Indices

LEVEL 1

1. Evaluate the following without using a calculator.

(a) $(4^5 + 7^9)^0$

(b) $6^8 \times 6^7 \div 6^{13}$

(c) $2^{15} \times 2^5 \div (4)^8$

(d) $(3^2)^2$

(e) $(5^0 + 5^1) \times 5^2$

(f) $(2^3)^2 + (2^3 \times 2^2)$

(g) $9^{\frac{1}{2}} + 9^2 + 9^{-1}$

(h) $27^{\frac{2}{3}}$

2. Evaluate the following without using a calculator.

(a) $16^{10} \times 16^{-8} \div \sqrt{16}$

(b) $5^{\frac{5}{2}} \times 5^2 \div 5^{\frac{3}{2}}$

(c) $25^{\frac{3}{2}}$

(d) $\left(\sqrt{64}\right)^{\frac{2}{3}}$

(e) $2^{-1} - 2^{-2} + 2^{-3}$

(f) $\left(\frac{2}{3}\right)^{-3}$

(g) $(-3)^2 + 3^{-2}$

(h) $\left(8^{\frac{2}{3}}\right)^{-2}$

3. Simplify the following and express your answers with positive indices.

(a) $(3x^4)^2$

(b) $5x^3 \times 3x^2$

(c) $24y^3 \div 8y^2$

(d) $(3x^5)^0$

(e) $\left(a^{\frac{2}{3}}\right)^6$

(f) $\left(\frac{z^8}{z^2}\right)^{\frac{2}{3}}$

(g) w^{-4}

(h) $(c^4)^{-2}$

4. Simplify the following and express your answers with positive indices.

(a) $15x^3y \div 3xy^4$

(b) $(p^2)^{-1} \times (q^3)^2$

(c) $\sqrt[3]{27s^6t^9}$

(d) $(3x^3y^2)^2 \times x^2y^4$

(e) $(2a^4b^{-3})^2(a^{-1}b)^5$

(f) $\frac{(n^2)^2}{m^6 \times n^7}$

(g) $\frac{6p^4 \times 7q^3}{14q^6 \times 3p^2}$

(h) $\left(\frac{x^4}{9y^6}\right)^{\frac{1}{2}}$

5. Solve the following equations.

(a) $6^x = 1$

(b) $3^x = 27$

(c) $2^x = \frac{1}{16}$

(d) $4^x = 2^{15}$

(e) $5^x = 25^{-8}$

(f) $\sqrt{7^x} = 49$

6. Solve the following equations.

(a) $3^{4x} = 9^{12}$ **(b)** $6^{2-x} = 36^4$

(c) $5^2 \times 5^{2x} = 5^2$ **(d)** $2^x \div 32 = 2^{-x}$

(e) $\sqrt[x]{7^2} = 7^6$ **(f)** $4^x - 1 = 0$

7. Express each of the following in standard form correct to 3 significant figures.

(a) 3245 **(b)** 6 782 450

(c) $0.034\,63 \times 10^7$ **(d)** $279\,825 \div 10^2$

(e) 0.006 752 **(f)** 0.000 046 4

(g) $0.034\,63 \times 10^{-5}$ **(h)** $4295 \div 10^{-8}$

8. Express each of the following as an integer or a decimal.

(a) 2.556×10^4 **(b)** 1.245×10^6

(c) 3.774×10^2 **(d)** $0.024\,5 \times 10^7$

(e) 6.583×10^{-3} **(f)** 5.378×10^{-5}

(g) 145.8×10^{-6} **(h)** 0.034×10^{-1}

9. Evaluate the following without using a calculator and express your answers in standard form.

(a) $(3.2 \times 10^2) \times (2.4 \times 10^7)$ **(b)** $(5.5 \times 10^{-6}) \times (4.8 \times 10^3)$

(c) $(7.6 \times 10^7) \div (1.9 \times 10^4)$ **(d)** $(2.5 \times 10^5) \div (5 \times 10^{-1})$

(e) $2.4 \times 10^6 + 3.6 \times 10^5$ **(f)** $2.1 \times 10^6 - 7.2 \times 10^4$

(g) $3.35 \times 10^{-3} - 4.46 \times 10^{-4}$ **(h)** $8.5 \times 10^{-4} + 2.1 \times 10^{-2}$

10. Given that $x = 7.5 \times 10^2$ and $y = 2.5 \times 10^{-2}$, find the value of each of the following in standard form correct to 3 significant figures.

(a) xy **(b)** x^3y^4

(c) $\frac{x}{y}$ **(d)** $\frac{x^5}{y^2}$

(e) $4x + y^{-2}$ **(f)** $y^{-2} - x$

11. A bank offers simple interest at the rate of 5% p.a. for savings accounts.

(a) Mr Tan deposited \$8000 with the bank. Find the amount of interest earned after 6 years.

(b) Mrs Lim deposited \$$x$ with the bank. Find the value of x if the total amount of interest earned after 4 years is \$800.

(c) Jenny deposited \$3000 with the bank for t years. Find the value of t if her deposit increased to \$3750 after t years.

12. Yun Ting invested \$20 000 at the interest rate of 6% p.a.

Find the total amount after 10 years if the interest is compounded

(a) yearly,

(b) half yearly,

(c) quarterly.

Give your answers correct to the nearest cent.

13. Eligible clients may take up a loan from a bank at 12% p.a. compound interest.

(a) May Yee borrowed \$3500 from the bank for 2 years. Find

(i) the total amount paid,

(ii) the total interest accumulated,

by the end of the loan period.

(b) Muthu borrowed \$$y$ from the bank for 3 years. In total, he repaid \$10 536.96 at the end of the loan period. Find the value of y.

LEVEL 2

14. Evaluate the following without using a calculator.

(a) $(-2)^{3r+1} + 2(-2)^{3r}$

(b) $\sqrt[3]{-27} \times \left(-\frac{3}{2}\right)^{-1} + 9^{\frac{3}{2}}$

(c) $8^{\frac{4}{3}} + \left(1\frac{9}{16}\right)^{-\frac{1}{2}} - 5^0$

(d) $27^{\frac{2}{3}} \times \left(\frac{32^2}{2^2 \times 4^3}\right) - (-3)^2$

15. Solve the following equations.

(a) $5^{x-1} - 25^{x+4} = 0$

(b) $3^{\frac{2}{3}} = 9x^{\frac{2}{3}}$

(c) $2^{2x+1} \times 8^{-x} = 4^{-1}$

(d) $2^x = \frac{32}{4^{\frac{3x}{2}}}$

(e) $\sqrt[3]{5^{2(x-4)}} = \left(\frac{1}{25}\right)^x$

16. Simplify the following and express your answers with positive indices.

(a) $\frac{2^4 q^{-2}}{3r} \div \left(\frac{2}{3qr^2}\right)^3$

(b) $\frac{(x+\sqrt{x})(x-\sqrt{x})}{(\sqrt{x})^8 - x^2}$

(c) $\frac{(yw)^{-2}}{y^2}$

(d) $(-2p^3q^{-2})^4(-p^{-2}q^2)^3$

(e) $\frac{(-x)^2 y^{-\frac{2}{5}}}{4x} \div \frac{x^{-3}}{8y^{-\frac{3}{5}}}$

17. **(a)** Find the positive value of x if $20(4x)^{-2} = \frac{1}{5}$.

(b) Show that $2^{2m+3} + 4^m$ is a multiple of 3 for all positive values of m.

18. **(a)** Solve for w if $4(3^w) = 9^2 + 9^{\frac{w}{2}} - 2(3^{w+1})$.

(b) **(i)** Show that the value of the expression $\left(\frac{(w^2)^{(-m+1)}\sqrt{w}}{3\sqrt{w^{-4m}}}\right)^2$ does not depend on m.

(ii) Hence, evaluate $\left(\frac{(w^2)^{(-m+1)}\sqrt{w}}{3\sqrt{x^{-4m}}}\right)^2$ when w is 6.

19. **(a)** Simplify $\sqrt[4]{m\left(\sqrt[3]{m\left(\sqrt{m}\right)}\right)}$, expressin your answer in positive index notation.

(b) Hence, evaluate y^4 if $y = \sqrt[4]{m\left(\sqrt[3]{m\left(\sqrt{m}\right)}\right)}$ and $m = 100$.

20. **(a)** Simplify $(-3a)^2 + (-3a)^{-2} \times (-3a^{-2})$.

(b) Solve for x if $5x^{-\frac{3}{2}} = 320$.

21. Evaluate the following without using a calculator.

(a) $\left(\frac{100}{81}\right)^{-\frac{1}{2}} \div \left(\frac{3^{-2}}{5}\right)^{-1}$

(b) $\sqrt{2.25} + \left(\frac{8}{125}\right)^{-\frac{2}{3}}$

22. The speed of sound is estimated to be 1188 km/h. The maximum speed of a fighter jet is 3.5 times the speed of sound.

(a) Find the maximum speed of the fighter jet. Give your answer in standard form.

(b) Convert the maximum speed of the fighter jet to m/s. Give your answer in standard form.

23. The equatorial radius of the Earth is 6370 km. Calculate the distance in km, that an orbiting satellite will cover when it travels 100 rounds around the equator at an altitude of 180 km. Give your answer in standard form correct to 3 significant figures.

24. Clients who participate in a particular investment scheme are paid 5% p.a. simple interest for the first 5 years. If they continue to participate in the scheme after 5 years, the clients will earn an additional 2% p.a. simple interest on their principal.

(a) Mrs Ling invested \$12 000 in the scheme. How much did Mrs Ling accumulate in the scheme after 4 years?

(b) Mr Han invested \$$x$ in the scheme. The amount he accumulated in the scheme after 8 years is \$1990 lower than the amount Mrs Ling accumulated after 4 years. Find the value of x.

25. The following table shows the details of 3 investment plans.

Plan	Interest rate p.a. (%)	Interest compounded
A	5%	yearly
B	4.95%	quarterly
C	4.9%	monthly

Mr Seow wants to invest \$100 000 into one of the 3 plans over a period of 5 years. Which plan should he invest in? Explain your answer.

26. A loan grows to \$8640 after 3 years and \$12 441.60 after 5 years with compound interest that is computed annually. The interest rate per annum, $r\%$, remains unchanged in these years. Find

(a) the value of r,

(b) the value of the original loan,

(c) the total interest in the first 4 years.

27. The original value of a machine is \$30 000. Each year, the value decreases by 12% of its value at the start of the year.

(a) Calculate the value of the machine at the end of 4 years. Give your answer to the nearest dollar.

(b) What is the overall percentage reduction in the value of the machine after 4 years?

LEVEL 3

28. Convert the following and express your answers in standard form.

(a) **(i)** 3.45 m^2 to mm^2

(ii) 3.45 m^2 to km^2

(b) A prankster sent a message to his friend that instructed its recipients to forward it to 5 other people on the next day for good luck. His friend then forwarded the message to 5 other people on the next day and each of the 5 people forwarded the message to 5 other people on the following day. Suppose that all recipients, including repeated recipients and the prankster, forwarded the message on the next day after they had received it. How many times would the message be forwarded on the 10th day if the day the message was first sent by the prankster is defined as the 1st day?

29. Under a controlled environment, the living population, P, of a certain microorganism one hour after defrosting can be modelled by the formula $P = N(2^t - 2^{t-1})$ where N is the initial number of living microorganisms and t is the time in hours after defrosting.

(a) Factorise the formula $P = N(2^t - 2^{t-1})$ completely.

(b) Hence, calculate the number of living microorganisms after 10 hours if there are 5 million of them initially. Give your answer in standard form.

30. The diameter of a water molecule is about 0.275 nanometres.

(a) Express 0.275 nanometres in centimetres, giving your answer in standard form.

(b) Suppose that water molecules are placed side by side to form a line segment of 1 cm. Find the number of water molecules on the line segment, expressing your answer in standard form, correct to 3 significant figures.

31. In June 2006, the total population of Singapore was 4.4839×10^6. In 2006, the Gross Domestic Product (GDP) of Singapore was $\$2.099\,909 \times 10^{11}$.

(a) Find the per capita GDP (i.e. GDP per person) of Singapore in 2006. Express your answer

(i) in standard form, correct to 4 significant figures,

(ii) correct to the nearest hundred dollar.

(b) The land area of Singapore is 704 km^2. Calculate the population density, in persons per km^2, of Singapore in June 2006. Give your answer correct to the nearest 10 persons/km^2.

(c) If 19.3% of the total population in June 2006 was below 15 years old, find the population below 15 years old. Give your answer correct to 3 significant figures in standard form.

32. If $2^{x+3} = 4^y$ and $6^x = 216^{\frac{y+1}{3}}$, find the values of x and y.

33. Mr Wee repaid a loan of \$10 000 at 10% p.a. compounded annually in 3 annual instalments of \$$w$, \$$2w$ and \$$3w$ respectively.

(a) Express, in terms of w, the balance of Mr Wee's loan just after his

(i) first repayment,

(ii) second repayment.

(b) **(i)** Form an equation in w and solve it. Give your answer correct to 2 decimal places.

(ii) Hence, express the total interest paid as a percentage of the original loan.

(c) Mr Wee repaid his loan with progressively higher annual repayments.
Give a possible reason for his action.

34. The wavelength, λ m, and the frequency, f Hz of a visible light have the relationship $c = f\lambda$, where c m/s, being the velocity of light, is a constant.

(a) The wavelength of an orange light beam is 6.0×10^{-7} m and its frequency is 5×10^{14} Hz. Find the velocity of light. Give your answer in standard form.

(b) The frequency of a violet light beam is 7.5×10^{14} Hz. Find the wavelength of this violet light beam. Give your answer in standard form.

(c) Are the wavelength and frequency of a visible light in direct proportion or inverse proportion? Explain your answer.

35. Mr Tan deposited an equal sum, \$$P$, at the beginning of each year for 4 years. He earned compound interest of 5% per annum. The total amount he got at the end of the fourth year is \$60 000. Find the value of P.

36. What is the last digit of each of the following numbers?

(a) 3^{1001} **(b)** 7^{238}

37. **(a)** Find the value of each of the following.

(i) $(11 - 2)^{\frac{1}{2}}$ **(ii)** $(1111 - 22)^{\frac{1}{2}}$

(iii) $(111111 - 222)^{\frac{1}{2}}$

(b) Suggest the value of $(11...11 - 22...2)^{\frac{1}{2}}$, in which there are $2n$ digits of 1's and n digits of 2's.

(c) Let $m = 11...1$, in which there are n digits of 1's.

(i) Express $22...2$, in which there are n digits of 2's, in terms of m.

(ii) Show that $10^n = 9m + 1$.

(iii) Express $11...11$, in which there are $2n$ digits of 1's, in terms of m.

(iv) Hence or otherwise, prove your suggested result in **(b)**.

38. If $a > 0$ and $b > 0$, and $a^b = b^a$, show that $\left(\frac{a}{b}\right)^{\frac{a}{b}} = a^{\frac{a}{b}-1}$.

39. It is given that $3^{3x+2} \times 7^{x-1} = 81^x \times 7^{2x}$, find 21^x.