H

NANYANG JUNIOR COLLEGE

JC1 BLOCK TEST

Higher 2

CANDIDATE NAME

CT CLASS | 1 | 9 | | |

MATHEMATICS **9758/01**

Paper 1

1st July 2019

2 Hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **70**.

Question	1	2	3	4	5	6	7	8	Total
Marks									

This document consists of **16** printed pages.

NANYANG JUNIOR COLLEGE
Internal Examinations

[Turn Over

1 A curve C has parametric equations

$$x = \mathrm{e}^{3t},\ y = t\mathrm{e}^{2t}.$$

(i) Find a cartesian equation of C. [2]

(ii) Sketch C, stating the coordinates of the turning point and any points of intersection with the axes. [2]

2 Without using a calculator, solve the inequality

$$x-1 \le 3+\frac{2}{x+1}.$$ [4]

[2]

3 Referred to the origin O, points A, B and P have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{p}$ respectively. Given that $\mathbf{p} = \mathbf{a} + \lambda\mathbf{b}$, where $\lambda \in \mathbb{R}$. Find the area of the triangle OAP in terms of λ and $|\mathbf{a} \times \mathbf{b}|$. [3]

It is now given that $\mathbf{a} = 4\mathbf{i} + \mathbf{k}$, $\mathbf{b} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ and C is a point on AB such that $2AC = 3CB$.

Find the

(i) position vector of C, [2]

(ii) size of angle OAB and [2]

(iii) exact length of projection of $\overrightarrow{AB}$ onto $\overrightarrow{OA}$. [2]

4 Functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{bx+a}{ax-b} \quad \text{for } x \in \mathbb{R}, x \neq \frac{b}{a}, a > 0, b < 0,$$

$$\mathrm{g}: x \mapsto -\frac{1}{2x} \quad \text{for } x \in \mathbb{R}, x < 0.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(ii) Hence or otherwise find $\mathrm{f}^{2}(x)$. [2]

(iii) Show that fg exists, justifying your answer. [2]

(iv) Find, in terms of a and b, the range of fg . [2]

5 **(a)** **(i)** Given that $\mathbf{a}$ and $\mathbf{b}$ are two non-zero vectors such that $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, what can be deduced about the vectors $\mathbf{a}$ and $\mathbf{b}$? [1]

(ii) Find a unit vector $\mathbf{r}$ such that $\mathbf{r} \times (\mathbf{i} + \mu\mathbf{j} + \mathbf{k}) = \mathbf{r} \times (\mu\mathbf{i} + \mathbf{j} + \mu\mathbf{k})$, where $\mu \in \mathbb{R}, \mu \neq 1$. [3]

(b) Referred to the origin O, points A, B and C have position vectors **a**, **b** and **c** respectively.

(i) Given that **b** and **c** are not parallel, $\mathbf{a}\cdot\mathbf{b}=0$ and $\mathbf{a}\cdot\mathbf{c}=0$. Prove that **a** is perpendicular to $\mu\mathbf{b}+\lambda\mathbf{c}$ for all $\mu, \lambda \in \mathbb{R}$. [1]

(ii) What is the geometrical interpretation of the result in part **(i)**? [1]

(iii) Given that **a** is a unit vector, $\mathbf{b}=\mathbf{i}+\alpha\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=\alpha\mathbf{i}+\mathbf{j}-\alpha\mathbf{k}$, where α is a constant. Find, in terms of α, the value of $|\mathbf{b}\times\mathbf{c}|$ and hence write down a possible vector **a**. [3]

6 A curve C has equation $9x^2 - ay^2 - 36x - 2aby + 9a - ab^2 + 36 = 0$, where a and b are positive constants. C passes through $(2, -2)$ and has an oblique asymptote with equation $y = \frac{3}{2}x - 8$.

(i) Find the values of a and b. [5]

(ii) Find the equation of the other oblique asymptote. [2]

(iii) Sketch C, stating clearly the coordinates of the vertices. [3]

7 A curve has equation $y = f(x)$, where

$$f(x)=\begin{cases}\cos x & \text{for } 0\le x\le \pi,\\ \dfrac{2}{\pi}x-3 & \text{for } \pi< x\le 2\pi,\end{cases}$$

and that $f(x)=f(x+2\pi)$ for all real values of x.

(i) Sketch the curve for $0\le x\le 3\pi$.

Hence state the number of solutions of the equation $f(x) = k$, where $-1<k<1$ and $0\le x\le 3\pi$. [4]

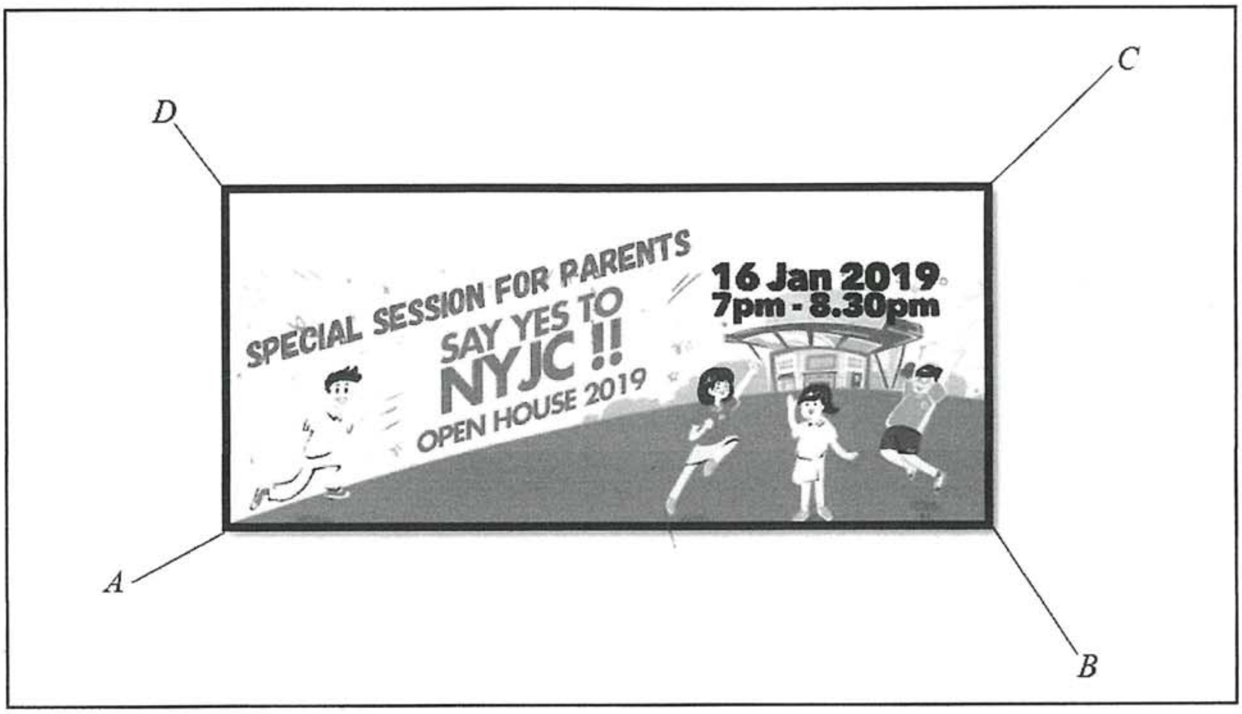
(ii) Sketch on separate diagrams, the graphs of

(a) $y=\left|f(x)\right|$ for $0\le x\le 3\pi$, [2]

(b) $y = \dfrac{1}{\mathrm{f}(x)}$ for $0 \le x \le 3\pi$ and [2]

(c) $y = -\mathrm{f}\left(2x + \dfrac{\pi}{2}\right)$ for $0 \le x \le \pi$. [3]

8



The diagram shows the front view of a banner, which will be hung up by student leaders at the Atrium of Nanyang Junior College. The four corners of the banner are tied to points A, B, C and D tautly with ropes of negligible thickness such that A, B, C and D are coplanar with the flat banner. The coordinates of A, B, C and D are (1, 2, 1), (1, 3, 0), $(\alpha, \beta, 10)$ and (0, 2, 8) respectively with respect to the origin O.

(i) Find a cartesian equation of the plane containing the banner and hence show that $7\alpha + \beta = 0$. [4]

It is further given that points A and B are on the floor and $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ is parallel to the floor.

(ii) Determine the acute angle that the banner makes with the floor. [3]

To better support the banner, the student leaders tie two extra ropes of negligible thickness tautly across the back of the banner. The two ends of one of the ropes are at points A and C and the two ends of the other rope are at points B and D.

(iii) Give a reason why the ropes will overlap. [1]

(iv) If $\beta = 4,$ find the coordinates of the point of overlap of the two ropes. [4]

<table>
<tr><th>Qn1</th><th>Suggested Answers</th><th>Guidance</th></tr>
<tr><td>(i)</td><td>

$x = e^{3t} \;\Rightarrow\; t = \frac{1}{3}\ln x$

$y = te^{2t} \;\Rightarrow\; y = \frac{1}{3}(\ln x)\left(e^{3t}\right)^{\frac{2}{3}} = \frac{1}{3}(\ln x)\left(x^{\frac{2}{3}}\right)$

The Cartesian equation is : $y = \frac{1}{3}(\ln x)\left(x^{\frac{2}{3}}\right)$.

Alternatively, using $e^{\ln f(x)} = f(x)$, we have $x = e^{3t} \;\Rightarrow\; t = \frac{1}{3}\ln x$, then

$y = te^{2t} = \frac{1}{3}(\ln x)\left(e^{2}\right) y = te^{2t}$

$\Rightarrow\; y = \frac{1}{3}(\ln x)e^{\frac{2}{3}(\ln x)} = \frac{1}{3}(\ln x)e^{\ln x^{\frac{2}{3}}} = \frac{1}{3}(\ln x)\left(x^{\frac{2}{3}}\right)$.

</td><td>

The objective is to remove all t in $y = te^{2t}$

1) From $x = e^{3t}$, make t the subject. $t = \frac{1}{3}\ln x$

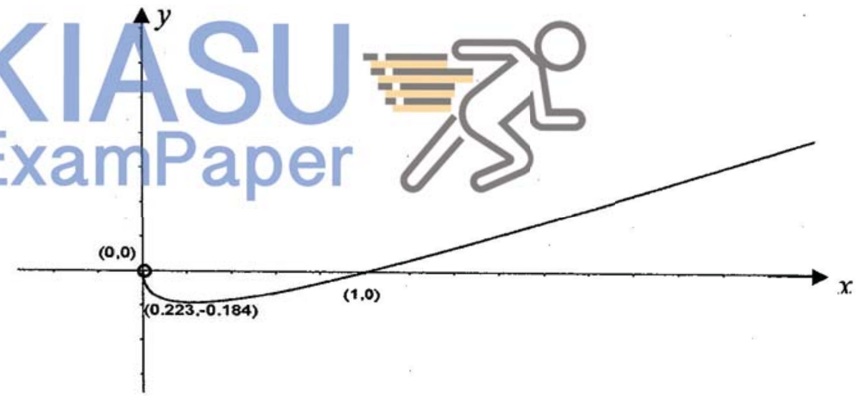
2) To remove e^{2t} in $y = te^{2t}$ as follows:

Since $x = e^{3t}$, $e^{2t} = \left(e^{3t}\right)^{\frac{2}{3}} = x^{\frac{2}{3}}$

Using 1 & 2, obtain the Cartesian equation.

Many unnecessarily took ln on both sides for $y = te^{2t}$, which complicated their workings. Some directly substituted $t = \frac{1}{3}\ln x$ into e^{2t} to obtain $e^{\frac{2}{3}\ln x}$ but did not simplify it further. (see alternative method)

</td></tr>
<tr><td>(ii)</td><td>



</td><td>

Using the Cartesian equation from (i), plot into the GC and find the coordinates of the respective needed points.

Note that (0,0) is excluded because $x = e^{3t}$ and $y = te^{2t}$ both cannot be 0. In fact, $t \to -\infty$, $x \to 0$, $y \to 0$.

If you use the parametric mode to plot, you will not be able to find the coordinates of the turning point. Also, the default value of t is from 0 to 6.283… you need to re-set the t values under "window", so that you obtain the full shape of the curve.

</td></tr>
</table>

<table>
<tr><th>Qn2</th><th>Suggested Answers</th><th>Guidance</th></tr>
<tr><td></td><td>$x-1\le 3+\frac{2}{x+1} \Rightarrow \frac{x^2-3x-6}{x+1}\le 0 \Rightarrow \frac{\left(x-\frac{3-\sqrt{33}}{2}\right)\left(x-\frac{3+\sqrt{33}}{2}\right)}{x+1}\le 0$

$x\le\frac{3-\sqrt{33}}{2}$ or $-1<x\le\frac{3+\sqrt{33}}{2}$</td><td>Make into a single fraction first. Thereafter, find the roots by setting $x^2-3x-6=0$ and solving the equation.
Alternatively, students can use completing the squares and factorise the numerator.
Then apply the test point method to solve the inequality, noting that -1 is not inclusive.</td></tr>
<tr><td></td><td>$|x|-1\le\frac{3|x|+5}{|x|+1} \Rightarrow |x|-1\le\frac{3(|x|+1)+2}{|x|+1} \Rightarrow |x|-1\le 3+\frac{2}{|x|+1}$
Compare with the above inequality, we can replace x by $|x|$ in the above results,
$|x|\le\frac{3-\sqrt{33}}{2}$ or $-1<|x|\le\frac{3+\sqrt{33}}{2}$.
It is no solution for $|x|\le\frac{3-\sqrt{33}}{2}$ because $\frac{3-\sqrt{33}}{2}<0$ but $|x|\ge 0$.
$-1<|x|\le\frac{3+\sqrt{33}}{2} \Rightarrow |x|\le\frac{3+\sqrt{33}}{2}$ as $|x|>-1$ for all real x.

$\therefore\ -\frac{3+\sqrt{33}}{2}\le x\le\frac{3+\sqrt{33}}{2}$</td><td>Observe that the inequality in part (i):
$x-1\le 3+\frac{2}{x+1} \Rightarrow x-1\le\frac{3x+5}{x+1}$.

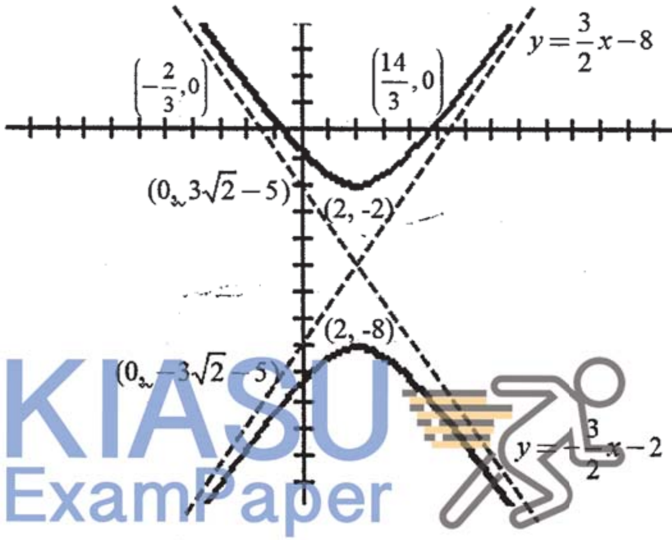
Note that $\frac{3-\sqrt{33}}{2}$ is negative.</td></tr>
</table>

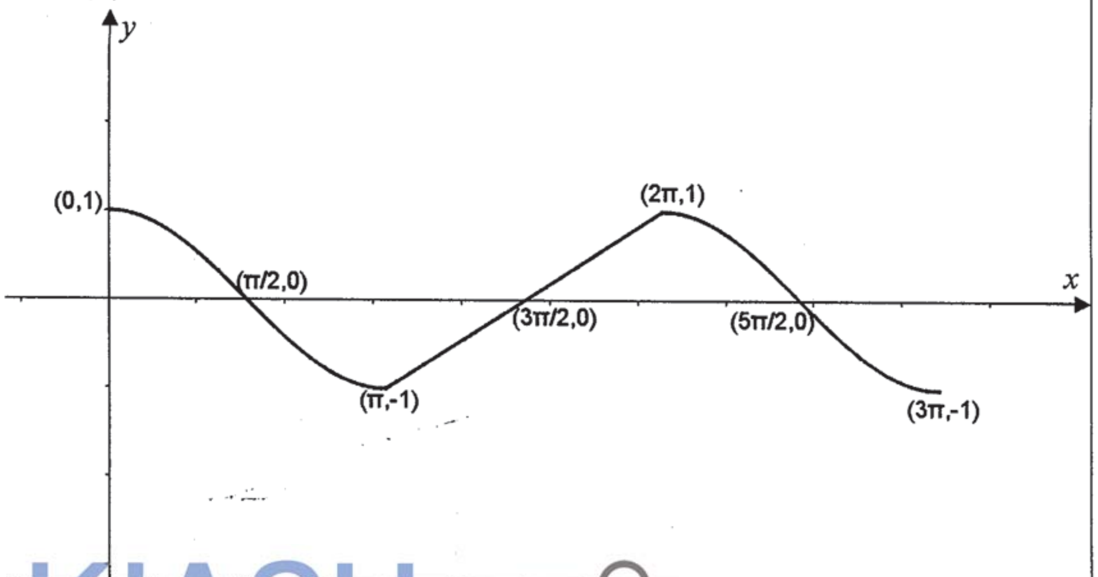
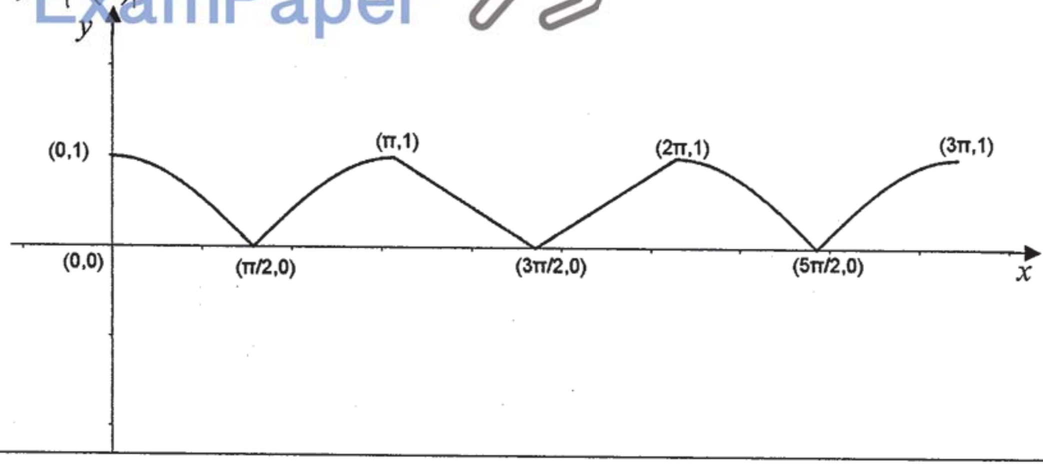
Qn3	Suggested Answers	Guidance
	Area of the triangle $OAP = \frac{1}{2}\lvert \mathbf{a}\times\mathbf{p}\rvert = \frac{1}{2}\lvert \mathbf{a}\times(\mathbf{a}+\lambda\mathbf{b})\rvert = \frac{1}{2}\lvert \mathbf{a}\times\mathbf{a}+\mathbf{a}\times(\lambda\mathbf{b})\rvert$ $= \frac{1}{2}\lvert \underset{\sim}{0}+\lambda(\mathbf{a}\times\mathbf{b})\rvert = \frac{1}{2}\lvert\lambda\rvert\lvert\mathbf{a}\times\mathbf{b}\rvert$	λ could be positive or negative. Hence the need for modulus when we separate it from $\lvert\mathbf{a}\times\mathbf{b}\rvert$. Question asked to leave in terms of $\lvert\mathbf{a}\times\mathbf{b}\rvert$ and λ. Hence $\frac{1}{2}\lvert\lambda(\mathbf{a}\times\mathbf{b})\rvert$ is not sufficient.
(i)	$AC{:}CB = 3{:}2$ $\overrightarrow{OC} = \frac{2\overrightarrow{OA}+3\overrightarrow{OB}}{2+3} = \frac{1}{5}\left[2\begin{pmatrix}4\\0\\1\end{pmatrix}+3\begin{pmatrix}4\\5\\6\end{pmatrix}\right] = \begin{pmatrix}4\\3\\4\end{pmatrix}$	Apply ratio theorem.
(ii)	$\overrightarrow{AB} = \overrightarrow{OB}-\overrightarrow{OA} = 5\mathbf{j}+5\mathbf{k}$ $\measuredangle OAB = \cos^{-1}\left(\frac{\overrightarrow{AO}\bullet\overrightarrow{AB}}{\lvert\overrightarrow{AO}\rvert\lvert\overrightarrow{AB}\rvert}\right) = \cos^{-1}\left(\frac{-5}{\sqrt{17}\times\sqrt{50}}\right) = 99.9^\circ$	O, A, B are fixed points, hence we should not have modulus for the dot product. Direction of vectors matters here. You can also use $\overrightarrow{OA}\bullet\overrightarrow{BA}$. Other dot products are not accepted.
(iii)	Length of projection of $\overrightarrow{AB}$ onto $\overrightarrow{OA} = \left\lvert\begin{pmatrix}0\\5\\5\end{pmatrix}\bullet\frac{1}{\sqrt{17}}\begin{pmatrix}4\\0\\1\end{pmatrix}\right\rvert = \frac{5}{\sqrt{17}}$	

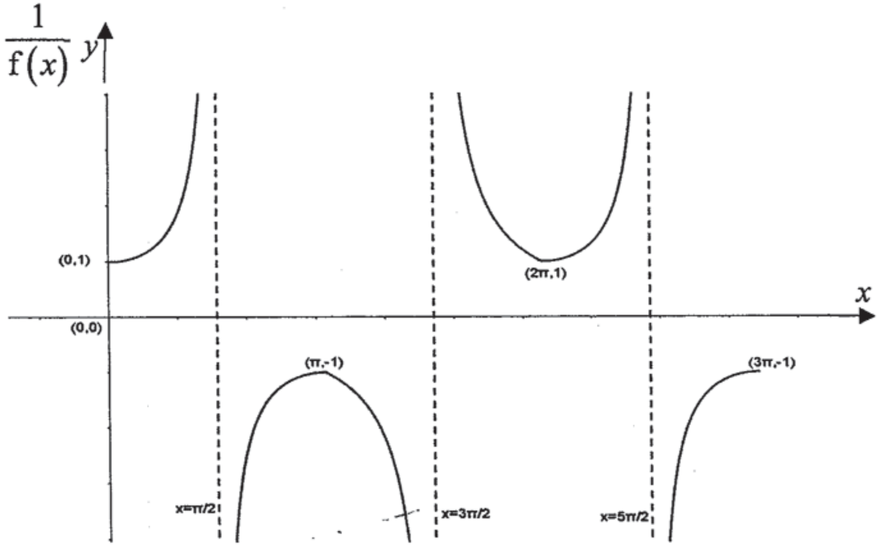
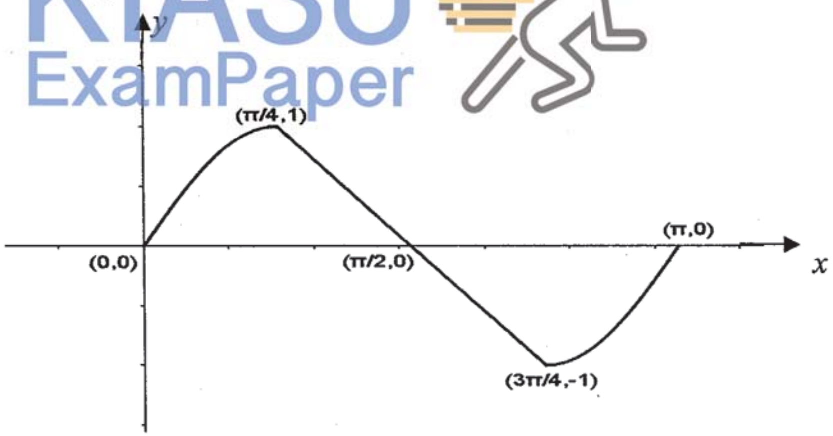
Qn4	Suggested Answers	Guidance
(i)	$y=\dfrac{bx+a}{ax-b} \Rightarrow y(ax-b)=bx+a \Rightarrow axy-by=bx+a$ $\Rightarrow axy-bx=by+a \Rightarrow x(ay-b)=by+a \Rightarrow x=\dfrac{by+a}{ay-b}$ $\therefore \mathrm{f}^{-1}(x)=\dfrac{bx+a}{ax-b}$ $\mathrm{D}_{\mathrm{f}^{-1}}=\mathrm{R}_\mathrm{f}=\mathbb{R}\setminus\left\{\dfrac{b}{a}\right\}$ or $\mathrm{D}_{\mathrm{f}^{-1}}=\mathrm{R}_\mathrm{f}=\left(-\infty,\dfrac{b}{a}\right)\cup\left(\dfrac{b}{a},\infty\right)$	Many students do not know how to write the domain in interval notation, especially the part on excluding $\dfrac{b}{a}$. Some students confused the usage of "OR" and "AND". $\left(-\infty,\dfrac{b}{a}\right)\cup\left(\dfrac{b}{a},\infty\right)$ means $\left(-\infty,\dfrac{b}{a}\right)$ OR $\left(\dfrac{b}{a},\infty\right)$. $\left(-\infty,\dfrac{b}{a}\right)\cap\left(\dfrac{b}{a},\infty\right)$ means $\left(-\infty,\dfrac{b}{a}\right)$ AND $\left(\dfrac{b}{a},\infty\right)$.
(ii)	Since $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$, then $\mathrm{f}^2(x)=x$	$\mathrm{f}^2(x)\neq \mathrm{f}(x)\times \mathrm{f}(x)$ Many students who substituted $\mathrm{f}(x)$ into $\mathrm{f}(x)$ made slips in simplifying the algebraic expression.
(iii)	For fg to exist, we need $\mathrm{R}_\mathrm{g}\subseteq \mathrm{D}_\mathrm{f}$ $\mathrm{R}_\mathrm{g}=(0,\infty)$ $\mathrm{D}_\mathrm{f}=\mathbb{R}\setminus\left\{\dfrac{b}{a}\right\}$ or $\left(-\infty,\dfrac{b}{a}\right)\cup\left(\dfrac{b}{a},\infty\right)$ Since $\dfrac{b}{a}<0$, $\mathrm{R}_\mathrm{g}\subseteq \mathrm{D}_\mathrm{f}$. Hence, fg exists.	R_g is a SUBSET of D_f i.e. $\mathrm{R}_\mathrm{g}\subseteq \mathrm{D}_\mathrm{f}$. Many students mistakenly wrote it as $\mathrm{R}_\mathrm{g}\in \mathrm{D}_\mathrm{f}$, which is wrong. Many failed to realise that the graph of $y=\mathrm{g}(x)$ has a vertical asymptote at $x=0$. Do not use the trace function in GC to find the "endpoints".
(iv)	$\mathrm{D}_{\mathrm{fg}}=(-\infty,0)\xrightarrow[\mathrm{g}]{}(0,\infty)\xrightarrow[\mathrm{f}]{}\left(\dfrac{b}{a},-\dfrac{a}{b}\right)$	Question asked for "range" not "rule". $\dfrac{b}{a}<-\dfrac{a}{b}$ since $a>0$ and $b<0$.

Qn5	Suggested Answers	Guidance
(a)(i)	**a** and **b** are parallel. OR $\mathbf{a} = k\mathbf{b}$, where $k \in \mathbb{R} \setminus \{0\}$ is a constant.	
(a)(ii)	$\mathbf{r}\times(\mathbf{i}+\mu\mathbf{j}+\mathbf{k})-\mathbf{r}\times(\mu\mathbf{i}+\mathbf{j}+\mu\mathbf{k})=\mathbf{0} \Rightarrow \mathbf{r}\times\left[(1-\mu)\mathbf{i}+(\mu-1)\mathbf{j}+(1-\mu)\mathbf{k}\right]=\mathbf{0}$ $\Rightarrow \mathbf{r}\times(1-\mu)(\mathbf{i}-\mathbf{j}+\mathbf{k})=\mathbf{0} \Rightarrow \mathbf{r}\times(\mathbf{i}-\mathbf{j}+\mathbf{k})=\mathbf{0}$ as $1-\mu\neq 0$ From part (i), $\mathbf{r}\,//\,(\mathbf{i}-\mathbf{j}+\mathbf{k})$ Unit vector $\mathbf{r}=\frac{1}{\sqrt{3}}(\mathbf{i}-\mathbf{j}+\mathbf{k})$ or $\mathbf{r}=-\frac{1}{\sqrt{3}}(\mathbf{i}-\mathbf{j}+\mathbf{k})$	Recognise from (i) that should make RHS=0. After factorising **r** out, again link to (i) that there are parallel. From there should get the unit vector out quickly.
(b)(i)	$\mathbf{a}\cdot(\mu\mathbf{b}+\lambda\mathbf{c})=\mu\mathbf{a}\cdot\mathbf{b}+\lambda\mathbf{a}\cdot\mathbf{c}=0$ Therefore, **a** is perpendicular to $\mu\mathbf{b}+\lambda\mathbf{c}$ for all $\mu, \lambda \in \mathbb{R}$	To show perpendicular, quickly show that $\mathbf{a}\cdot(\mu\mathbf{b}+\lambda\mathbf{c})=0$
(b)(ii)	**a** is a normal vector of the plane containing the origin and the points B and C. Or **a** is a normal vector of a plane // vectors **b** and **c**.	
(b)(iii)	$\mathbf{b}\times\mathbf{c}=\begin{pmatrix}1\\\alpha\\1\end{pmatrix}\times\begin{pmatrix}\alpha\\1\\-\alpha\end{pmatrix}=\begin{pmatrix}-\alpha^2-1\\-(-\alpha-\alpha)\\1-\alpha^2\end{pmatrix}=\begin{pmatrix}-\alpha^2-1\\2\alpha\\1-\alpha^2\end{pmatrix}$ $\lvert\mathbf{b}\times\mathbf{c}\rvert=\sqrt{(-\alpha^2-1)^2+(2\alpha)^2+(1-\alpha^2)^2}=\sqrt{2}\left(\alpha^2+1\right)$ Unit vector $\mathbf{a}=\frac{1}{\sqrt{2}\left(\alpha^2+1\right)}\begin{pmatrix}-\alpha^2-1\\2\alpha\\1-\alpha^2\end{pmatrix}$ or $-\frac{1}{\sqrt{2}\left(\alpha^2+1\right)}\begin{pmatrix}-\alpha^2-1\\2\alpha\\1-\alpha^2\end{pmatrix}$.	Vector product involving unknowns should be the same. Likewise for finding magnitude. Note that $\left\lvert\begin{pmatrix}-\alpha^2-1\\2\alpha\\1-\alpha^2\end{pmatrix}\right\rvert \neq \left\lvert\begin{pmatrix}\alpha^2+1\\2\alpha\\1+\alpha^2\end{pmatrix}\right\rvert$

Qn6	Suggested Answers	Guidance
(i)	$9x^2 - ay^2 - 36x - 2aby + 9a - ab^2 + 36 = 0$ The curve passes throught the point $(2,-2)$, then $9(2^2) - a(-2)^2 - 36(2) - 2ab(-2) + 9a - ab^2 + 36 = 0$ $\Rightarrow\ 36 - 4a - 72 + 4ab + 9a - ab^2 + 36 = 0$ $\Rightarrow\ 5a + 4ab - ab^2 = 0 \quad \Rightarrow \quad b^2 - 4b - 5 = 0$ since $a \neq 0$ $\therefore\quad b = 5$ or -1 (rejected as $b > 0$) When $b = 5$, we have $9x^2 - ay^2 - 36x - 10ay + 9a - 25a + 36 = 0$ $\Rightarrow\quad 9(x^2 - 4x + 4) - a(y^2 + 10y + 25) + 9a = 0$ $\Rightarrow\ 9(x-2)^2 - a(y+5)^2 + 9a = 0 \quad \Rightarrow \quad \frac{(y+5)^2}{9} - \frac{(x-2)^2}{a} = 1$ The equations of the 2 oblique asymptotes are obtained by : $\frac{(y+5)^2}{9} - \frac{(x-2)^2}{a} = 0$ $\Rightarrow \frac{(y+5)^2}{9} = \frac{(x-2)^2}{a} \Rightarrow y + 5 = \pm\sqrt{\frac{9}{a}}(x-2)$. i.e. $y = \frac{3}{\sqrt{a}}x - \frac{6}{\sqrt{a}} - 5$ or $y = -\frac{3}{\sqrt{a}}x + \frac{6}{\sqrt{a}} - 5$ Given that an oblique asymptote has equation $y = \frac{3}{2}x - 8$, then we compare it with equation $y = \frac{3}{\sqrt{a}}x - \frac{6}{\sqrt{a}} - 5$, we have $\frac{3}{\sqrt{a}} = \frac{3}{2} \quad \Rightarrow \quad a = 4$	You should substitute the value $(2,-2)$ into the given equation of curve before doing anything manipulation. Need to take note of the following point in completing of square. 1. Note the sign changes. Eg $-ay^2 - 10ay + \cdots = -a(y^2 + 10y) + \cdots$ 2. If you add a term to complete the square, you need to subtract the same term to balance the expression Eg $9(x^2 - 4x) = 9(x^2 - 4x + 4) - 9(4)$

(ii)	Substitute a = 4 into another equation $y=-\frac{3}{\sqrt{a}}x+\frac{6}{\sqrt{a}}-5$, we have $y=-\frac{3}{\sqrt{4}}x+\frac{6}{\sqrt{4}}-5=-\frac{3}{2}x-2$ Thus, equation of the other asymptote is $y=-\frac{3}{2}x-2$	
(iii)		Note that: The graph should approach the asymptotes Symmetry about the centre $(2,-5)$ which is also the point of intersection of the 2 asymptotes. State the coordinates of the 2 vertices , which are $(2,-2)$ and $(2,-8)$ on the graph Also the equations of asymptotes etc …

Qn7	Suggested Answers	Guidance
(i)	$y = \mathrm{f}(x)$ 	The graph of $y = \frac{2}{\pi} - 3$ is a straight line. $\mathrm{f}(x) = \mathrm{f}(x + 2\pi)$ means the period of the graph is 2π. Draw the graph within the domain stated.
	For equation $\mathrm{f}(x) = k$, where $-1 < k < 1$, number of solutions = 3	
(ii)(a)	$y = \lvert \mathrm{f}(x) \rvert$ 	

(ii)(b)	$y=\dfrac{1}{f(x)}$ 	Label the asymptotes and the coordinates of the critical points clearly.
(ii)(c)	$y=-f\left(2x+\dfrac{\pi}{2}\right)$ 	Translate the graph of $y = \mathrm{f}(x)$ by $-\dfrac{\pi}{2}$ in the direction of the x-axis followed by a scaling parallel to the x-axis by scale factor $\dfrac{1}{2}$. Finally reflect it about the x-axis. Remember to draw the graph only in the domain $0 \le x \le \pi$.

Qn8	Suggested Answers	Guidance
(i)	Normal of plane required $=\overrightarrow{BA}\times\overrightarrow{DA}$ $=\left[\begin{pmatrix}1\\2\\1\end{pmatrix}-\begin{pmatrix}1\\3\\0\end{pmatrix}\right]\times\left[\begin{pmatrix}1\\2\\1\end{pmatrix}-\begin{pmatrix}0\\2\\8\end{pmatrix}\right]=\begin{pmatrix}0\\-1\\1\end{pmatrix}\times\begin{pmatrix}1\\0\\-7\end{pmatrix}=\begin{pmatrix}7\\1\\1\end{pmatrix}$ Equation of plane is $\mathbf{r}\cdot\begin{pmatrix}7\\1\\1\end{pmatrix}=\begin{pmatrix}1\\2\\1\end{pmatrix}\cdot\begin{pmatrix}7\\1\\1\end{pmatrix}\Rightarrow\mathbf{r}\cdot\begin{pmatrix}7\\1\\1\end{pmatrix}=10\Rightarrow 7x+y+z=10$ C lies on the plane, thus we have: $7\alpha+\beta+10=10 \Rightarrow 7\alpha+\beta=0$	Cartesian equation always refers to one that connects x, y and z together. We need to find scalar product form before proceeding. To find the normal, we will find any two vectors parallel to the plane and take the cross product.
(ii)	Normal to floor $=\overrightarrow{BA}\times\begin{pmatrix}3\\2\\1\end{pmatrix}=\begin{pmatrix}0\\-1\\1\end{pmatrix}\times\begin{pmatrix}3\\2\\1\end{pmatrix}=\begin{pmatrix}-3\\3\\3\end{pmatrix}/\!/\begin{pmatrix}-1\\1\\1\end{pmatrix}$ Angle required $=\cos^{-1}\dfrac{\left\lvert\begin{pmatrix}7\\1\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}7\\1\\1\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}-1\\1\\1\end{pmatrix}\right\rvert}=\cos^{-1}\dfrac{5}{\sqrt{51}\sqrt{3}}=66.2^\circ$	The question did not say that the floor is a line! Vector given is just parallel to the floor. The floor forms a plane. Understanding this will reduce this question to finding angle between two planes.

(iii)	The 2 ropes will overlap since they are non-parallel and coplanar.	It is insufficient just saying the lines are non-parallel. It can be skew lines.
(iv)	Since $\beta = 4, \alpha = -\frac{4}{7}$, Equation of line AC is $\mathbf{r} = \begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda\left[\begin{pmatrix}-\frac{4}{7}\\4\\10\end{pmatrix} - \begin{pmatrix}1\\2\\1\end{pmatrix}\right] = \begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda\begin{pmatrix}-\frac{11}{7}\\2\\9\end{pmatrix}, \lambda \in \mathbb{R}$ or $\mathbf{r} = \begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda\left[\begin{pmatrix}\alpha\\4\\10\end{pmatrix} - \begin{pmatrix}1\\2\\1\end{pmatrix}\right] = \begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda\begin{pmatrix}\alpha-1\\2\\9\end{pmatrix}, \lambda \in \mathbb{R}$ Equation of line BD is $\mathbf{r} = \begin{pmatrix}1\\3\\0\end{pmatrix} + \mu\left[\begin{pmatrix}0\\2\\8\end{pmatrix} - \begin{pmatrix}1\\3\\0\end{pmatrix}\right] = \begin{pmatrix}1\\3\\0\end{pmatrix} + \mu\begin{pmatrix}-1\\-1\\8\end{pmatrix}$ At point of overlap,	We can always use the GC to cut down computations needed. Note also that the question asked for coordinates of point NOT position vector of point. They are different.

	$\begin{pmatrix}1\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}-\frac{11}{7}\\2\\9\end{pmatrix}=\begin{pmatrix}1\\3\\0\end{pmatrix}+\mu\begin{pmatrix}-1\\-1\\8\end{pmatrix}$ $\Rightarrow \lambda=\frac{7}{25}, \mu=\frac{11}{25}$ or $\begin{pmatrix}1\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}\alpha-1\\2\\9\end{pmatrix}=\begin{pmatrix}1\\3\\0\end{pmatrix}+\mu\begin{pmatrix}-1\\-1\\8\end{pmatrix}$ $\Rightarrow \lambda=\frac{7}{25}, \mu=\frac{11}{25}, \alpha=-\frac{4}{7}$ Coordinates of point of overlap $=\left(\frac{14}{25},\frac{64}{25},\frac{88}{25}\right)$	