NANYANG JUNIOR COLLEGE

JC2 COMMON TEST

Higher 2

CANDIDATE NAME

CT CLASS

1	9		

MATHEMATICS 9758/01

Paper 1

26 Jun 2020

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For examiner's use only	
Question number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

This document consists of **5** printed pages and **0** blank page.

NANYANG JUNIOR COLLEGE
Internal Examinations

1 Without using a calculator, solve the inequality

$$\frac{2}{7x-1} \geq \frac{1}{3x-4},$$

giving your answer in exact form.

Hence, solve $\frac{2}{7x^2-1} \geq \frac{1}{3x^2-4}$. [4]

2 Find the equations of tangents to the curve $y^2 + x^2 = \frac{1+x}{y}$ that are parallel to the x-axis. [6]

3 Referred to the origin O, the points B and C have position vectors **b** and **c** respectively such that

$\mathbf{b} = \alpha\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} - \alpha\mathbf{j} + 4\mathbf{k}$, where α is a constant.

(i) Find $(\mathbf{b} + \mathbf{c}) \times (\mathbf{b} - \mathbf{c})$ in terms of α. [2]

(ii) Given that the **i**- and **k**-components of the answer to part **(i)** are equal, find the possible values of the area of triangle OBC. [4]

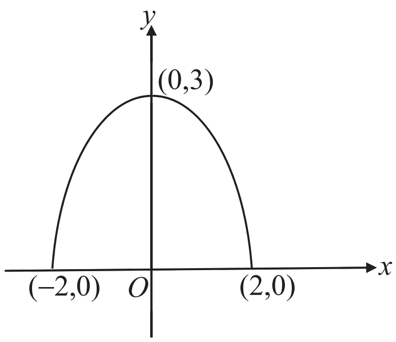
4 A curve is defined by the parametric equations

$$x = \frac{t}{1+t^2},\ y = \frac{t}{1-t^2},\ \text{where } t \in \mathbb{R},\ t \neq -1,\ 1.$$

(i) Show that the tangent to the curve at any point with parameter t has equation $\left(1-t^2\right)^3 y = \left(1+t^2\right)^3 x - 4t^3$. [4]

(ii) Find the gradient of the tangent to the curve at $t = \frac{1}{\sqrt{2}}$. Hence determine the acute angle between this tangent and the line $y = x + 3$. [3]

5



The diagram above shows a part of an ellipse with equation $y = \mathrm{f}(x) = 3\sqrt{1-\frac{x^2}{2^2}}$.

(i) Sketch the graph of $y = \mathrm{f}\left(\frac{1}{2}x - 1\right) - 1$, labelling the coordinates of the points where the curve crosses the axes, end points and any stationary points. [4]

(ii) On a separate diagram, sketch the graph of $y = \mathrm{g}(x)$ where $\mathrm{g}(x) = \dfrac{1}{\mathrm{f}\left(\frac{1}{2}x - 1\right) - 1}$, labelling the coordinates of the point(s) where the curve crosses the axes and the equations of any asymptotes and stationary point(s). [3]

(iii) Write down the solution set for $\mathrm{f}(x) > \mathrm{g}(x)$. [1]

6 **(a)** **(i)** Show that $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$ using the addition formula from the List of Formulae (MF26). [1]

(ii) By using the substitution $x = \tan\theta$, or otherwise, find the exact value of $\displaystyle\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right) \mathrm{d}x$. [5]

(b) Find the exact value of $\displaystyle\int_{-1}^{1} \frac{2+|x|}{2+x} \mathrm{d}x$. [4]

7 Given that $\mathrm{f}(r) = \dfrac{r+1}{3^{r-1}}$ where $r \geq 0$, show that

$$\mathrm{f}(r) - \mathrm{f}(r+1) = \frac{2r+1}{3^r}. \quad [1]$$

Use the above result to find $\dfrac{1}{3} + \dfrac{3}{3^2} + \dfrac{5}{3^3} + \ldots + \dfrac{2n-1}{3^n}$, giving your answer in the form $A - \dfrac{Bn+C}{3^n}$, where A, B and C are constants to be determined.

Hence find $\displaystyle\sum_{r=n+1}^{\infty} \frac{2r-1}{3^r}$ in terms of n, justifying your answer. [9]

8 **(i)** By using the substitution $x = a\sin\theta$, show $\displaystyle\int_0^a \sqrt{1 - \frac{x^2}{a^2}} \, \mathrm{d}x = \frac{a\pi}{4}$. [3]

(ii) The finite region, R, in the first quadrant is bounded by the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and the line $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where $a > b > 0$. Find the exact value of the area of R. [3]

(iii) By writing down the equation of the resulting curve when $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is translated b units in the direction of the negative y-axis, show that the volume of the region R when it is rotated through 2π radians about the line $y = b$ is given by $\dfrac{\pi ab^2}{6}(3\pi - 8)$. [4]

9 **Do not use a calculator in answering this question.**

(a) The complex numbers u and v are given by $1+\mathrm{i}\sqrt{3}$ and $-k-\mathrm{i}k$ respectively, where k is a positive real number. Find $\dfrac{(u^*)^7}{v^{10}}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where r is a positive constant in terms of k and $-\pi<\theta\le\pi$. [5]

(b) **(i)** The roots of the equation $z^2-9-40\mathrm{i}=0$ are z_1 and z_2. Find z_1 and z_2 in cartesian form $x+\mathrm{i}y$, showing your working. [4]

(ii) Hence find in cartesian form the roots w_1 and w_2 of the equation $w^2+5w+4-10\mathrm{i}=0$. [3]

10 The motion of a particle is described by the differential equation $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}-4\dfrac{\mathrm{d}x}{\mathrm{d}t}+4x=\mathrm{e}^{2t}$, where x is the displacement of the particle in metres along the x-axis with respect to origin O at time t seconds. By using the substitution $x=u\mathrm{e}^{2t}$, show that this differential equation may be reduced to the form $\dfrac{\mathrm{d}^2u}{\mathrm{d}t^2}=a$, where a is a constant to be determined.

Hence, find the displacement of the particle at time t if the particle moves off from the initial point $x = 1$ with a velocity of 1m/s. [7]

After some time, the particle is returned to its initial point and set into a new motion along the x-axis. The motion of the particle is described by the differential equation $\dfrac{\mathrm{d}v}{\mathrm{d}t}=2\mathrm{e}^{-v}$, where v is the velocity of the particle in metres per second at time t seconds after it starts on this new motion. If the particle is at rest just before it begins this new motion, find an expression for t in terms of v.

Show that $2x=3+\mathrm{e}^{v}(v-1)$, where x is the horizontal displacement of the particle in metres. [6]

11 A team of engineers is drawing a three-dimensional blueprint to plan the installation of underground water pipes to connect water sources to different locations in a new town. Coordinates (x, y, z) are defined relative to the operations office at (0, 0, 0), where units are metres. Water pipes are installed in straight lines and the thickness of the pipes can be neglected.

A water source is located at S (2, 3, −1) and the engineers plan to install a pipe from S to a location A (8, 5, −2) in the town. Another water source is located at $T\left(8, a, -\dfrac{7}{2}\right)$ and the engineers plan to install another pipe from this source passing through a site at C (5, 6, −2).

(i) Given that the two pipes will connect, show that $a = 11$. [4]

The engineers also want to find a new location at B in the town to install additional water pipes to connect the water sources to it.

(ii) To connect the water source from S to B, the engineers need to extend a new pipe from A to B such that the pipes connecting S, A and B lie in a straight line and the length of the pipe from S to A is twice that of the length from A to B. Find the coordinates of B. [2]

(iii) To connect the water source from T to B, an inexperienced engineer proposed to install a pipe from T to C and a pipe from C to B. However, the chief engineer discovers that T, C and B lie directly beneath a layer of rocks and it is very difficult to drill through it to lay the pipes. Assuming that the layer of rocks is part of a plane with negligible thickness, find the cartesian equation of this plane. [3]

(iv) The chief engineer finally decides to install a pipe to connect T to the existing pipe from S to B such that the length of the pipe from T to this existing pipe is the shortest. Show that the coordinates of the point of connection of the two pipes are $\left(\frac{409}{41}, \frac{232}{41}, -\frac{191}{82}\right)$ and find the length of this pipe. [5]

BLANK PAGE

NANYANG JUNIOR COLLEGE

JC2 COMMON TEST

Higher 2

CANDIDATE NAME

CT CLASS | 1 | 9 | | |

MATHEMATICS 9758/02

Paper 2

30 Jun 2020

2 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For examiner's use only	
Question number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
Total	70

This document consists of **5** printed pages and **0** blank page.

NANYANG JUNIOR COLLEGE
Internal Examinations

Section A: Pure Mathematics [35 marks]

1 **(a)** Given that $y=\left[\cos^{-1}(3x)\right]^2$.

(i) Show that $\left(1-9x^2\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=36y$ and $\left(1-9x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-9x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=18$. [3]

(ii) By further differentiation, find the Maclaurin series for y in terms of π, up to and including the term in x^3. [3]

(b) Using the standard series of $\sin x$ from the List of Formulae (MF26), find the exact value of

$$\sum_{r=1}^{\infty}\frac{(-1)^r\left(\frac{\pi}{4}\right)^{2r+1}}{(2r+1)!}.$$ [2]

2 The function f is defined by

$$\mathrm{f}:x\mapsto\begin{cases} a\mathrm{e}^{a-x} & \text{for } 0\le x<a,\\ \left|\frac{1}{a}(x-a)^2-a\right| & \text{for } a\le x\le 2a.\end{cases}$$

(i) Define f^{-1} in similar form. [4]

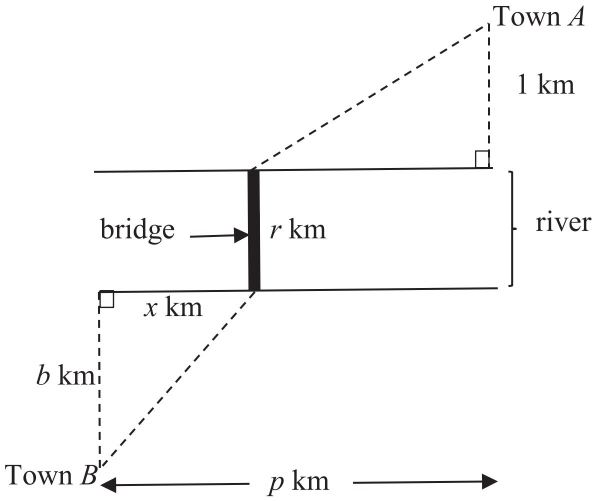
(ii) Sketch the graph of $y=\mathrm{f}^{-1}(x)$, indicating clearly the axial intercepts. [2]

(iii) The function g is defined by

$$\mathrm{g}:x\mapsto\ln x \quad \text{for } x>0.$$

Determine if gf^{-1} exists. [2]

3 The diagram below shows a city map of two towns, A and B separated by a river. A bridge is to be built between the two towns, which are on opposite sides of a straight river of uniform width r km, and the two towns are p km apart measured along the riverbank. Town A is 1 km from the riverbank, and Town B is b km away from riverbank. A bridge is to be built perpendicular to the riverbank at a distance of x km from Town B, measured along the riverbank, allowing traffic to flow between the two towns.



Find the distance x, in terms of b and p, such that the distance of travel between Town A and Town B can be minimised if $b > 1$. (It is not necessary to verify that the distance is minimum.) [9]

4 **(a)** An arithmetic progression has first term α and non-zero common difference β. Given that the 6th term is 37 and the sum of the first 10 terms is twice the 22nd term of the sequence, find the values of α and β. [4]

(b) The first three terms of a sequence of complex numbers are $a+\mathrm{i}\sqrt{b}$, $\left(a^2-a\sqrt{b}\right)+\mathrm{i}\left(a\sqrt{b}+a^2\right)$ and $-2a^2\sqrt{b}+\mathrm{i}\left(2a^3\right)$, where a and b are real numbers.

Prove that these three terms form a geometric progression.

It is given that a geometric progression has $a+\mathrm{i}\sqrt{b}$, $\left(a^2-a\sqrt{b}\right)+\mathrm{i}\left(a\sqrt{b}+a^2\right)$ and $-2a^2\sqrt{b}+\mathrm{i}\left(2a^3\right)$ as its first three terms, where $a=1$ and $b=3$. Find the sum of first eight terms in this geometric progression, giving your answer exactly. [6]

Section B: Probability and Statistics [35 marks]

5 A bag contains 5 black cards and 3 white cards, which are indistinguishable apart from colour. In a game, a player draws 2 cards at random, without replacement, from the bag. If the cards are of a different colour, three fair coins are tossed and the score is equal to the number of heads obtained. If the cards are of the same colour, two fair coins are tossed and the score is equal to twice the number of heads obtained. The score is denoted by X.

(i) Show that $\mathrm{P}(X=2)=\dfrac{97}{224}$. [2]

The probability distribution for X is given below:

x	0	1	2	3	4
$\mathrm{P}(X=x)$	$\dfrac{41}{224}$	$\dfrac{45}{224}$	$\dfrac{97}{224}$	$\dfrac{15}{224}$	$\dfrac{13}{112}$

(ii) If X is odd, the player loses \$3. Otherwise, he wins \$$x$. Find his expected winnings. [3]

(iii) If X_1 and X_2 are two independent observations of X, find $\mathrm{P}(X_1-X_2\geq 3\mid X_2 \text{ is even})$. [2]

6 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

To spray paint a car in a car-servicing workshop, one coat of paint A, two coats of paint B and three coats of paint C are used. The quantity of each type of paint used per coat follows a normal distribution. The following table gives the means and standard deviations of these quantities measured in litres.

	Mean	Standard deviation
Each coat of paint A	2.0	0.29
Each coat of paint B	1.8	0.21
Each coat of paint C	1.0	0.12

Assuming that the quantities of paint used for each coat are independent, calculate the

(i) least value of a such that the probability that the total amount of paint received by a car differs from its mean by not more than a litres exceeds 0.9. [4]

(ii) probability that the cost of spray-painting a car exceeds \$2000 if the cost per litre of paint A, B and C are \$350, \$200 and \$150 respectively. [3]

7 During Orientation 2020, a game is played by arranging cards. There are 20 cards, consisting of 4 suits of 5 coloured cards: Red, Orange, Yellow, Green and Blue. The suits are Nova, Yvex, Juno and Cleo.

The 20 cards are arranged in a row.

(i) In how many different ways can the 20 cards be arranged so that the 5 cards of each suit are next to each other? [2]

(ii) In how many different ways can the cards be arranged so that all 4 Red cards are next to each other, all 5 Juno cards are next to each other and all 5 Cleo cards are next to each other? [3]

The cards are now arranged in a circle.

(iii) Find the probability that no two Yvex cards are next to each other. [4]

8 In a large shipment of surgical masks, it is known that, on average, a proportion p of the masks is faulty. The masks are sold in boxes of 200. The number of faulty surgical masks in a randomly chosen box is the random variable X.

(i) State, in context of the question, two assumptions needed to model X by a binomial distribution. [2]

You are now given that X can be modelled by a binomial distribution.

(ii) Given that the probability of a box containing not more than two faulty surgical masks box is 0.9, find p. [2]

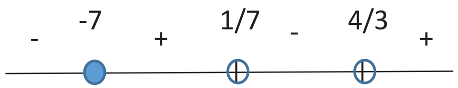
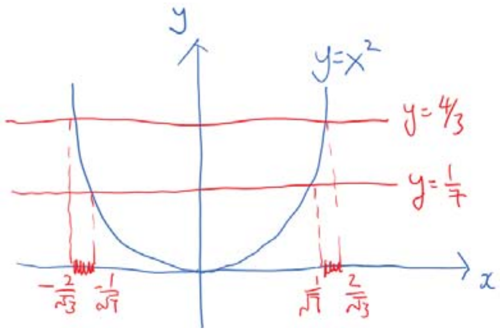
(iii) As part of a quality control process, surgical masks are selected from a randomly chosen box for inspection. Find the probability that the 180th surgical mask is the 3rd surgical mask that is faulty. [2]

Each week, a distributor sends in four shipments each containing 50 boxes of surgical masks.

(iv) A box will be rejected if it contains more than two faulty surgical masks. Find the probability that, in a shipment, more than seven boxes are rejected. [3]

(v) For each shipment, the distributor will have to pay a compensation of \$1250 if at least eight boxes are rejected. Find the probability that the distributor has to pay a compensation of less than \$3000. [3]

BLANK PAGE

Q1	Suggested Answers	Guidance
	$\frac{2}{7x-1} \geq \frac{1}{3x-4}$ $\frac{-7-x}{(7x-1)(3x-4)} \geq 0$ $\frac{x+7}{(7x-1)(3x-4)} \leq 0$  $x \leq -7$ or $\frac{1}{7} < x < \frac{4}{3}$ $\frac{2}{7x^2-1} \geq \frac{1}{3x^2-4}$ $x^2 \leq -7$ (reject) or $\frac{1}{7} < x^2 < \frac{4}{3}$ $\therefore -\frac{2}{\sqrt{3}} < x < -\frac{1}{\sqrt{7}}$ or $\frac{1}{\sqrt{7}} < x < \frac{2}{\sqrt{3}}$	- We must exclude values that make the denominator zero. - To solve inequalities like $\frac{1}{7} < x^2 < \frac{4}{3}$, a graph is useful to see the correct range.  - Any attempts to solve $x^2 \leq -7$ by complex numbers is wrong.

Q2	Suggested Answers	Guidance
	$y^2+x^2=\frac{1+x}{y}$ $y^3+yx^2=1+x$ $3y^2\frac{dy}{dx}+x^2\frac{dy}{dx}+2xy=1$ --- (1) $\frac{dy}{dx}=\frac{1-2xy}{3y^2+x^2}$ When tangents parallel to x-axis, $\frac{dy}{dx}=0$ $2xy-1=0$ $x=\frac{1}{2y}$ Substitute into equation of curve: $y^3+y\left(\frac{1}{2y}\right)^2-\frac{1}{2y}=1$ $y^3+\frac{1}{4y}-\frac{1}{2y}=1$ $4y^4-1=4y$ $4y^4-4y-1=0$ $y=1.07$ or $y=-0.246$	- Students who did not manipulate the equation often made careless mistakes when doing quotient rule. - It is not necessary to make $\frac{dy}{dx}$ the subject as the question does not require. You can sub $\frac{dy}{dx}=0$ directly into (1). - When the tangent is parallel to x-axis, the tangent is a **horizontal line** with gradient zero, hence the y-coordinate of the points which the tangent makes with the curve will give the equation of the tangent.

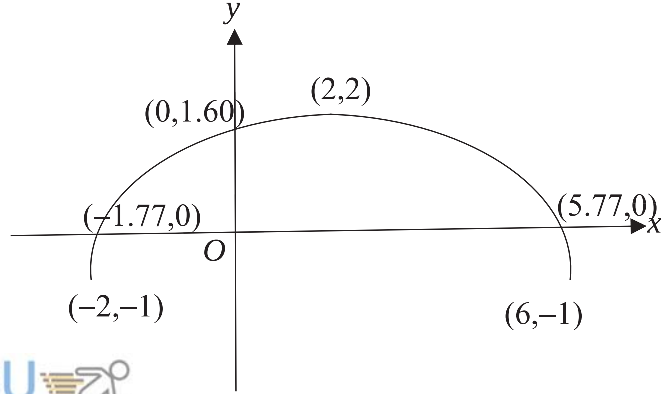
<table>
<tr><th>Q3</th><th>Suggested Answers</th><th>Guidance</th></tr>
<tr><td>(i)</td><td>$\mathbf{b}=\begin{pmatrix}\alpha\\-1\\1\end{pmatrix}$ $\mathbf{c}=\begin{pmatrix}2\\-\alpha\\4\end{pmatrix}$
$(\mathbf{b}+\mathbf{c})\times(\mathbf{b}-\mathbf{c})$
$=\mathbf{b}\times\mathbf{b}-\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{b}-\mathbf{c}\times\mathbf{c}$
$=2\,\mathbf{c}\times\mathbf{b}$
$=2\begin{pmatrix}2\\-\alpha\\4\end{pmatrix}\times\begin{pmatrix}\alpha\\-1\\1\end{pmatrix}=2\begin{pmatrix}4-\alpha\\4\alpha-2\\\alpha^2-2\end{pmatrix}$
Alternatively,
$(\mathbf{b}+\mathbf{c})\times(\mathbf{b}-\mathbf{c})$
$=\left[\begin{pmatrix}\alpha\\-1\\1\end{pmatrix}+\begin{pmatrix}2\\-\alpha\\4\end{pmatrix}\right]\times\left[\begin{pmatrix}\alpha\\-1\\1\end{pmatrix}-\begin{pmatrix}2\\-\alpha\\4\end{pmatrix}\right]$
$=\begin{pmatrix}\alpha+2\\-1-\alpha\\5\end{pmatrix}\times\begin{pmatrix}\alpha-2\\\alpha-1\\-3\end{pmatrix}=\begin{pmatrix}8-2\alpha\\8\alpha-4\\2\alpha^2-4\end{pmatrix}=2\begin{pmatrix}4-\alpha\\4\alpha-2\\\alpha^2-2\end{pmatrix}$</td><td>A number of students used the alternative method. It is tedious, bringing much grief and pain with serious algebraic errors.

Note:
• b x c ≠ c x b
• b x b ≠ | b |
• $2\begin{pmatrix}4-\alpha\\4\alpha-2\\\alpha^2-2\end{pmatrix}\neq\begin{pmatrix}4-\alpha\\4\alpha-2\\\alpha^2-2\end{pmatrix}$

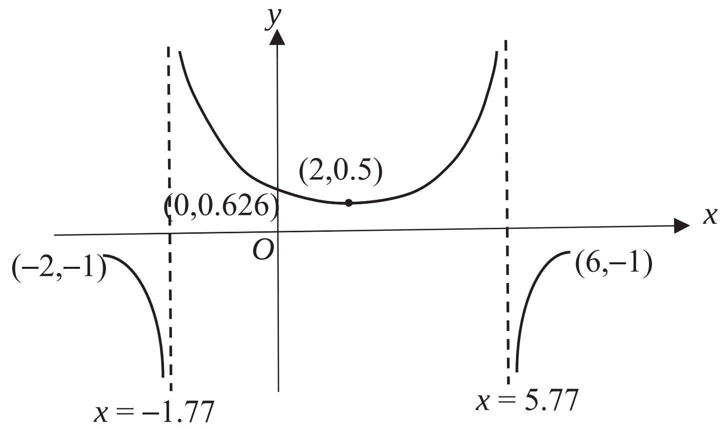
A minor error that was not penalised is the Greek symbol used in question was α and not σ or a. It would be very confusing if one interchange α with σ in a statistics question.</td></tr>
<tr><td>(ii)</td><td>Since the i- and k-components are equal,
$4-\alpha=\alpha^2-2$
$\alpha^2+\alpha-6=0$
$\alpha=-3$ or 2
Area of triangle OBC $=\frac{1}{2}\left|\mathbf{c}\times\mathbf{b}\right|$
$=\frac{1}{2}\left|\begin{pmatrix}4-\alpha\\4\alpha-2\\\alpha^2-2\end{pmatrix}\right|=\frac{1}{2}\left|\begin{pmatrix}2\\6\\2\end{pmatrix}\right|$ or $\frac{1}{2}\left|\begin{pmatrix}7\\-14\\7\end{pmatrix}\right|$</td><td>Area of triangle OBC
$\neq\frac{1}{2}\left|(\mathbf{b}+\mathbf{c})\times(\mathbf{b}-\mathbf{c})\right|$
$\neq\frac{1}{2}\left|\mathbf{b}\right|\left|\mathbf{c}\right|$
× is the cross product and not multiplication.
Careless mistakes of equating the coefficient of j and k or i and j components instead.</td></tr>
</table>

	$=\frac{1}{2}\sqrt{44}$ or $\frac{1}{2}\sqrt{294}=\sqrt{11}$ or $\frac{7}{2}\sqrt{6}$ $= 3.32$ or 8.57	

Q4	Suggested Answers	Guidance
(i)	$\frac{dx}{dt}=\frac{1-t^2}{(1+t^2)^2}$, $\frac{dy}{dt}=\frac{1+t^2}{(1-t^2)^2}$ $\frac{dy}{dx}=\left(\frac{1+t^2}{1-t^2}\right)^3$ $y-\frac{t}{1-t^2}=\left(\frac{1+t^2}{1-t^2}\right)^3\left(x-\frac{t}{1+t^2}\right)$ $(1-t^2)^3 y=(1+t^2)^3 x-t(1+t^2)^2+t(1-t^2)^2$ $=(1+t^2)^3 x-4t^3$	Students should use quotient rule. Quite a number of students used $y=mx+c$, resulting in tedious algebraic manipulation. Should use $y-y_1=m(x-x_1)$ instead. Many students didn't show result clearly.
(ii)	When $t=\frac{1}{\sqrt{2}}$, $\frac{dy}{dx}=27$ Let A and B be the angles that the tangent and the line $y=x+3$ make with the positive x-axis respectively. $A=\tan^{-1}27=87.879°$ $B=\tan^{-1}(1)=45°$ $\alpha=A-B\approx 42.9°$	Students should learn how to make use of the expression in (i) to find $\frac{dy}{dx}$

<table>
<tr><th>Q5</th><th>Suggested Answers</th><th>Guidance</th></tr>
<tr><td>(i)</td><td>$y = \mathrm{f}(x) = 3\sqrt{1-\frac{x^2}{2^2}}$
$y = \mathrm{f}\left(\frac{1}{2}x-1\right)-1 = 3\sqrt{1-\frac{\left(\frac{1}{2}x-1\right)^2}{2^2}}-1$
</td><td>It's easier to do the translation first followed by the scaling. This would enable you to get the max. point and the 2 end points right away.

But since the equation is given, you could use your GC to sketch the curve. In addition, the intercepts could be obtained from the GC directly.

The shape of the ellipse should be drawn such that it is stretched horizontally, i.e. the major axis is the longer diameter from (-2,-1) to (6,-1).</td></tr>
<tr><td>(ii)</td><td></td><td>The curve must be drawn to approach the two vertical asymptotes. On the other hand, the curve must not be drawn to approach the x-axis as there are two end points (-2,-1) and (6,-1).
Several students forgot to take reciprocal for the y-intercept.
All values should be simplified and given as a decimal value rounded off to 3 s.f. since no exact answers are mentioned in the question.</td></tr>
</table>

(iii)	Solution set $=\{x\in\mathbb{R}: -2<x<-1.77 \text{ or } -1.47<x<1.97\}$	This is simply a GC question where the x-values could be found from the intersection of the two curves. Many students forgot how to write solution in set notation.

Q6	Suggested Answers	Guidance
(a)(i)	$\tan(\theta+\theta)=\dfrac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}=\dfrac{2\tan\theta}{1-\tan^2\theta}$	Do not use the double angle formula here as the question specifically stated to use the "addition formula".
(ii)	$x=\tan\theta \Rightarrow \dfrac{dx}{d\theta}=\sec^2\theta$ $x=0,\ \theta=0$ and $x=1,\ \theta=\dfrac{\pi}{4}$ $\displaystyle\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx=\int_0^{\frac{\pi}{4}}\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\left(\sec^2\theta\right)d\theta$ $\displaystyle=\int_0^{\frac{\pi}{4}}\tan^{-1}(\tan 2\theta)\left(\sec^2\theta\right)d\theta$ $\displaystyle=\int_0^{\frac{\pi}{4}}2\theta\sec^2\theta\ d\theta$ $\displaystyle=\left[2\theta\tan\theta\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}2\tan\theta\ d\theta$ $=2\left(\dfrac{\pi}{4}\right)-2\left[\ln\sec\theta\right]_0^{\frac{\pi}{4}}$ $=\dfrac{\pi}{2}-2\ln\sqrt{2}$ $=\dfrac{\pi}{2}-\ln 2$ "Otherwise" Let $u=\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ and $\dfrac{dv}{dx}=1$	Do remember to change the limit. Many failed to realise that $\tan^{-1}(\tan 2\theta)=2\theta$. Final answer is to be left in the simplified form.

	$\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2}\cdot\frac{2(1-x^2)-(-2x)(2x)}{(1-x^2)^2}$ and $v=x$ $=\frac{2+2x^2}{(1-x^2)^2+(2x)^2}$ $=\frac{2(1+x^2)}{(1+x^2)^2}$ $=\frac{2}{1+x^2}$ $\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)\mathrm{d}x=\left[x\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]_0^1-\int_0^1\frac{2x}{1+x^2}\mathrm{d}x$ $=\frac{\pi}{2}-\left[\ln(1+x^2)\right]_0^1$ $=\frac{\pi}{2}-\ln 2$	
(b)	$\int_{-1}^1 \frac{2+\lvert x\rvert}{2+x}\mathrm{d}x=\int_{-1}^0\frac{2-x}{2+x}\mathrm{d}x+\int_0^1\frac{2+x}{2+x}\mathrm{d}x$ $=\int_{-1}^0 -1+\frac{4}{2+x}\mathrm{d}x+\int_0^1 1\,\mathrm{d}x$ $=\left[-x+4\ln\lvert 2+x\rvert\right]_{-1}^0+\left[x\right]_0^1$ $=4\ln 2$	Many do not know how to deal with $\lvert x\rvert$. $\lvert x\rvert=\begin{cases}x, & \text{for } x>0\\ -x, & \text{for } x<0\end{cases}$

Q7	Suggested Answers	Guidance
	$\mathrm{f}(r)-\mathrm{f}(r+1)=\frac{r+1}{3^{r-1}}-\frac{r+2}{3^r}=\frac{3r+3}{3^r}-\frac{r+2}{3^r}=\frac{2r+1}{3^r}$ $\frac{1}{3}+\frac{3}{3^2}+\frac{5}{3^3}+\ldots\ldots+\frac{2n-1}{3^n}=\sum_{r=1}^{n}\frac{2r-1}{3^r}=\sum_{r=1}^{n}\left(\frac{2r+1}{3^r}-\frac{2}{3^r}\right)$ $=\sum_{r=1}^{n}\frac{2r+1}{3^r}-\sum_{r=1}^{n}\frac{2}{3^r}$ $=\sum_{r=1}^{n}[\mathrm{f}(r)-\mathrm{f}(r+1)]-2\sum_{r=1}^{n}\left(\frac{1}{3}\right)^r$, where $\mathrm{f}(r)=\frac{r+1}{3^{r-1}}$ $=[\mathrm{f}(1)-\mathrm{f}(2)]$ $+[\mathrm{f}(2)-\mathrm{f}(3)]$ $+\ldots\ldots\ldots\ldots$ $+[\mathrm{f}(n)-\mathrm{f}(n+1)] \quad - \quad 2\left[\frac{\frac{1}{3}\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}\right]$ $=[\mathrm{f}(1)-\mathrm{f}(n+1)] \quad - \quad \left(1-\frac{1}{3^n}\right)$ $=\frac{2}{3^0}-\frac{n+2}{3^n}-1+\frac{1}{3^n}$ $=1-\left(\frac{n+2}{3^n}-\frac{1}{3^n}\right)=1-\frac{n+1}{3^n}$	Since it is a show question, please list out all the steps clearly and don't skip any steps. Quite a number of students just assume $\frac{1}{3}+\frac{3}{3^2}+\frac{5}{3^3}+\ldots\ldots+\frac{2n-1}{3^n}$ $=\sum_{r=1}^{n}[\mathrm{f}(r)-\mathrm{f}(r+1)]$ without first checking to see if the series is correct. $\mathrm{f}(r)$ is $\frac{r+1}{3^{r-1}}$ while a number of students confused $\mathrm{f}(r)$ as $\frac{2r+1}{3^r}$

As $n \to \infty$, $\frac{n+1}{3^n} \to 0$, $\sum_{r=1}^{\infty} \frac{2r-1}{3^r} = 1$

Therefore $\sum_{r=n+1}^{\infty} \frac{2r-1}{3^r} = \sum_{r=1}^{\infty} \frac{2r-1}{3^r} - \sum_{r=1}^{n} \frac{2r-1}{3^r}$

$= 1 - \left(1 - \frac{n+1}{3^n}\right)$

$= \frac{n+1}{3^n}$

Alternative method:

$\frac{1}{3}\left[\mathrm{f}(r) - \mathrm{f}(r+1)\right] = \frac{2r+1}{3^{r+1}}$

$$\frac{1}{3} + \frac{3}{3^2} + \frac{5}{3^3} + \text{........} + \frac{2n-1}{3^n} = \sum_{r=0}^{n-1} \frac{2r+1}{3^{r+1}}$$

$$= \frac{1}{3}\sum_{r=0}^{n-1}\left[\mathrm{f}(r) - \mathrm{f}(r+1)\right]$$

$$= \frac{1}{3}[\mathrm{f}(0) - \mathrm{f}(1)$$

$$+\mathrm{f}(1) - \mathrm{f}(2)$$

.

.

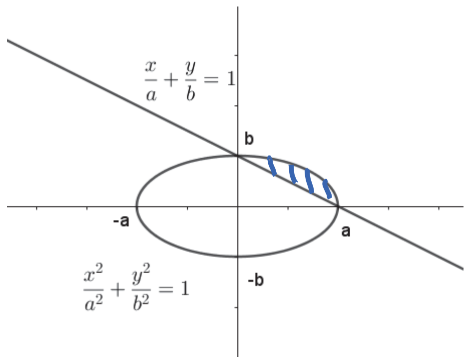
$$+\mathrm{f}(n-1) - \mathrm{f}(n)]$$

$$= \frac{1}{3}[\mathrm{f}(0) - \mathrm{f}(n)]$$

$$= \frac{1}{3}\left[\frac{1}{3^{-1}} - \frac{n+1}{3^{n-1}}\right] = 1 - \frac{n+1}{3^n}$$

	As $n \to \infty$, $\frac{n+1}{3^n} \to 0$, $\sum_{r=0}^{\infty} \frac{2r+1}{3^{r+1}} = 1$ Therefore $\sum_{r=n+1}^{\infty} \frac{2r-1}{3^r} = \sum_{r+1=n+1}^{\infty} \frac{2(r+1)-1}{3^{r+1}}$ $= \sum_{r=n}^{\infty} \frac{2r+1}{3^{r+1}} = \sum_{r=0}^{\infty} \frac{2r+1}{3^{r+1}} - \sum_{r=0}^{n-1} \frac{2r+1}{3^{r+1}}$ $= 1 - \left(1 - \frac{n+1}{3^n}\right)$ $= \frac{n+1}{3^n}$	

<table>
<tr><th>Q8</th><th>Suggested Answers</th><th>Guidance</th></tr>
<tr><td>(i)</td><td>$\int_0^a \sqrt{1-\frac{x^2}{a^2}}\,\mathrm{d}x = \int_0^{\frac{\pi}{2}} \sqrt{1-\frac{(a\sin\theta)^2}{a^2}}\,a\cos\theta\,\mathrm{d}\theta$
$= \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2\theta}\;a\cos\theta\,\mathrm{d}\theta$
$= a\int_0^{\frac{\pi}{2}} \frac{\cos 2\theta+1}{2}\,\mathrm{d}\theta$
$= \frac{a}{2}\left[\frac{\sin 2\theta}{2}+\theta\right]_0^{\frac{\pi}{2}}$
$= \frac{a\pi}{4}$</td><td></td></tr>
<tr><td>(ii)</td><td>$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow y = b\sqrt{1-\frac{x^2}{a^2}}$ (for first quadrant)

Area of $R = \int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,\mathrm{d}x - \frac{1}{2}ab$
$= b\int_0^a \sqrt{1-\frac{x^2}{a^2}}\,\mathrm{d}x - \frac{1}{2}ab$
$= b\left(\frac{\pi a}{4}\right)-\frac{1}{2}ab$
$= \frac{ab}{4}(\pi-2)$</td><td>Student should draw correct graphs to identify the correct region.
</td></tr>
</table>

<table>
<tr>
<td>(iii)</td>
<td>

New equation is $\dfrac{x^2}{a^2}+\dfrac{(y+b)^2}{b^2}=1$

Rearranging $\dfrac{x^2}{a^2}+\dfrac{(y+b)^2}{b^2}=1$

$\Rightarrow y=-b+b\sqrt{1-\dfrac{x^2}{a^2}}$ (reject $y=-b-b\sqrt{1-\dfrac{x^2}{a^2}}$ since $y>-b$)

Required volume $=\dfrac{1}{3}\pi ab^2-\pi\displaystyle\int_0^a\left(-b+b\sqrt{1-\dfrac{x^2}{a^2}}\right)^2\mathrm{d}x$

$=\dfrac{1}{3}\pi ab^2-\pi\displaystyle\int_0^a\left[b^2-2b^2\sqrt{1-\dfrac{x^2}{a^2}}+\left(b\sqrt{1-\dfrac{x^2}{a^2}}\right)^2\right]\mathrm{d}x$

$=\dfrac{1}{3}\pi ab^2-\pi\displaystyle\int_0^a\left(2b^2-\dfrac{b^2x^2}{a^2}\right)\mathrm{d}x+2b^2\pi\displaystyle\int_0^a\sqrt{1-\dfrac{x^2}{a^2}}\,\mathrm{d}x$

$=\dfrac{1}{3}\pi ab^2-\pi b^2\left[2x-\dfrac{x^3}{3a^2}\right]_0^a+2\pi b^2\left(\dfrac{a\pi}{4}\right)$

$=\dfrac{1}{3}\pi ab^2-\pi b^2\left(\dfrac{5a}{3}\right)+\left(\dfrac{ab^2\pi^2}{2}\right)$

$=\dfrac{\pi^2ab^2}{2}-\dfrac{4\pi ab^2}{3}$

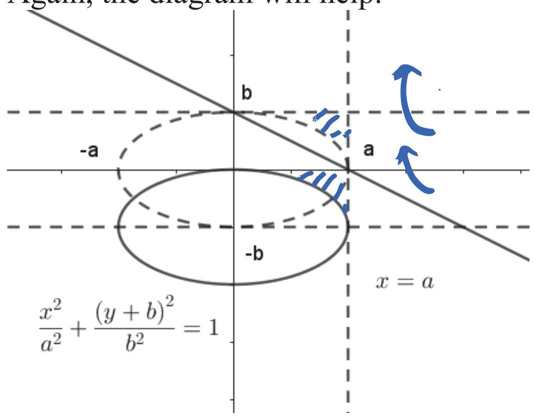
$=\dfrac{\pi ab^2}{6}(3\pi-8)$

</td>
<td>

Interpretation of the question is a problem.

The question wanted to find the volume rotated about $y=b$ by the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$. This is done by rotating about x-axis $(y=0)$ by $\dfrac{x^2}{a^2}+\dfrac{(y+b)^2}{b^2}=1$.

Again, the diagram will help.



</td>
</tr>
</table>

Q9	Suggested Answers	Guidance
(a)	$u=1+\mathrm{i}\sqrt{3}=2\mathrm{e}^{\frac{\mathrm{i}\pi}{3}}$ $v=-k-\mathrm{i}k=k(-1-\mathrm{i})=k\sqrt{2}\mathrm{e}^{\frac{-\mathrm{i}3\pi}{4}}$ $\frac{(u^*)^7}{v^{10}}=\frac{\left(2\mathrm{e}^{\frac{-\mathrm{i}\pi}{3}}\right)^7}{\left(k\sqrt{2}\mathrm{e}^{\frac{-\mathrm{i}3\pi}{4}}\right)^{10}}$ $=\frac{4}{k^{10}}\mathrm{e}^{\mathrm{i}\left(\frac{-7\pi}{3}+\frac{30\pi}{4}\right)}$ $=\frac{4}{k^{10}}\mathrm{e}^{\mathrm{i}\left(\frac{31\pi}{6}\right)}=\frac{4}{k^{10}}\mathrm{e}^{\mathrm{i}\left(\frac{31\pi}{6}-6\pi\right)}=\frac{4}{k^{10}}\mathrm{e}^{\frac{-\mathrm{i}5\pi}{6}}$	<u>Please use GC to find/check the argument.</u> Recall ABS. Always note the position of the point representing the complex number on the Argand diagram to determine the magnitude and sign of the argument.
(b)(i)	$z^2-9-40\mathrm{i}=0$ $\Rightarrow z^2=9+40\mathrm{i}$ Let $z=x+\mathrm{i}y$ $(x+\mathrm{i}y)^2=9+40\mathrm{i}$ $x^2+2\mathrm{i}xy-y^2=9+40\mathrm{i}$ Comparing real and imaginary parts, $x^2-y^2=9$ --------------------------------(1) $2xy=40\Rightarrow xy=20\Rightarrow y=\frac{20}{x}$ --------(2) Subst (2) into (1), $x^2-\left(\frac{20}{x}\right)^2=9$ $x^4-9x^2-400=0$ $x^2=-16$ or $x^2=25$	Note that it is specifically stated that the use of calculator is not allowed for this question. Else we could use GC to find $\sqrt{9+40\mathrm{i}}$. Always find the complex number z directly. There are 2 common scenarios where we would write $z=x+y\mathrm{i}$ and solve for x and y by comparing real and imaginary parts: (1) Solving an equation of the form $z^2=\alpha$ where α is a complex number. (2) Solving an equation which involves 2 or more of the terms z, z^*, and $\|z\|$.

	Since x is real, $x^2 = 25 \Rightarrow x = \pm 5, y = \pm 4$ $z_1 = 5+4\text{i}$, $z_2 = -5-4\text{i}$ Alternatively, $z^2 - 9 - 40\text{i} = 0$ $z = \frac{\pm\sqrt{-4(-9-40\text{i})}}{2} = \pm\sqrt{9+40\text{i}}$ $\Rightarrow z^2 = 9+40\text{i}$ then let $z = x + \text{i}y$ and solve for x and y as before.	
(ii)	$w^2 + 5w + 4 - 10\text{i} = 0$ Standard (recommended) method to solve quadratic equation: $w = \frac{-5 \pm \sqrt{25 - 4(4-10\text{i})}}{2} = \frac{-5 \pm \sqrt{9+40\text{i}}}{2}$ $w_1 = \frac{-5+(5+4\text{i})}{2}$ and $w_2 = \frac{-5-(5+4\text{i})}{2}$ $w_1 = 2\text{i}$ and $w_2 = -5-2\text{i}$ Alternative method: $w^2 + 5w + 4 - 10\text{i} = 0$ $\left(w+\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 4 - 10\text{i} = 0$ $\left(w+\frac{5}{2}\right)^2 - \frac{9}{4} - 10\text{i} = 0$ $4\left(w+\frac{5}{2}\right)^2 - 9 - 40\text{i} = 0$ $(2w+5)^2 - 9 - 40\text{i} = 0$	"Hence": need to apply result in (i) to solve the equation.

	Comparing with equation in (i), $z = 2w + 5 \Rightarrow w = \dfrac{z-5}{2}$ Hence $w = \dfrac{5+4\text{i}-5}{2} = 2\text{i}$ or $w = \dfrac{-5-4\text{i}-5}{2} = -5-2\text{i}$	

Q10	Suggested Answers	Guidance
	Differentiating $x = ue^{2t}$ wrt t gives $\frac{dx}{dt} = 2ue^{2t} + e^{2t}\frac{du}{dt}$ ---- (1) and $\frac{d^2x}{dt^2} = 2\left(2ue^{2t} + e^{2t}\frac{du}{dt}\right) + \left(e^{2t}\frac{d^2u}{dt^2} + 2e^{2t}\frac{du}{dt}\right) = e^{2t}\frac{d^2u}{dt^2} + 4e^{2t}\frac{du}{dt} + 4ue^{2t}$ Substituting into $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = e^{2t}$ gives $e^{2t}\frac{d^2u}{dt^2} + 4e^{2t}\frac{du}{dt} + 4ue^{2t} - 4\left(2ue^{2t} + e^{2t}\frac{du}{dt}\right) + 4ue^{2t} = e^{2t}$ Simplifying, we have $\frac{d^2u}{dt^2} = 1$.	Many candidates did not infer that u is a variable and differentiate $x = ue^{2t}$ without the use of Product Rule. Some manipulated the expression to make u the subject instead, which is not necessary.
	Solving, $\frac{du}{dt} = t + c$ and $u = \frac{t^2}{2} + ct + d$ Therefore, $x = e^{2t}\left(\frac{t^2}{2} + ct + d\right)$ When $t = 0$, $x = 1$, gives $d = 1$ and $u = 1$ Also when $t = 0$, $\frac{dx}{dt} = 1$, gives $\frac{du}{dt} = 1 - 2 = -1$ and $c = -1$ Therefore, $x = e^{2t}\left(\frac{t^2}{2} - t + 1\right) = \frac{e^{2t}}{2}\left(t^2 - 2t + 2\right)$.	Many candidates did not proceed to solve $\frac{d^2u}{dt^2} = 1$, although the question states 'Hence find …'. Some who did, missed out one of the two required arbitrary constants.
	$\frac{dv}{dt} = 2e^{-v} \Rightarrow \int e^{v}\,dv = \int 2\,dt$ Therefore, $e^{v} = 2t + k$, where k is an arbitrary constant.	Some candidates did not manage to separate the variables correctly. Many,

	When $t=0$, $v=0$ gives $k=1$. Therefore, $t=\dfrac{e^{v}-1}{2}$. Also $v=\ln(2t+1)\Rightarrow\dfrac{dx}{dt}=\ln(2t+1)$ Using integration by parts, $x=\int\ln(2t+1)\ dt=t\ln(2t+1)-\int\dfrac{2t}{2t+1}\ dt$ Solving, $x=t\ln(2t+1)-\int 1-\dfrac{1}{2t+1}\ dt=t\ln(2t+1)-t+\dfrac{1}{2}\ln(2t+1)+k_1$ where k_1 is an arbitrary constant. When $t=0$, $x=1$ (original position), $k_1=1$. Therefore, $x=\left(t+\dfrac{1}{2}\right)\ln(2t+1)-t+1$ ie, $x=\left(\dfrac{e^{v}-1}{2}+\dfrac{1}{2}\right)v-\dfrac{e^{v}-1}{2}+1=\dfrac{e^{v}}{2}v-\dfrac{e^{v}}{2}+\dfrac{1}{2}+1$ and this gives $2x=3+e^{v}(v-1)$ Alternatively, $\dfrac{dv}{dt}=2e^{-v}\Rightarrow\dfrac{dx}{dt}\dfrac{dv}{dx}=2e^{-v}\Rightarrow v\dfrac{dv}{dx}=2e^{-v}$ By separating the variables, $\int ve^{v}dv=\int 2\ dx$ $\Rightarrow ve^{v}-\int e^{v}dv=2x+C\Rightarrow ve^{v}-e^{v}=2x+C$ When $t=0$, $v=0$, $x=1$, we have $C=-3$ and this gives $2x=3+e^{v}(v-1)$	who managed to solve, did not express the final answer as t in terms of v. This second part is a 'new motion' and is unrelated to the motion in the first part of the question. Some students attempt to use the solution from the first part to find the displacement. Most who solve using integration by parts successfully, managed to find the value of the constant. All steps to attain the final expression must be shown clearly as this is an AG(answer given) part. You may be keen to view the alternative solution offered as it uses an alternative expression for which acceleration $\dfrac{dv}{dt}$ may be expressed. This provides a more elegant way to solve this part.

Q11	Suggested Answers	Guidance
(i)	$\overrightarrow{OS}=\begin{pmatrix}2\\3\\-1\end{pmatrix}$, $\overrightarrow{OA}=\begin{pmatrix}8\\5\\-2\end{pmatrix}$ $\overrightarrow{SA}=\begin{pmatrix}8\\5\\-2\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}6\\2\\-1\end{pmatrix}$ Equation of pipe from S to A is $\underset{\sim}{r}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}6\\2\\-1\end{pmatrix}, \lambda\in\mathbb{R}$ $\overrightarrow{OT}=\begin{pmatrix}8\\a\\-\frac{7}{2}\end{pmatrix}$, $\overrightarrow{OC}=\begin{pmatrix}5\\6\\-2\end{pmatrix}$ $\overrightarrow{CT}=\begin{pmatrix}8\\a\\-\frac{7}{2}\end{pmatrix}-\begin{pmatrix}5\\6\\-2\end{pmatrix}=\begin{pmatrix}3\\a-6\\-\frac{3}{2}\end{pmatrix}$ Equation of pipe from T to C is	

	$\underset{\sim}{r} = \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ a-6 \\ -\frac{3}{2} \end{pmatrix}, \mu \in \mathbb{R}$ Since the 2 pipes will connect, $2+6\lambda = 5+3\mu \quad \Rightarrow 6\lambda - 3\mu = 3 \quad ----(1)$ $3+2\lambda = 6+\mu(a-6) \quad \Rightarrow 2\lambda = 3+\mu(a-6) ----(2)$ $-1-\lambda = -2-\frac{3}{2}\mu \quad \Rightarrow -\lambda + \frac{3}{2}\mu = -1 \quad ----(3)$ Solving (1) and (3), $\lambda = \frac{1}{4}, \quad \mu = -\frac{1}{2}$ Subst into (2), $\frac{1}{2} = 3 - \frac{1}{2}(a-6)$ $\Rightarrow a = 11$	
(ii)	 Using ratio theorem, $\overrightarrow{OA} = \frac{2\overrightarrow{OB} + \overrightarrow{OS}}{3}$ $\overrightarrow{OB} = \frac{3\overrightarrow{OA} - \overrightarrow{OS}}{2} = \frac{1}{2}\left[3\begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}\right] = \begin{pmatrix} 11 \\ 6 \\ -\frac{5}{2} \end{pmatrix}$	Use ratio theorem for quick manipulation to get coordinates of B

	B is located at $\left(11, 6, -\frac{5}{2}\right)$	
(iii)	$\overrightarrow{CB} = \begin{pmatrix} 11 \\ 6 \\ -\frac{5}{2} \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ -\frac{1}{2} \end{pmatrix}$ $\overrightarrow{CT} = \begin{pmatrix} 3 \\ 5 \\ -\frac{3}{2} \end{pmatrix}$ $\overrightarrow{CB} \times \overrightarrow{CT} = \begin{pmatrix} 6 \\ 0 \\ -\frac{1}{2} \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ -\frac{3}{2} \end{pmatrix}$ which is parallel to $\begin{pmatrix} 1 \\ 3 \\ 12 \end{pmatrix}$. Equation of plane is $\underset{\sim}{r} \bullet \begin{pmatrix} 1 \\ 3 \\ 12 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 12 \end{pmatrix} = -1$ Cartesian equation of plane is $x + 3y + 12z = -1$	Use GC to find a vector parallel to the normal instead of working out the vector product
(iv)	Let F be a point on pipe SB such that the distance from T to SB is the shortest. $\overrightarrow{OF} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}$, for some $\lambda \in \mathbb{R}$	If F is on pipe SB, we can use the equation of pipe SA to avoid using coordinates of B since S, A and B lie in a straight line

$$\overrightarrow{TF} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 8 \\ 11 \\ -\frac{7}{2} \end{pmatrix} = \begin{pmatrix} -6+6\lambda \\ -8+2\lambda \\ \frac{5}{2}-\lambda \end{pmatrix}$$

$\overrightarrow{TF}$ is perpendicular to $\begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} -6+6\lambda \\ -8+2\lambda \\ \frac{5}{2}-\lambda \end{pmatrix} \bullet \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} = 0$$

$$-36+36\lambda-16+4\lambda-\frac{5}{2}+\lambda=0$$

$$\lambda = \frac{109}{82}$$

Alternatively, $\left|\overrightarrow{TF}\right|^2 = (-6+6\lambda)^2 + (-8+2\lambda)^2 + \left(\frac{5}{2}-\lambda\right)^2$

To get minimum distance, $\dfrac{d\left(\left|\overrightarrow{TF}\right|^2\right)}{d\lambda} = 0$

$$12(-6+6\lambda)+4(-8+2\lambda)-2\left(\frac{5}{2}-\lambda\right)=0$$

$$-72+72\lambda-32+8\lambda-5+2\lambda=0$$

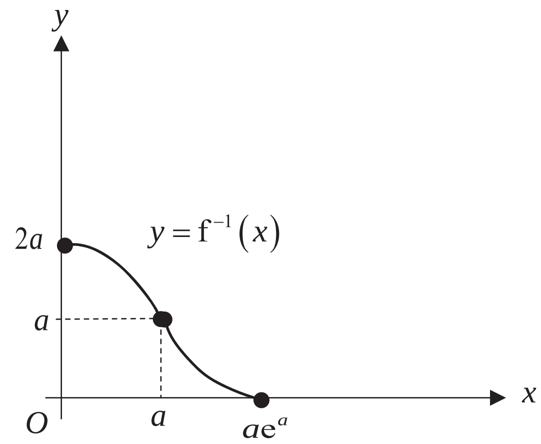
	$\lambda = \frac{109}{82}$ $\overrightarrow{OF} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \frac{109}{82}\begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{409}{41} \\ \frac{232}{41} \\ -\frac{191}{82} \end{pmatrix}$ The pipes will connect at $\left(\frac{409}{41}, \frac{232}{41}, -\frac{191}{82}\right)$ $\overrightarrow{TF} = \begin{pmatrix} -6+6\left(\frac{109}{82}\right) \\ -8+2\left(\frac{109}{82}\right) \\ \frac{5}{2}-\left(\frac{109}{82}\right) \end{pmatrix} = \begin{pmatrix} \frac{81}{41} \\ -\frac{219}{41} \\ \frac{48}{41} \end{pmatrix}$ Or $\overrightarrow{TF} = \overrightarrow{OF} - \overrightarrow{OT} = \begin{pmatrix} \frac{409}{41} \\ \frac{232}{41} \\ -\frac{191}{82} \end{pmatrix} - \begin{pmatrix} 8 \\ 11 \\ -\frac{7}{2} \end{pmatrix} = \begin{pmatrix} \frac{81}{41} \\ -\frac{219}{41} \\ \frac{48}{41} \end{pmatrix}$ Length of pipe	Even if you cannot find given result, use it to get vector $\overrightarrow{TF}$ and hence length of required pipe *TF*.

	$= \left\|\overrightarrow{TF}\right\| = \frac{1}{41}\sqrt{81^2+219^2+48^2}$ $= \frac{1}{41}\sqrt{56826} = \frac{3}{41}\sqrt{6314} \approx 5.81$	

Q1	Suggested Answers	Guidance
(i)	$y=\left[\cos^{-1}(3x)\right]^2$ $\frac{dy}{dx}=2\left[\cos^{-1}(3x)\right]\frac{-1}{\sqrt{1-9x^2}}(3)$ $\sqrt{1-9x^2}\frac{dy}{dx}=-6\sqrt{y}------(*)$ $\therefore\ (1-9x^2)\left(\frac{dy}{dx}\right)^2=36y$ (shown) $(1-9x^2)2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2(-18x)=36\left(\frac{dy}{dx}\right)$ $(1-9x^2)2\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)(-18x)=36$ $\therefore\ (1-9x^2)\frac{d^2y}{dx^2}-9x\left(\frac{dy}{dx}\right)=18$	You should differentiate the equation implicitly. Those who attempt to make $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ the subject made many careless mistakes.
(ii)	$(1-9x^2)\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}(-18x)-9x\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)(-9)=0$ When $x=0$, $y=\frac{\pi^2}{4}$ $\frac{dy}{dx}=\frac{-2\left[\cos^{-1}(0)\right](3)}{\sqrt{1-9(0)^2}}=-3\pi$ or $\frac{dy}{dx}=-6\sqrt{\frac{\pi^2}{4}}=-3\pi$ from (*) $\frac{d^2y}{dx^2}=18$ $\frac{d^3y}{dx^3}=-27\pi$	To find the value of $\frac{dy}{dx}$, many substitute into $(1-9x^2)\left(\frac{dy}{dx}\right)^2=36y$ which gives $\frac{dy}{dx}=\pm 3\pi$. However, from (*), we know that $\frac{dy}{dx}=\frac{-6\sqrt{y}}{\sqrt{1-9x^2}}<0$

	$\therefore\ y=\frac{\pi^2}{4}-3\pi x+9x^2-\frac{9\pi}{2}x^3+\ldots$	Hence, $\frac{\mathrm{d}y}{\mathrm{d}x}=-3\pi$.
(iii)	$\sum_{r=1}^{\infty}\frac{(-1)^r(\frac{\pi}{4})^{2r+1}}{(2r+1)!}=-\frac{(\frac{\pi}{4})^3}{3!}+\frac{(\frac{\pi}{4})^5}{5!}-\frac{(\frac{\pi}{4})^7}{7!}+\ldots.$ $=\sin\frac{\pi}{4}-\frac{\pi}{4}$ $=\frac{1}{\sqrt{2}}-\frac{\pi}{4}$	Many left this part blank. You should expand out the first few terms and draw parallel to the standard series of $\sin x$.

Q2	Suggested Answers	Guidance
(i)	Let $y=\mathrm{f}(x)\Rightarrow \mathrm{f}^{-1}(y)=x$ 1. $y=a-\frac{1}{a}(x-a)^2$ $(x-a)^2=a^2-ay$ $x=a\pm\sqrt{a^2-ay}$ $\left(\text{reject } x=a-\sqrt{a^2-ay} \because x\ge a\right)$ $\therefore x=a+\sqrt{a^2-ay}$ 2. $y=ae^{a-x}$ $a-x=\ln\left(\frac{y}{a}\right)$ $x=a-\ln\left(\frac{y}{a}\right)$ $\therefore \mathrm{f}^{-1}:x\mapsto\begin{cases} a+\sqrt{a^2-ax} & \text{for } 0\le x\le a \\ a-\ln\left(\frac{x}{a}\right) & \text{for } a<x\le ae^a \end{cases}$	For the modulus portion, take note that the portion of the curve is negative. Hence, removing the mod will give $y=a-\frac{1}{a}(x-a)^2$. When making x the subject, do not forget to reject $x=a-\sqrt{a^2-ay} \because x\ge a$. This other portion $y=ae^{a-x}$ is relativey easy. Note the inequalities for the domains. Many students were confused about the signs. Some also mixed up the two domains. It would be good to work out the domains in ascending order i.e. $0\le x\le a$ followed by $a<x\le ae^a$.

(ii)		The concavity of the curve should be drawn clearly. Some students drew straight lines instead of a curve. Other diagrams are ridiculously drawn i.e. it's not even a function! There are 2 intercepts in this curve. There is no need to put in coordinates form as the question did not specify to do so. In addition, the point (a, a) needs to be shown to ensure continuity of the curve.
(iii)	$R_{f^{-1}} = [0, 2a]$ $D_g = (0, \infty)$ $\because R_{f^{-1}} \not\subset D_g$ $\therefore\ gf^{-1}$ does not exist.	$R_{f^{-1}}$ could be easily found by looking at the domain of f which is given in the question i.e. $[0, 2a]$. This needs to be specified, making comparison with the domain of g, instead of simply saying $R_{f^{-1}} \not\subset D_g$.

Q3	Suggested Answers	Guidance
	Let l be the total distance travelled between City A and B. $l=\sqrt{x^2+b^2}+r+\sqrt{(p-x)^2+1^2}$ $\frac{\mathrm{d}l}{\mathrm{d}x}=\frac{1}{2}(x^2+b^2)^{-\frac{1}{2}}(2x)+\frac{1}{2}\left[(p-x)^2+1\right]^{-\frac{1}{2}}\left[-2(p-x)\right]$	Use Pythagoras Theorem to find the distance since the question asks for least distance Note that b, r and p are constants.
	At stationary point, $\frac{\mathrm{d}l}{\mathrm{d}x}=0$ $\frac{1}{2}(x^2+b^2)^{-\frac{1}{2}}(2x)+\frac{1}{2}\left[(p-x)^2+1\right]^{-\frac{1}{2}}\left[-2(p-x)\right]=0$ $\frac{1}{2}(x^2+b^2)^{-\frac{1}{2}}(2x)=\frac{1}{2}\left[(p-x)^2+1\right]^{-\frac{1}{2}}\left[2(p-x)\right]$ $\frac{x}{\sqrt{x^2+b^2}}=\frac{p-x}{\sqrt{(p-x)^2+1}}$ $x^2[(p-x)^2+1]=(p-x)^2(x^2+b^2)$ $x^2(p-x)^2+x^2=x^2(p-x)^2+b^2(p-x)^2$ $x^2-b^2(x^2-2px+p^2)=0$ $(1-b^2)x^2+2b^2px-b^2p^2=0$	Do not expand. Try to factorise.
	$x=\frac{-2b^2p\pm\sqrt{(2b^2p)^2-4(1-b^2)(-b^2p^2)}}{2(1-b^2)}$ $=\frac{-2b^2p\pm\sqrt{4b^4p^2+4b^2p^2-4b^4p^2}}{2(1-b^2)}$ $=\frac{-2b^2p\pm 2bp}{2(1-b^2)}=\frac{b^2p-bp}{(b^2-1)}$ or $\frac{b^2p+bp}{(b^2-1)}$ Since $\frac{b^2p+bp}{(b^2-1)}=\frac{bp}{(b-1)}>p$, $\therefore x=\frac{bp(b-1)}{(b+1)(b-1)}=\frac{bp}{(b+1)}$	Do not assume that if there is '-' in front then the expression will be negative

	Or $x^2 = b^2(p-x)^2$ $x = \pm b(p-x)$ $x = bp - bx$ or $x = -bp + bx$ $x = \dfrac{bp}{1+b}$ or $x = \dfrac{bp}{b-1}$ since x is minimum and $b > 1$ $x = \dfrac{bp}{1+b}$	

Q4	Suggested Answers	Guidance
(a)	$\frac{10}{2}(2\alpha+9\beta)=2(\alpha+21\beta)$ $10\alpha+45\beta=2\alpha+42\beta$ $8\alpha=-3\beta$ $\alpha=-\frac{3}{8}\beta \quad ---(1)$ $\alpha+5\beta=37 \quad ---(2)$ Sub (1) into (2) $-\frac{3}{8}\beta+5\beta=37$ $\beta=8$ sub into (1), $\alpha=-3$	
(b)	$\frac{a^2+a\sqrt{b}\mathrm{i}-a\sqrt{b}+a^2\mathrm{i}}{a+\sqrt{b}\mathrm{i}}$ $=\frac{a(a+\sqrt{b}\mathrm{i})-\frac{a}{\mathrm{i}}(\sqrt{b}\mathrm{i}+a)}{a+\sqrt{b}\mathrm{i}}$	Generally when the question ask to prove something is GP you need to prove $\frac{U_n}{U_{n-1}}$ is a constant. But this question has

	$= a - \frac{a}{\mathrm{i}} = a + a\mathrm{i}$ $\frac{-2a^2\sqrt{b}+2a^3\mathrm{i}}{a^2+a\sqrt{b}\mathrm{i}-a\sqrt{b}+a^2\mathrm{i}}$ $= \frac{2a^2(-\sqrt{b}+a\mathrm{i})}{a(a+\sqrt{b}\mathrm{i})-a(\sqrt{b}-a\mathrm{i})}$ $= \frac{2a^2\mathrm{i}(\frac{-\sqrt{b}}{\mathrm{i}}+a)}{a(a+\sqrt{b}\mathrm{i})-a\mathrm{i}(\frac{\sqrt{b}}{\mathrm{i}}-a)}$ $= \frac{2a^2\mathrm{i}(\sqrt{b}\mathrm{i}+a)}{a(a+\sqrt{b}\mathrm{i})-a\mathrm{i}(-\sqrt{b}\mathrm{i}-a)}$ $= \frac{2a^2\mathrm{i}(\sqrt{b}\mathrm{i}+a)}{a(a+\sqrt{b}\mathrm{i})+a\mathrm{i}(\sqrt{b}\mathrm{i}+a)}$ $= \frac{2a^2\mathrm{i}}{a+a\mathrm{i}}\cdot\frac{a-a\mathrm{i}}{a-a\mathrm{i}}$ $= \frac{2a^2\mathrm{i}(a-a\mathrm{i})}{a^2+a^2}$ $= \mathrm{i}(a-a\mathrm{i}) = a + a\mathrm{i}$ Hence the sequence is a GP.	only 3 terms so you just need to prove $\frac{U_2}{U_1}=\frac{U_3}{U_2}$ and not $\frac{U_n}{U_{n-1}} = \text{const}$. Although you can sub for example $a=1, b=2$ to check for the simplified terms of $\frac{-2a^2\sqrt{b}+2a^3\mathrm{i}}{a^2+a\sqrt{b}\mathrm{i}-a\sqrt{b}+a^2\mathrm{i}}$ but just simply just say by GC, $\frac{-2a^2\sqrt{b}+2a^3\mathrm{i}}{a^2+a\sqrt{b}\mathrm{i}-a\sqrt{b}+a^2\mathrm{i}} = a+\mathrm{i}a$ does not constitute a proof. You need to do the algebra to show.
	If $a=1$ and $b=3$, then the geometric progression is	

	$1+i\sqrt{3}$, $1+\sqrt{3}i-\sqrt{3}+i$, $-2\sqrt{3}+2i$ So the first term is $1+i\sqrt{3}$ and the common ratio is $1+i$ So $S_8=\dfrac{(1+i\sqrt{3})[(1+i)^8-1]}{(1+i)-1}=\dfrac{(1+i\sqrt{3})15}{i}=15(\sqrt{3}-i)$	

Q5	Suggested Answers	Guidance
(i)	Method 1: $\mathrm{P}(X=2)$ $= \mathrm{P}(\text{Black, White and 2 heads from 3 tosses})$ $+\mathrm{P}(\text{Black, Black and 1 head from 2 tosses})$ $+\mathrm{P}(\text{White, White and 1 head from 2 tosses})$ $=\frac{5}{8}\left(\frac{3}{7}\right)2\left({}^3C_2\right)\left(\frac{1}{2}\right)^3+\frac{5}{8}\left(\frac{4}{7}\right)2\left(\frac{1}{2}\right)^2+\frac{3}{8}\left(\frac{2}{7}\right)2\left(\frac{1}{2}\right)^2$ $=\frac{45}{224}+\frac{5}{28}+\frac{3}{56}$ $=\frac{97}{224}$ Method 2: $\mathrm{P}(X=2)$ $=\mathrm{P}(\text{cards are different and 2 heads from 3 tosses})$ $+\mathrm{P}(\text{cards are same and 1 head from 2 tosses})$ $=\frac{{}^5C_1{}^3C_1}{{}^8C_2}\left({}^3C_2\right)\left(\frac{1}{2}\right)^3+\frac{{}^5C_2+{}^3C_2}{{}^8C_2}\left({}^2C_1\right)\left(\frac{1}{2}\right)^2$ $=\frac{45}{224}+\frac{13}{56}$ $=\frac{97}{224}$	As this is an AG question, all working leading to the final given answer must be shown. Many wrote $\frac{5}{8}\left(\frac{3}{7}\right)2\left({}^3C_2\right)\left(\frac{1}{2}\right)^3$ as $\frac{5}{8}\left(\frac{3}{7}\right)\times 2\times\left(\frac{3}{8}\right)$ without any explanation and this is not accepted for an AG part. Likewise, similar working or statement was missing in the second part of the equation.

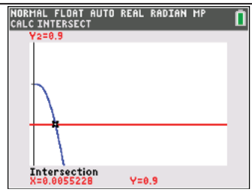
(ii)	E(winnings) $= -3\left(\frac{45}{224}+\frac{15}{224}\right)+0\left(\frac{41}{224}\right)+2\left(\frac{97}{224}\right)+4\left(\frac{13}{112}\right)$ $=0.52678$ Expected winnings = \$0.53 (to 2 d.p.)	Many students did not understand what the \$$x$ meant for this part of the question. Those who interpreted the question correctly mostly obtained full credit.
(iii)	$\mathrm{P}\left(X_1-X_2\geq 3 \mid X_2 \text{ is even}\right)$ $=\frac{\mathrm{P}\left(X_1-X_2\geq 3 \text{ and } X_2 \text{ is even}\right)}{\mathrm{P}\left(X_2 \text{ is even}\right)}$ $=\frac{\mathrm{P}\left(X_2=0, X_1=3,4\right)}{\mathrm{P}\left(X_2=0,2,4\right)}$ $=\frac{\frac{41}{224}\left(\frac{15}{224}+\frac{13}{112}\right)}{\frac{41}{224}+\frac{97}{224}+\frac{13}{112}}$ $=\frac{1681}{50176}\div\frac{41}{56}=\frac{41}{896}$ or 0.0458	Writing the conditional probability into the correct form was a challenge for many students, writing it as a product of two probabilities for the numerator and subsequently cancelling $\mathrm{P}\left(X_2 \text{ is even}\right)$ with the denominator, was common. Another group of students who assume '0' is not an even number, did not have much working to warrant any credit. The use of a table of outcome for this part was also applied incorrectly as students ignored the values given in the probability distribution table, and proceeded to count the number of outcomes to obtain the required probabilities.

Q6	Suggested Answers	Guidance
(i)	Let X, Y and W be the random variables denoting the amounts of paint A, B and C used on a car respectively. Then, $X \sim \text{N}(2.0,\ 0.29^2)$; $Y \sim \text{N}(1.8,\ 0.21^2)$ and $W \sim \text{N}(1.0,\ 0.12^2)$. The total quantity of paint, $T = X + Y_1 + Y_2 + W_1 + W_2 + W_3$ $\text{E}(T) = 2.0 + 2\times1.8 + 3\times1.0 = 8.6$ $\text{Var}(T) = 0.29^2 + 2\times0.21^2 + 3\times0.12^2 = 0.2155$ $T \sim N(8.6,\ 0.2155)$ Method 1: $\text{P}(\lvert T-8.6\rvert < a) > 0.9$ $\Rightarrow \text{P}(8.6-a < T < 8.6+a) > 0.9$ $\Rightarrow \text{P}(T < 8.6-a) < 0.05$ or $\text{P}(T > 8.6+a) < 0.05$ $8.6-a < 7.836426419$ or $8.6+a > 9.363573581$ $a > 0.763573581$ $\Rightarrow$ least $a = 0.764$ Method 2: $\text{P}(\lvert T-8.6\rvert < a) > 0.9$ $\Rightarrow \text{P}\left(\frac{-a}{\sqrt{0.2155}} < Z < \frac{a}{\sqrt{0.2155}}\right) > 0.9$ $\frac{a}{\sqrt{0.2155}} > 1.644853626$ $a > 0.7635735811$ $\Rightarrow$ least $a = 0.764$	Total amount of paint used is $T = X + Y_1 + Y_2 + W_1 + W_2 + W_3$ and **not** $T = X + 2Y + 3W$ Wrong definition of the random variable will affect the value of Var(T) and the value of Var(cost) in part (iii). The word "its mean" refers to the mean (or expectation) of the total amount of paint used and **not** $T = \frac{X + Y_1 + Y_2 + W_1 + W_2 + W_3}{6}$ Do not use the "table" in the GC to find value of a since it is not an integer.

(ii)	Let C be the random variable denoting the cost of spray-painting a car. $C = 350X + 200(Y_1 + Y_2) + 150(W_1 + W_2 + W_3)$ $C \sim N(1870, 14802.25)$ Probability required $= \mathrm{P}(C > 2000)$ ≈ 0.14264 $= 0.143$ (3 s.f.)	

Q7	Suggested Answers	Guidance
(i)	Number of different arrangements $=(5!)^4\,4!$ $=4,976,640,000$	Remember to arrange the 5 coloured cards for each of the 4 suits. Apply multiplication principle as we arrange each suit in sequence: $5!5!5!5!=(5!)^4$.
(ii)	Number of different arrangements $=\left[(2!)^2(4!)^2\right]9!$ $=836,075,520$ E.g. $\underbrace{\left[(\underbrace{B_JO_JY_JG_J}_{4!}R_J)(\underbrace{R_NR_Y}_{2!})(R_C\underbrace{O_CY_CG_CB_C}_{4!})\right]}_{9!}O_NY_NG_NB_NO_YY_YG_YB_Y$ (2! for swapping R_J and R_C) First, arrange R_NR_Y : 2! ways Next arrange R_J and R_C : $2!$ ways Then arrange the remaining 4 Juno cards ($B_JO_JY_JG_J$): $4!$ Then arrange the remaining 4 Cleo cards ($O_CY_CG_CB_C$): $4!$ Group $\left[(B_JO_JY_JG_JR_J)(R_NR_Y)(R_CO_CY_CG_CB_C)\right]$ as one unit and arrange this unit together with the remaining 8 cards: $9!$ ways	
(iii)	Probability that no two Yvex cards are next to each other	"No two Yvex cards are next to each other" is the same as all 5 Yvex cards are separated (by at least one

	$=\frac{(15-1)! \times {}^{15}C_5 \times 5!}{(20-1)!}$ $=\frac{1001}{3876}$ or 0.258	other non-Yvex card). Apply insertion method. It is not advisable to apply complementation principle as doing so for this question leads to more cases which are individually challenging.

Q8	Suggested Answers	Guidance
(i)	**Assumptions:** The probability of a randomly chosen surgical mask is faulty is constant. Whether a surgical mask is faulty or not is independent of other surgical masks.	Note that when describing independence we always refer to events.
(ii)	$X \sim \mathrm{B}(200, p)$ $\mathrm{P}(X \le 2) = 0.9$ Using GC, $p = 0.0055228 \approx 0.00552$ 	Proportion is a number between 0 and 1. Some students thought that it number of faulty masks and computed the probability wrongly as $\frac{p}{200}$.
(iii)	Probability required $= \left[{}^{179}C_2 (0.0055228)^2 (1-0.0055228)^{177} \right] 0.0055228$ $\approx 0.0010069 \approx 0.00101$	
(iv)	$\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \le 2) = 1 - 0.9 = 0.1$ Let Y be number of boxes with more than 2 faulty surgical masks, out of 50 boxes. $Y \sim \mathrm{B}(50, 0.1)$ $\mathrm{P}(Y > 7) = 1 - \mathrm{P}(Y \le 7)$ $\approx 0.12215 \approx 0.122$	The probability of having more than 2 faulty surgical masks in a box is found using the information given in question. It is not dependent on value found in (ii). Also, a number of students struggled at $\mathrm{P}(Y > 7) = 1 - \mathrm{P}(Y \le 7)$
(v)	Let W be the number of shipments of 50 boxes with at least 8 boxes that are rejected out of 4 shipments. $W \sim \mathrm{B}(4, 0.12215)$ Probability required $= \mathrm{P}(W \le 2) \approx 0.99338 \approx 0.993$	