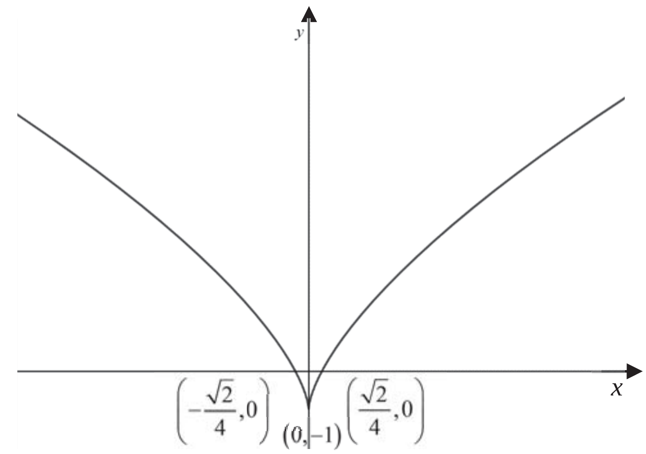
1	Let x, y & z be the number of \$2, \$5 and \$10 vouchers collected respectively. $2x+5y+10z=400 \quad \cdots\cdots\cdots(1)$ $x-z=10 \quad \cdots\cdots\cdots(2)$ $y=x+z$ $\Rightarrow x-y+z=0 \quad \cdots\cdots\cdots(3)$ Solving equations (1), (2) and (3) simultaneously: $x=25,\ y=40,\ z=15$ Hence, there are 25 \$2 vouchers, 40 \$5 vouchers and 15 \$10 vouchers collected.	Common error: • Did not define the variables used • Formed wrong eqns such as $x+y+z=400$ $x=10z$ Read the qn carefully. "x is 10 more than z" is different from "x is 10 times more than z"
2	$\frac{x^2-2}{x+3}\geq 1-x$ $\frac{x^2-2}{x+3}+x-1\geq 0$ $\frac{x^2-2+(x-1)(x+3)}{x+3}\geq 0$ $\frac{2x^2+2x-5}{x+3}\geq 0$ Let $2x^2+2x-5=0$ $x=\frac{-2\pm\sqrt{2^2-4(2)(-5)}}{4}$ $x=\frac{-2\pm\sqrt{44}}{4}$ $x=\frac{-1\pm\sqrt{11}}{2}$  $\therefore\ -3<x\leq\frac{-1-\sqrt{11}}{2}$ or $x\geq\frac{-1+\sqrt{11}}{2}$	Points to note: • Please be extra careful in handling + and – signs. $\frac{x^2-2}{x+3}-1+x=\frac{x^2-2+(-1+x)(x+3)}{x+3}$, NOT $\frac{x^2-2-(1+x)(x+3)}{x+3}$. • Qn clearly stated that the use of calculator is not allowed. So can't use a calculator to solve the eqn $2x^2+2x-5=0$. • You must draw the number line and show the working for a sign test clearly. • Since the denominator is $(x+3)$ and it can't be 0, so $x\neq -3$.

3 (i)	$y=\frac{2x-7}{x-3}$ $=\frac{2(x-3)-1}{x-3}$ $=2+\frac{-1}{x-3}$	This is only a 1-mark question. Do not use long and tedious working. Most efficient method is to "juggle" or do long division.
(ii)		• Remember to state the coordinates of the points of intersection with x- and y-axes. • The curve must approach the asymptotes. • Do not draw bended straight lines.
(iii	$y=2-\frac{1}{x-3}\xrightarrow{(1)}y=-\frac{1}{x-3}\xrightarrow{(2)}y=-\frac{1}{x}$ The sequence of transformations is (1) Translation of 2 units in the negative y-direction (2) Translation of 3 units in the negative x-direction Note: (1) and (2) can be swapped.	• Do not use layman terms such as "shift" or "move". • Read the qn carefully. Some did the **wrong order**, transforming the graph of $-\frac{1}{x}$ onto the graph of $2-\frac{1}{x-3}$.

4 (a)	$y=\mathrm{f}(-x)+3$ 	Original graph given:  The sequence of transformations is: (1) Translation of 3 units in the positive y-direction. (2) Reflection of the graph about the y-axis. The order of the transformation may be swapped since one involves transformation in the y-direction, while the other in x-direction.

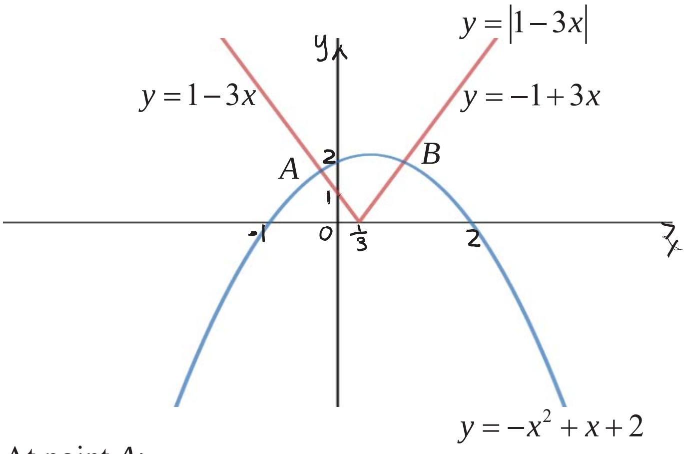
4 (b)	$y=\mathrm{f}(\lvert x\rvert-1)$ Either  OR  	Both transformations involved the *x*-direction. Thus we need to do translation first. 1. Translate the graph of $y=\mathrm{f}(x)$ by 1 unit in the positive x-direction to obtain $y=\mathrm{f}(x-1)$. 2. Keep the "positive-x portion" of $y=\mathrm{f}(x-1)$, discard the "negative-x portion" of $y=\mathrm{f}(x-1)$. 3. Do a mirror image of the "positive x portions" of $y=\mathrm{f}(x-1)$ in the y-axis Note: The graph of $y=\mathrm{f}(\lvert x\rvert-1)$ has a **sharp point** at the y-intercept

4 (c)	$y=\dfrac{1}{f(x)}$ 	Referring to the graph of $y=f(x)$,  Note that we take reciprocal of all y-coordinates. Thus • The original x-inter cepts →Vertical asymptotes. • New horizontal asymptote: $y=-\frac{1}{2}$ • Min pt $B(0,-4)$ becomes max pt $B'\left(0,-\frac{1}{4}\right)$. Thus B' lies above the horizontal asymptote $y=-\frac{1}{2}$.

5 a (i)	$x = t^3, \quad y = 2t^2 - 1$ for $t \in \mathbb{R}$ 	Did you • Change the window setting on the values of t to include -ve values since $t \in \mathbb{R}$? • Find the EXACT values of the coordinates of the x-intercepts? You may give the coordinates of the x-intercepts as $\left(-\frac{1}{2\sqrt{2}}, 0\right)$ and $\left(\frac{1}{2\sqrt{2}}, 0\right)$.
(ii)	Substitute $x = t^3$, $y = 2t^2 - 1$ into $y = -x - 1$: $2t^2 - 1 = -t^3 - 1$ $t^3 + 2t^2 = 0$ $t^2(t+2) = 0$ $t = 0$ or $t = -2$ When $t = 0$, $x = 0$ and $y = -1$ When $t = -2$, $x = -8$ and $y = 7$ Length of $AB = \sqrt{(0-(-8))^2 + (-1-7)^2} = \sqrt{128} = 8\sqrt{2}$ **<u>Alternative:</u>** Converting the parametric eqn into Cartesian eqn by using $t = x^{1/3}$, thus $y = 2x^{\frac{2}{3}} - 1$. Equate this with $y = -x - 1$ to find the points of intersection. $2x^{\frac{2}{3}} - 1 = -x - 1$ $x^{\frac{2}{3}}\left(2 + x^{\frac{1}{3}}\right) = 0$ $x^{\frac{2}{3}} = 0$ or $2 + x^{\frac{1}{3}} = 0$ $x = 0$ or $x = -8$ $\therefore\ y = -1$ or $y = 7$ Length of $AB = \sqrt{(0-(-8))^2 + (-1-7)^2} = \sqrt{128} = 8\sqrt{2}$	You cannot use GC to find the coordinates of A and B, as this approach is not rigorous enough for a "show" question. What is the formula for finding distance between two points?

(iii	$x=t^3, \qquad y=2t^2-1$ Since point D has parameter $-1 \Rightarrow D(-1,1)$ Similarly, E has parameter $p \Rightarrow E\left(p^3, 2p^2-1\right)$ $\therefore F\left(\frac{p^3-1}{2}, \frac{1+2p^2-1}{2}\right)=\left(\frac{p^3-1}{2}, p^2\right)$ Let $x=\frac{p^3-1}{2} \Rightarrow p=(2x+1)^{\frac{1}{3}} \qquad \ldots(1)$ and $y=p^2 \qquad \ldots(2)$ Subsititue (1) into (2) $y=\left[(2x+1)^{\frac{1}{3}}\right]^2$ $y=(2x+1)^{\frac{2}{3}}$ **<u>Alternative:</u>** Let $x=\frac{p^3-1}{2} \Rightarrow 2x=p^3-1 \quad \ldots(1)$ and $y=p^2 \Rightarrow p=\pm\sqrt{y} \qquad \ldots(2)$ Subsititue (2) into (1), $2x=\left(\pm\sqrt{y}\right)^3-1$ $2x+1=\pm y^{\frac{3}{2}}$ $(2x+1)^2=y^3$ $y=(2x+1)^{\frac{2}{3}}$	**<u>Important note:</u>** • After finding the coordinates of the midpoint F, you need to convert these coordinates from parametric to Cartesian form. • If you are taking square root to express p in terms of y, remember to include ± sign and eventually combine both equations into a single Cartesian equation

6 (a)



At point A:

Equation $1-3x=-x^2+x+2$

$$x^2-4x-1=0$$

$$x=\frac{4\pm\sqrt{16+4}}{2}$$

$$x=2-\sqrt{5} \text{ or } x=2+\sqrt{5} \text{ (rejected } \because x<0\text{)}$$

At point B:

Equation $-1+3x=-x^2+x+2$

$$x^2+2x-3=0$$

$$(x-1)(x+3)=0$$

$$x=1 \quad \text{or} \quad x=-3 \text{ (rejected } \because x>0\text{)}$$

For $x^2 \le x+2-|1-3x|$

$$|1-3x| \le -x^2+x+2$$

From the graphs, the required solution set is

$$\left\{x \in R : 2-\sqrt{5} \le x \le 1\right\}$$

Alternative:

$$|1-3x|=-x^2+x+2$$

$$(1-3x)^2=\left(-x^2+x+2\right)^2$$

$$1-6x+9x^2=x^4-2x^3-3x^2+4x+4$$

$$x^4-2x^3-12x^2+10x+3=0$$

$$(x+3)(x-1)\left(x^2-4x-1\right)=0$$

$$x=-3 \text{ or } x=1 \text{ or } x^2-4x-1=0$$

$$x=\frac{4\pm\sqrt{16+4}}{2}$$

$$x=2-\sqrt{5} \text{ or } x=2+\sqrt{5}$$

Since $-1<x<2$, $\therefore x=1$ or $x=2-\sqrt{5}$.

For $x^2 \le x+2-|1-3x|$

$$|1-3x| \le -x^2+x+2$$

From the graphs, the required solution set is

$$\left\{x \in R : 2-\sqrt{5} \le x \le 1\right\}$$

Important Note:

- The sharp pt of the modulus graph is at $(\frac{1}{3},0)$, NOT $\left(-\frac{1}{3},0\right)$.
- The maximum of the parabola DOES NOT lie on the y-axis.
- The word HENCE means you need to make use of the graph to solve the inequality. So, find the x-coordinates of A & B by equating $-x^2+x+2$ with the respective linear eqns.
- "Giving answers in exact form" means you CANNOT use GC to find the coordinates of the intersection points. You need to use an algebraic method to find the exact x-coordinates.
- Express the final answer in SET FORM.

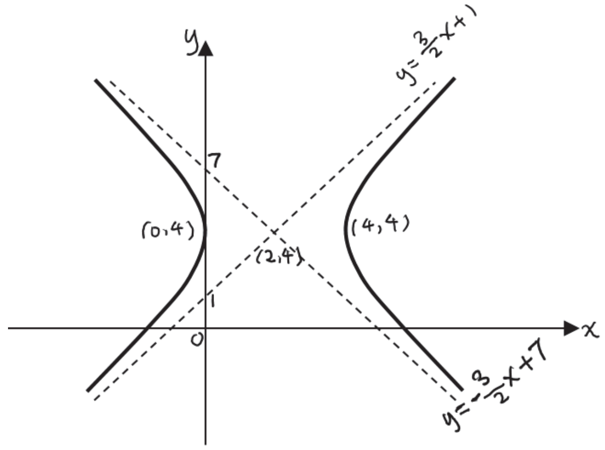
(b)

$$9x^2-4y^2-36x+32y-64=0$$

$$9\left(x^2-4x\right)-4\left(y^2-8y\right)-64=0$$

$$9\left[\left(x-2\right)^2-4\right]-4\left[\left(y-4\right)^2-16\right]-64=0$$

$$\frac{\left(x-2\right)^2}{2^2}-\frac{\left(y-4\right)^2}{3^2}=1$$



(i) $k\geq\frac{3}{2}$ or $k\leq-\frac{3}{2}$

Questions:

- Did you realise that the line $y-4=k\left(x-2\right)$ passes through the centre (2,4) of the hyperbola?
- Did you use (2, 4) as a **pivoting point** to rotate the line with equation $y-4=k\left(x-2\right)$ as the gradient k varies?
- Are you able to identify the range of values of k for which the line does not intersect the hyperbola?

Questions:

- Did you realise that this is an eqn of a hyperbola and therefore see the need to **complete the squares** to express the equation in the standard form $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$?
- Did you pay special attention to the $+$ and $-$ signs in the process of completing the squares?
- Did you find the eqns of the asymptotes and realise that both asymptotes have POSITIVE y-intercepts?
- Did you label the centre and vertices of the hyperbola?

<table>
<tr>
<td></td>
<td>(ii) $m=2$

Questions:

Did you realise that the graph of $\frac{(x-2)^2}{m^2}+(y-4)^2=1$ is an ellipse? What is the centre of this ellipse?
What is the length of the horizontal axis of the ellipse in terms of m?
How will you draw the ellipse such that it intersects the hyperbola at two distinct points?

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<tr>
<td>7
(a)</td>
<td>
$\sum_{r=0}^{n}(6r+1)(2r-1)$
$=\sum_{r=0}^{n}\left(12r^2-4r-1\right)$
$=12\sum_{r=0}^{n}r^2-4\sum_{r=0}^{n}r-\sum_{r=0}^{n}1$
$=12\left(\sum_{r=1}^{n}r^2\right)-4\sum_{r=1}^{n}r-\sum_{r=0}^{n}1$
$=12\cdot\frac{n}{6}(n+1)(2n+1)-4\cdot\frac{n}{2}(1+n)-(n+1)$
$=(n+1)\left[2n(2n+1)-2n-1\right]$
$=(n+1)(2n+1)(2n-1)$
OR
$\sum_{r=0}^{n}(6r+1)(2r-1)$

$=(1)(-1)+\sum_{r=1}^{n}\left(12r^2-4r-1\right)$
$=-1+12\sum_{r=1}^{n}r^2-4\sum_{r=1}^{n}r-\sum_{r=1}^{n}1$
$=-1+12\cdot\frac{n}{6}(n+1)(2n+1)-4\cdot\frac{n}{2}(n+1)-n$
$=(n+1)\left[2n(2n+1)-2n-1\right]$
$=(n+1)(2n+1)(2n-1)$
</td>
<td>
Common Error:
$\sum_{r=0}^{n}r^2=\sum_{r=1}^{n+1}r^2$
This is WRONG coz
LHS=$0^2+1^2+2^2+\cdots+n^2$
RHS=$1^2+2^2+\cdots+n^2+(n+1)^2$
LHS $\neq$ RHS

You need to know that:

$\sum_{r=0}^{n}r^2=0^2+\sum_{r=1}^{n}r^2=\sum_{r=1}^{n}r^2$
$\sum_{r=0}^{n}r=0+1+2+\ldots+n=\sum_{r=1}^{n}r$
$\sum_{r=1}^{n}r=\frac{n}{2}(1+n)$
$\sum_{r=0}^{n}1=\underbrace{1+1+\ldots+1}_{(n+1)\text{ times}}=n+1$

</td>
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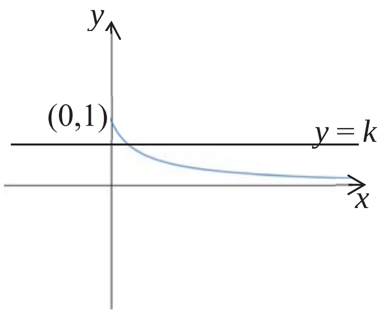
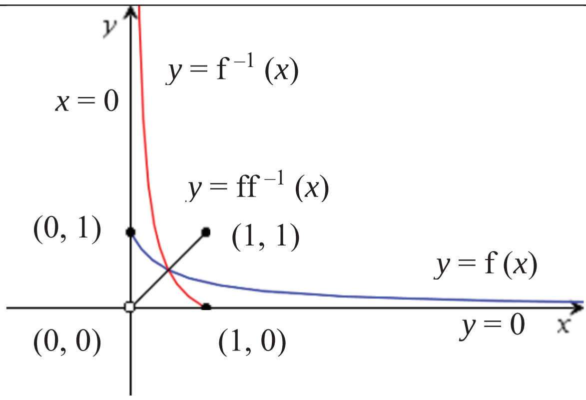
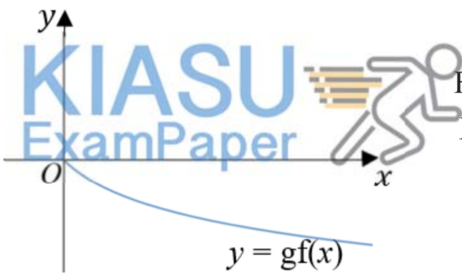
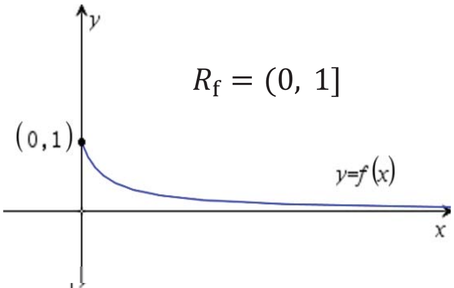
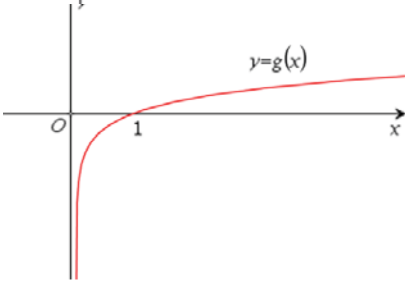
<tr>
<td>b
(i)</td>
<td>$\frac{1}{2n-1}-\frac{1}{2n+1}=\frac{2n+1-(2n-1)}{(2n-1)(2n+1)}=\frac{2}{4n^2-1}$</td>
<td></td>
</tr>
</table>

b (ii)	$\displaystyle\sum_{n=2}^{N}\frac{1}{4n^2-1}=\frac{1}{2}\sum_{n=2}^{N}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$ $=\dfrac{1}{2}\left(\begin{array}{rcccc} & \frac{1}{3} & - & \cancel{\frac{1}{5}} \\ + & \cancel{\frac{1}{5}} & - & \cancel{\frac{1}{7}} \\ + & \cancel{\frac{1}{7}} & - & \cancel{\frac{1}{9}} \\ + & & \vdots & \\ + & \cancel{\frac{1}{2N-3}} & - & \cancel{\frac{1}{2N-1}} \\ + & \cancel{\frac{1}{2N-1}} & - & \frac{1}{2N+1} \end{array}\right)$ $=\dfrac{1}{6}-\dfrac{1}{2(2N+1)}$	Use Method of Difference to solve this part.
b (iii	$\dfrac{1}{99}+\dfrac{1}{143}+\dfrac{1}{195}+\cdots+\dfrac{1}{16N^2-1}$ $=\displaystyle\sum_{n=5}^{2N}\frac{1}{4n^2-1}$ $=\displaystyle\sum_{n=2}^{2N}\frac{1}{4n^2-1}-\sum_{n=2}^{4}\frac{1}{4n^2-1}$ $=\left(\dfrac{1}{6}-\dfrac{1}{2(4N+1)}\right)-\left(\dfrac{1}{6}-\dfrac{1}{2(8+1)}\right)$ $=\dfrac{1}{18}-\dfrac{1}{2(4N+1)}$	Question: • What is the link between the given series and $\displaystyle\sum_{n=2}^{N}\frac{1}{4n^2-1}$? • Did you notice the following? $99=4n^2-1\Rightarrow n=5$ i.e. $99=4(5^2)-1$ $143=4(6^2)-1$ $\vdots$ $16N^2-1=4(2N)^2-1$

8 i(a)	$a=150,\quad ar^{29}=10$ $r^{29}=\frac{1}{15}$ $r=\left(\frac{1}{15}\right)^{1/29}$ $S_{10}=\dfrac{150\left[1-\left(\frac{1}{15}\right)^{10/29}\right]}{1-\left(\frac{1}{15}\right)^{1/29}}$ $\approx 1021\text{ cm}$ (nearest cm)	Question: • Did you apply the correct formulae for u_n and S_n in a GP? • You may press your calculator to find the value of r, but keep it to 5 s.f. or more to avoid any loss of accuracy in finding S_{10} • Give your answer for S_{10} correct to the nearest cm.
i(b)	$S_\infty=\dfrac{150}{1-\left(\frac{1}{15}\right)^{1/29}}$ $=1682.49<1683$ Thus total length of all bars is less than 1683 cm	You have done similar questions before in Tut5B. A concluding statement is necessary.
ii	Let the length of the first bar be b cm. $u_{13}=b+12d=120\quad\cdots\cdots(1)$ Total length of 25 bars $=1752$ cm **<u>Method 1:</u>** $\frac{13}{2}[b+120]\times 2-120=1752$ $b=24$ Substituting $b=24$ into eqn (1), $d=8$ **<u>Method 2:</u>** $\frac{13}{2}[2b+12d]\times 2-120=1752$ $13b+78d=936\quad\cdots\cdots(2)$ Solving eqns (1) & (2), $b=24,\quad d=8$ Length of the 1st bar = length of 25th bar = 24 cm Total length = 48 cm.	**<u>Questions:</u>** • The 25 table bars are symmetrical in length with the 13th bar being the centre bar. Why is this so? • What are the formulae for u_n and S_n in an AP? **<u>Note:</u>** Since the table bars are in symmetry, the sum of 25 bars may be expressed as $2\times S_{13}-u_{13}=1752$ or $2\times S_{12}+u_{13}=1752.$

9 (i)	 **Any** horizontal line $y = k$, $(k \in \mathbb{R})$ cuts the graph of f **at most once** so f is a one – one function and therefore function f^{-1} exists. OR **Any** horizontal line $y = k$, $0 < k \le 1$ cuts the graph of f **exactly once** so f is a one – one function and therefore function f^{-1} exists.	**Common Error:** • Failed to sketch the graph based on the domain given. • Incorrect statement in explaining why f^{-1} exists.
(ii)	Let $y = \dfrac{1}{2x+1}$ $2x+1 = \dfrac{1}{y}$ $x = \dfrac{1}{2}\left(\dfrac{1}{y} - 1\right)$ $x = \dfrac{1-y}{2y}$ $\mathrm{f}^{-1}(x) = \dfrac{1-x}{2x}$ $\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} = (0,1]$ $\mathrm{R}_{\mathrm{f}^{-1}} = \mathrm{D}_{\mathrm{f}} = [0,\infty)$	This part was well done by most students. Remember to simplify the expression for $\mathrm{f}^{-1}(x)$.

(iii		**Note:** • A wrong graph in part (i) will naturally affect the results in part (iii) • It is important to clearly indicate inclusion/exclusion of the end points using closed/open circles and state the coordinates of these end points. • The line $y = \mathrm{ff}^{-1}(x)$ is NOT a full line, but a line segment, as it takes the domain of f^{-1}. The origin is thus be excluded.
(iv)	$\mathrm{R_f} = (0,1]$ $\mathrm{D_g} = (0,\infty)$ Since $\mathrm{R_f} \subseteq \mathrm{D_g}$, gf exists.	This part was well done.
(v)	$\mathrm{g}: x \mapsto \ln\sqrt{x},\ \ x \in \mathbb{R},\ \ x > 0.$ $\mathrm{gf}(x) = \mathrm{g}\left(\dfrac{1}{2x+1}\right)$ $= \ln\sqrt{\dfrac{1}{2x+1}}$ $= -\dfrac{1}{2}\ln(2x+1)$ $\mathrm{D_{gf}} = \mathrm{D_f} = [0,\infty)$ **Method ①**  From the graph, $\mathrm{R_{gf}} = (-\infty, 0]$	**Note:** • $\mathrm{gf}(x) \neq \mathrm{fg}(x)$ $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$ • Recall that there are two methods of finding R_{gf}. ① By sketching the graph of $y = \mathrm{gf}(x)$ **Note** : When writing intervals $(a, b), [a, b), (a, b]$ or $[a, b]$, the smaller value is always on the LEFT, the larger on the RIGHT. So it is WRONG to write $R_{\mathrm{gf}} = [0, -\infty)$.

	Method ②   From the graph of g(x), when $0 < x \leq 1, y \leq 0$. Thus $R_{\mathrm{gf}} = (-\infty, 0]$.	② By using the range of f as a “sub-domain” in the graph of g, find the corresponding range.