Worksheet 7A   Direct proportion

1. The height of a stack of 30 identical textbooks is 48 cm.
   Find the height of
   (i) one such textbook,
   (ii) 35 textbooks.

2. A chef makes 15 dumplings in two minutes.
   Find the number of dumplings he makes in 20 minutes.

3. 100 g of curry powder costs $4.
   Find the cost of 0.24 kg of curry powder.

4. A law firm charges $C for x hours of consultation.
   Find an expression, in terms of C, x and y, for the amount it charges for y hours of consultation.

5. A shop sells fish food by the kilogram.
   5 kg of fish food costs $53.50.
   Sherrie has two $50-notes.
   Determine how much change Sherrie will receive if she buys 8 kg of fish food.

6. The fuel capacity of a van is 65 litres.
   A van travels at a constant speed along an expressway.
   Its fuel consumption is directly proportional to the distance travelled.
   The van uses 1 litre of fuel to travel 15 km.
   Calculate the maximum distance the van can travel on a half-full tank of fuel.

7. This is a nutrition label found on a packet of cheese snacks.

NUTRITION INFORMATION

Serving size: 15 g
Servings per pack: 4

	Per 100 g	Per serving
Energy	567 kcal	85 kcal
Carbohydrate	~~~~	7.9 g
Protein	~~~~	1.0 g
Fat	37.4 g	~~~~
Sodium	1083 mg	~~~~

Part of the label has been scratched off.
Determine the values corresponding to the components that are not shown.

8. A car rental company charges $84 per day or $550 per week for renting a 5-seater car.
Joseph wants to rent a car for 10 days.
Calculate the minimum amount of rental he has to pay.

9. On average, Alicia can type 24 words per minute.
An author gives Alicia a 12 000-word manuscript to type.
He pays her $6 for every 360 words she types.
(i) Can Alicia finish typing the manuscript in 1 hour?
(ii) Find out how much the author pays Alicia to type the full manuscript.

Worksheet 7B   Algebraic and graphical representations of direct proportion

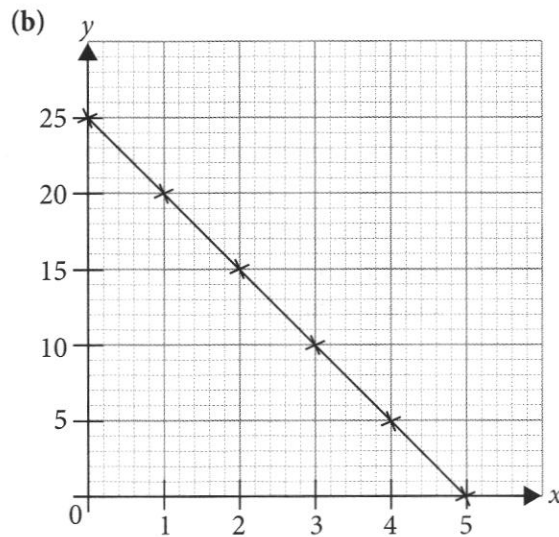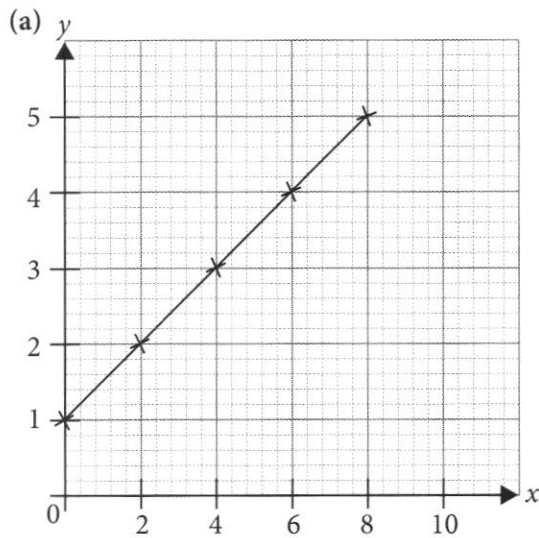1. Determine whether each set of values shows x and y in direct proportion.

    (a)

x	2	4	7
y	18	36	54

    (b)

x	3	5	11
y	5.1	8.5	18.7

2. State whether each of the following graphs shows x and y in direct proportion. Explain your answer.

    (a)

    (b)

3. The tables show some values of x and the corresponding values of y.
   Show that the variables x and y are in direct proportion.

   (a)

x	8	9	12
y	56	63	84

   (b)

x	0.3	2.5	3.9
y	5.4	45	70.2

4. In each of the following, state whether the variables x and y are directly proportional to each other.
   (a) $y = 5x$
   (b) $y = x - 1$

   (c) $y = 2x^3$
   (d) $y = \dfrac{1}{4}x$

5. Given that y is directly proportional to x, find the values of p and q.

x	5	8	q
y	4	p	9.6

6. It is given that y is directly proportional to x and $y = 14$ when $x = 2$.
   Find
   (a) an equation connecting x and y,
   (b) the value of y when $x = 3$,
   (c) the value of x when $y = 20$.

7. y is proportional to x and $y = 8$ when $x = 12$.
   Find y when $x = 4$.

Worksheet 7C   Other forms of direct proportion

1. Determine whether this set of values shows x^3 and y in direct proportion.

x	2	4	6
y	2	16	48

2. The table shows some values of x and the corresponding values of y.
   Show that the variables x and y^2 are in direct proportion.

x	3	12	27
y	3	6	9

3. In each of the following, state the two variables which are directly proportional to each other.
   (a) $y = 4x^3$ (b) $6y = x$

   (c) $y^3 = \sqrt{x}$ (d) $\sqrt[3]{y} = \dfrac{2}{x}$

4. Given that y is directly proportional to x^2, find the values of a and b.

x	4	10	b
y	a	50	128

5. The variables x and y are connected by the equation $y = k\sqrt{x+1}$, where k is a constant.

x	3	15	q
y	8	p	20

   Find the values of k, p and q.

6. y is directly proportional to x^3.
    Given that $y = 64$ when $x = 8$, find
    (a) an expression for y in terms of x,
    (b) the value of y when $x = \dfrac{1}{2}$.

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7. y is directly proportional to $\sqrt{x}$.
    When $x = 9$, $y = 5$.
    Find the value of y when $x = 36$.

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8. It is given that y is directly proportional to the cube root of x.
    When $x = 27$, $y = 9$.
    (a) Find
        (i) an equation connecting x and y,
        (ii) the value of y when $x = 1000$,
        (iii) the value of x when $y = 2$.
    (b) Sketch the graph of y against the cube root of x.

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9. It is given that y is directly proportional to $(2x + 1)^3$.
    When $x = 3$, $y = 49$.
    (a) Find
        (i) an equation connecting x and y,
        (ii) the value of y when $x = \dfrac{1}{2}$,
        (iii) the value of x when $y = 16\,807$.
    (b) Sketch the graph of y against $(2x + 1)^3$.

10. y^3 is directly proportional to x^2.
    When $x = 4$, $y = 2$.
    Find the values of x when $y = 8$.

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11. y is directly proportional to x^2.
    When x has a certain value, $y = 7$.
    Find the value of y when x is doubled.

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12. It is given that y is directly proportional to the square of x.
    The difference between the values of y when $x = 3$ and $x = 6$ is 9.
    Find the value of y when $x = 9$.

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13. The mass, m grams, of a ball bearing is directly proportional to the cube of the radius, r cm.
    A ball bearing with a radius of 0.7 cm has a mass of 10.78 g.
    Find
    (a) an equation connecting m and r,
    (b) the mass of a ball bearing with a radius of 0.8 cm.

14. A particle, initially at rest, is released from a height 80 m above the ground.
    The distance travelled, s metres, is directly proportional to the square of the travelling time, t seconds.
    In the first 3 seconds, the particle travels 45 m.
    (a) Find the equation for s in terms of t.
    (b) Find the value of s when $t = 4$ and explain the significance of the value in the context of the question.

15. The period, T seconds, of a pendulum is directly proportional to the square root of the length, L metres, of
    the pendulum.
    When the pendulum is 0.36 m long, the period is 1.2 s.
    (a) Find an equation connecting T and L.
    (b) Find the length of the pendulum when the period is 1.1 s.
    (c) For two identical pendulums, the ratio of the periods is $3 : 2$.
        Find the ratio of the lengths of the pendulums.

CHALLENGE MYSELF!

16. Given the equation $y = x^2 + 1$, which of the following statements are true? Explain your answer.
    (a) y is directly proportional to x^2.
    (b) x^2 is directly proportional to $y - 1$.
    (c) y is linearly related to x.
    (d) x is linearly related to $\sqrt{y}$.

Worksheet 7D   Inverse proportion

1. Five robots take 3 hours to mow a lawn.
    How many robots are needed to mow the lawn in half the time?

2. It takes 20 workers 4 days to harvest a tomato plantation.
    Given that the workers harvest at the same rate, how long will it take 25 workers to harvest the same plantation?

3. The time taken to paint a wall mural is inversely proportional to the number of painters.
    Four painters take 3.5 hours.
    Find the time it would have taken seven painters to paint the mural.

4. A 3D-printer prints a smartphone case in 20 minutes.
    (i)   How long does it take to print 50 smartphone cases?
    Eight 3D-printers are deployed to print the 50 smartphone cases.
    (ii) Calculate the number of hours and minutes needed.

5. Eight identical pipes can empty a full tank in half an hour.
    Two of the pipes are choked and cannot be used.
    The tank is to be emptied in 40 minutes.
    Determine whether the remaining pipes can empty the tank in 40 minutes.

6. Three seamstresses can sew 6 blouses in 2 hours.
Given that all the seamstresses sew at the same rate, find the time it will take two of them to sew 15 blouses.

7. Twelve workers each working 6 hours a day can fix the electrical wiring in a building in 10 days.
   (i) If the job is to be completed in 8 days instead, find the additional number of workers needed for the job.
   (ii) State the assumption you have made in part (i).

CHALLENGE MYSELF!

8. Three landscape artists agree to revamp a pavilion from 12.30 p.m. to 7 p.m., with a half hour break in between.
Each landscape artist charges $45 for each hour of work, excluding the break, from 12.30 p.m. to 5 p.m.
After 5 p.m., each landscape artist charges $30 for each half hour of work.
At 2.30 p.m., one landscape artist is injured and stops work, leaving the other two landscape artists to complete the project.
Estimate the amount of money the owner of the pavilion has to pay, giving your answer to a sensible degree of accuracy.
State the assumption you have made.

Worksheet 7E   Algebraic and graphical representations of inverse proportion

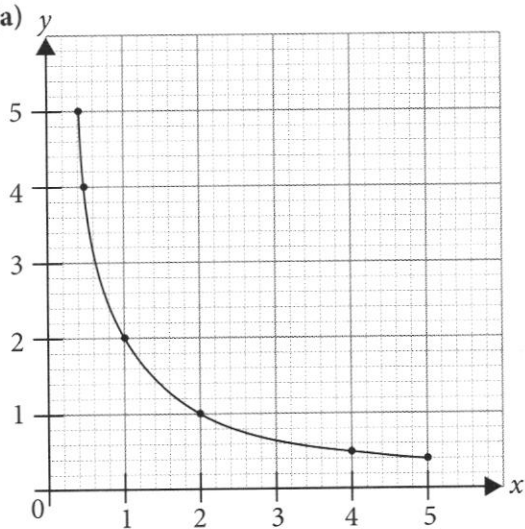1. Determine whether each set of values shows x and y in inverse proportion.

(a)

x	5	10	15
y	18	9	6

(b)

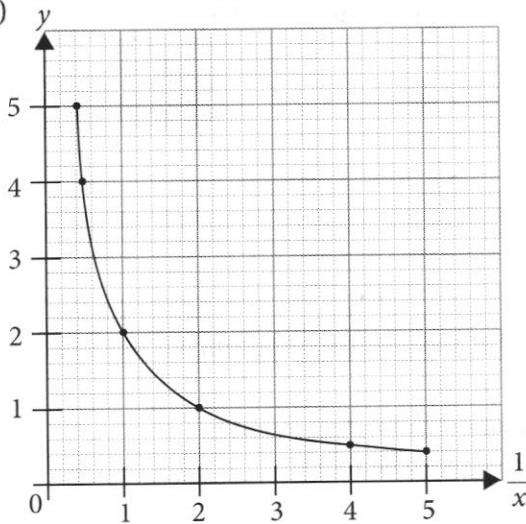| x	6	9	12
y	1.2	0.9	0.6

2. State whether each of the following graphs shows x and y in inverse proportion.

(a)

(b)

3. The tables show some values of x and the corresponding values of y.
    Show that the variables x and y are in inverse proportion.

    (a)

x	2	4	8
y	12	6	3

    (b)

x	5	8	11
y	$7\frac{1}{5}$	$4\frac{1}{2}$	$3\frac{3}{11}$

4. In each of the following, state whether the variables x and y are inversely proportional to each other.

    (a) $y = \dfrac{5}{x}$

    (b) $y = \dfrac{5}{x} + 1$

    (c) $xy = 2$

    (d) $\dfrac{y}{x} = \pi$

5. Given that y is inversely proportional to x, find the values of p and q.

x	3	4.8	q
y	8	p	1.5

6. It is given that y is inversely proportional to x and $y = 5$ when $x = 10$.
    Find
    (a) an equation connecting x and y,
    (b) the value of y when $x = 4$,
    (c) the value of x when $y = 20$.

7. y is inversely proportional to x and $y = 8$ when $x = 12$.
    Find y when $x = 4$.

8. It is given that p is inversely proportional to q and $p = \frac{1}{2}$ when $q = \frac{3}{4}$.
   (a) Find
      (i) the value of p when $q = \frac{5}{6}$,
      (ii) the value of q when $p = \frac{7}{8}$.
   (b) Sketch the graph of p against q.

9. y is inversely proportional to x.
   When x has a certain value, $y = \frac{4}{5}$.
   Find the value of y when x is tripled.

10. A sack of corn can feed N chickens for x days such that N is inversely proportional to x.
    When $N = 50$, $x = 12$.
    (a) Find an equation connecting N and x.
    (b) Find the number of days a sack of corn can feed 120 chickens.
    (c) A sack of corn lasts for a week. Find the number of chickens on the farm.

11. The volume, V m^3, of a fixed mass of gas is inversely proportional to the pressure, P pascals.
    When $V = 2.5$, $P = 240$.
    (a) Find the volume when the pressure is 180 pascals.
    (b) Find the pressure when the volume is 3.5 m^3.

12. The work done, W joules, on an object is equal to the product of the force, F newtons, applied to it and the distance, d m, that the object moves in the direction of the force.

The work done is to be kept constant.

When $d = 1.8$, $F = 6$.

Find

(a) an equation connecting d and F,

(b) the force needed to be applied for the object to move a distance of 3.2 m,

(c) the distance moved by the object when a force of 5 newtons is applied to it.

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13. The time, T hours, taken by x workers to build 20 identical stands at a convention centre is recorded in the table.

Number of workers, x	2	5	10
Time taken, T h	3.75	1.5	0.75

(a) Show that T is inversely proportional to x.

(b) Hence, write down an equation connecting T and x.

(c) Calculate the amount of time taken by 6 workers to build 20 stands.

(d) Hence, state the amount of time taken by 6 workers to build 500 stands.

CHALLENGE MYSELF!

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14. $340 is paid for a packing service which Patricia can complete in 3 hours, Queenie in 8 hours and Rosa in 4 hours.

Given that all three ladies work together, how much money should each of them receive?

Worksheet 7F   Other forms of inverse proportion

1. Determine whether this set of values shows x and y^2 in inverse proportion.

x	4	25	50
y	5	2	1

2. The table shows some values of x and the corresponding values of y.

   Show that the variables $\sqrt{x}$ and y are in inverse proportion.

x	1	16	49
y	4	1	$\dfrac{4}{7}$

3. In each of the following, state the two variables which are in inverse proportion.

   (a) $y = \dfrac{5}{x^3}$

   (b) $y = \dfrac{1}{4\sqrt{x}}$

   (c) $y^3 = \dfrac{7}{x}$

   (d) $\sqrt[3]{y} = \pi x$

4. Given that y is inversely proportional to $\sqrt[3]{x}$, find the values of a and b.

x	27	64	b
y	1.5	a	0.45

5. The variables x and y are connected by the equation $y = \dfrac{k}{(2x-1)^2}$, where k is a constant.

x	2	3	q
y	$\dfrac{2}{3}$	p	600

   Find the values of k, p and q.

6. y is inversely proportional to the cube of x.
    Given that $y = 5$ when $x = 2$, find y when $x = 3$.

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7. y is inversely proportional to x^2.
    When $x = 3$, $y = 9$.
    (a) Find an equation connecting x and y.
    (b) Find the value of y when $x = 6$.
    (c) Find the values of x when $y = 100$.

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8. It is given that y is inversely proportional to $\sqrt{x}$.
    When $x = 64$, $y = \dfrac{1}{4}$.
    (a) Find
        (i)   an equation connecting x and y,
        (ii) the value of y when $x = 256$,
        (iii) the value of x when $y = 9$.
    (b) Sketch the graph of y against $\dfrac{1}{\sqrt{x}}$.

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9. It is given that p^2 is inversely proportional to $q + 1$.
    When $q = 8$, $p = 2$.
    (a) Find
        (i)   an equation connecting p and q,
        (ii) the values of p when $q = 3$,
        (iii) the value of q when $p = \dfrac{1}{5}$.
    (b) Sketch the graph of p^2 against $q + 1$.