1 Question A

12 In this question you should state clearly the values of the parameters of any normal distribution you use.

Over a one-month period, Mark makes X minutes of telephone calls and uses Y gigabytes of data on his mobile phone. X and Y are independent random variables with the distribution $X \sim \mathrm{N}(\mu, 16^2)$ and $Y \sim \mathrm{N}(1, 0.8^2)$ respectively.

(i) Find the probability that over a one-month period, Mark uses less than 2 gigabytes of data. [1]

(ii) There is a 25% chance that Mark makes more than 110.8 minutes of telephone call in a month. Show that $\mu = 100.0$, correct to 1 decimal place. [2]

(iii) Find the probability that, over three one-month periods, the total number of minutes of telephone calls made by Mark in the first two months is more than thrice the number of minutes made by him in the third month. [3]

State an assumption needed for the calculation above to be valid. [1]

Mark opted for a Pay As You Use plan. For this payment plan, there is no monthly subscription. Mobile phone's calls cost \$0.05 per minute and data costs \$0.02 per megabyte.

[One gigabyte is equal to 1000 megabytes]

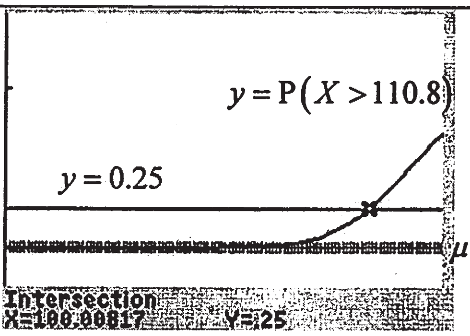
(iv) Find the probability that, over a one-month period, the total cost of Mark's mobile phone bill exceeds \$30. [3]

(v) Explain, without evaluating, whether your answer in **(iv)** is greater than or less than the probability that, over a one-month period, the cost of Mark's telephone calls exceeds \$5 and the cost of his data use exceeds \$25. [2]

1.1 Answer A

12i	$\text{P}(Y<2)=0.894$	
ii	$\text{P}(X>110.8)=0.25$ Method 1: $\text{P}\left(Z>\dfrac{110.8-\mu}{16}\right)=0.25$ $\dfrac{110.8-\mu}{16}=0.67449$ $\mu=100.008$ $\approx 100.0(1\text{dp})$ Method 2: By GC, Plot Y_1 = normalcdf(110.8,E99.X,16) $Y_2=0.25$	For show questions, we show the rounding off to the required degree of accuracy.

1.2 Answer A

Qn	Solution	Comments
	 $\mu = 100.008$ $\approx 100.0\,(1\text{dp})$	
iii	$\mathrm{P}(X_1 + X_2 > 3X_3) = \mathrm{P}(X_1 + X_2 - 3X_3 > 0)$ $\mathrm{E}(X_1 + X_2 - 3X_3) = 100.0 + 100.0 - 300.0 = -100.0$ $\mathrm{Var}(X_1 + X_2 - 3X_3) = 16^2 + 16^2 + (-3)^2 16^2 = 2816$ $\therefore X_1 + X_2 - 3X_3 \sim \mathrm{N}(-100.0, 2816)$ $\mathrm{P}(X_1 + X_2 - 3X_3 > 0) = 0.0298 \quad (\text{or } 0.0297)$ The number of minutes of telephone calls made during a one-month period is independent of that during other one-month periods.	The first month is independent of the second month, so we do not say $2X > 3X$ Assumptions should be given in the context of the question.
Iv	Let $\$T$ be the cost of Mark's mobile phone bill $T = 0.05X + 0.02(1000)Y = 0.05X + 20Y$ $\mathrm{E}(T) = 0.05(100.0) + 20(1) = 25$ $\mathrm{Var}(T) = 0.05^2(16^2) + 20^2(0.8^2) = 256.64$ $\therefore T \sim \mathrm{N}(25, 256.64)$ $\mathrm{P}(T > 30) = 0.377$	Note the difference in units between megabytes and gigabytes
v	The answer in part (iv) is greater because the event "cost of Marcus' telephone calls exceeds \$5 and the cost of his data use exceeds \$25" is a subset of the event "total cost of Marcus's mobile phone bill exceeds \$30".	

2 Question B,C

10 The masses, in kilograms, of rock melons and honeydew melons sold at a market are normally distributed. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are given in the following table.

	Mean Mass (kg)	Standard Deviation (kg)	Selling Price (\$ per kg)
Rock melon	2.50	0.20	1.80
Honeydew melon	2.20	0.05	1.50

(i) 1 rock melon and 1 honeydew melon are randomly chosen. Find the probability that the mass of the rock melon is more than that of the honeydew melon by less than 0.4 kg. [3]

(ii) 2 rock melons and 3 honeydew melons are randomly chosen. Find the probability that the selling price of the melons is at most \$20. [4]

Due to climate change, the sizes of rock melons change such that among a new batch of rock melons, a randomly chosen rock melon now has mass that is normally distributed with mean μ kg and standard deviation σ kg. 10% of the new batch of rock melons have masses more than 2.2 kg and at least 25% of the new batch of rock melons have masses within 0.1 kg of its mean.

(iii) Find the maximum value of σ and the corresponding value of μ, correct to 2 decimal places. [5]

11 A new radio station broadcasts only popular hit songs from the 1980s and 1990s. Tuck Wah, who is very familiar with songs from that era, is able to correctly identify 80% of the songs aired by the station.

(i) Find the smallest value of n such that, out of n songs aired by the station, the probability that Tuck Wah is unable to correctly identify at most one song is less than 50%. [3]

The radio station has a regular feature – an uninterrupted sequence of 9 randomly-selected songs aired without advertisement breaks. 4 such sequences are played daily.

(ii) Find the probability that Tuck Wah is able to correctly identify at least 5 out of 9 songs in a randomly selected sequence. [2]

(iii) Find the probability that Tuck Wah is unable to correctly identify any two consecutive songs in a randomly selected sequence, given that he is able to correctly identify at least 5 out of 9 songs in that sequence. [3]

(iv) Find the probability that in a month of 30 days, Tuck Wah is able to correctly identify an average of at least 7 songs in a sequence. [5]

2.1 Answer B,C

10 The masses, in kilograms, of rock melons and honeydew melons sold at a market are normally distributed. The means and standard deviations of these distributions, and the selling prices, in $ per kilogram, are given in the following table.

	Mean Mass (kg)	Standard Deviation (kg)	Selling Price ($ per kg)
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(i) 1 rock melon and 1 honeydew melon are randomly chosen. Find the probability that the mass of the rock melon is more than that of the honeydew melon by less than 0.4 kg. [3]

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Due to climate change, the sizes of rock melons change such that among a new batch of rock melons, a randomly chosen rock melon now has mass that is normally distributed with mean μ kg and standard deviation σ kg. 10% of the new batch of rock melons have masses more than 2.2 kg and at least 25% of the new batch of rock melons have masses within 0.1 kg of its mean.

(iii) Find the maximum value of σ and the corresponding value of μ, correct to 2 decimal places. [5]

[Solution]

Let R be the mass of a randomly chosen rock melon, $R \sim \mathrm{N}\left(2.5, 0.2^2\right)$

Let H be the mass of a randomly chosen honeydew melon, $H \sim \mathrm{N}\left(2.2, 0.05^2\right)$

(i) $R - H \sim \mathrm{N}\left(2.5 - 2.2\ ,\ 0.2^2 + 0.05^2\right)$

$R - H \sim \mathrm{N}\left(0.3, \sqrt{0.0425}^2\right)$

$\mathrm{P}\left(0 < R - H < 0.4\right) = 0.61338 \approx 0.613$

(ii) Let C be the selling price of the fruits. $C = 1.8\left(R_1 + R_2\right) + 1.5\left(H_1 + H_2 + H_3\right)$

$C \sim \mathrm{N}\left(1.8(2.5 \times 2) + 1.5(2.2 \times 3)\ ,\ 1.8^2(0.2^2 \times 2) + 1.5^2(0.05^2 \times 3)\right)$

$C \sim \mathrm{N}\left(18.9, \sqrt{0.276075}^2\right)$

$\mathrm{P}\left(C \le 20\right) = 0.98185 \approx 0.982$

2.2 Answer B,C

(iii) Let W be the mass of a randomly chosen rock melon from the new batch.

$W \sim \mathrm{N}\left(\mu, \sigma^2\right)$

$\mathrm{P}(W > 2.2) = 0.1$

$\mathrm{P}\left(Z > \dfrac{2.2-\mu}{\sigma}\right) = 0.1$ standardize

From GC, $\dfrac{2.2-\mu}{\sigma} = 1.28155$ ----- (1)

$\mathrm{P}(\mu - 0.1 < W < \mu + 0.1) \geq 0.25$

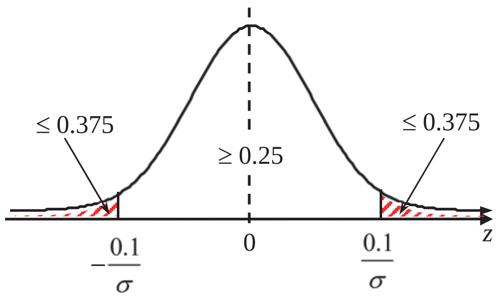
$\mathrm{P}\left(-\dfrac{0.1}{\sigma} < Z < \dfrac{0.1}{\sigma}\right) \geq 0.25$ standardize

From GC, $\dfrac{0.1}{\sigma} \geq 0.31864$

$\sigma \leq 0.31383$

Maximum $\sigma \approx 0.31$ (2 d.p.)

Sub in (1), $\mu \approx 1.80$ (2 d.p.)



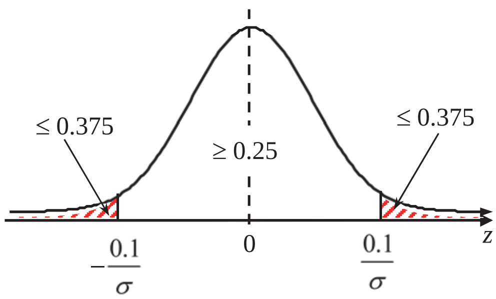
<u>Old GC:</u>

$\mathrm{P}\left(-\dfrac{0.1}{\sigma} < Z < \dfrac{0.1}{\sigma}\right) \geq 0.25$

$\mathrm{P}\left(Z < -\dfrac{0.1}{\sigma}\right) \leq 0.375$

$-\dfrac{0.1}{\sigma} \leq -0.31864$

$\sigma \leq 0.31383$



<u>Comments</u>

(i) Generally, most students did not take into account that since $R > H$, we have $R - H > 0$. They just worked with $\mathrm{P}(R - H < 0.4)$ instead of $\mathrm{P}(0 < R - H < 0.4)$.

2.3 Answer B,C

(ii) Common mistakes made include

- $2R$ & $3H$ instead of $R_1 + R_2$ & $H_1 + H_2 + H_3$ resulting in wrong variances
- Wrongly calculating Var (1.8 (R_1 + R_2)) = 1.8 × 2 Var (R) instead of $1.8^2 \times 2$ Var (R)

(iii) Students need to take care in interpreting word problems into mathematical expressions, e.g.

- "within" : wrongly writing $\mathrm{P}(\mu - 0.1 \le W \le \mu + 0.1)$ instead of $\mathrm{P}(\mu - 0.1 < W < \mu + 0.1)$. While this does not affect the answer, it would for problems involving binomial distribution.
- "at least" : wrongly writing $\mathrm{P}(\mu - 0.1 < W < \mu + 0.1) > 0.25$ or $= 0.25$ instead of $\mathrm{P}(\mu - 0.1 < W < \mu + 0.1) \ge 0.25$

Some students were careless or became confused when they tried to do standardization.

Many students did not know how to proceed after standardization to $\mathrm{P}\left(-\frac{0.1}{\sigma} < Z < \frac{0.1}{\sigma}\right) \ge 0.25$ to obtain σ or wrongly concluding that $\mathrm{P}\left(Z < \frac{0.1}{\sigma}\right) \ge 0.25 \div 2$

Quite a number of students did not read question carefully to **leave their answers in two decimal places**

2.4 Answer B,C

11 A new radio station broadcasts only popular hit songs from the 1980s and 1990s. Tuck Wah, who is very familiar with songs from that era, is able to correctly identify 80% of the songs aired by the station.

(i) Find the smallest value of n such that, out of n songs aired by the station, the probability that Tuck Wah is unable to correctly identify at most one song is less than 50%. [3]

The radio station has a regular feature – an uninterrupted sequence of 9 randomly-selected songs aired without advertisement breaks. 4 such sequences are played daily.

(ii) Find the probability that Tuck Wah is able to correctly identify at least 5 out of 9 songs in a randomly selected sequence. [2]

(iii) Find the probability that Tuck Wah is unable to correctly identify any two consecutive songs in a randomly selected sequence, given that he is able to correctly identify at least 5 out of 9 songs in that sequence. [3]

(iv) Find the probability that in a month of 30 days, Tuck Wah is able to correctly identify an average of at least 7 songs in a sequence. [5]

[Solution]

(i) Let X be the no. of songs Tuck Wah is unable to correctly identify in a sequence of n songs.

$X \sim \mathrm{B}(n, 0.2)$

$\mathrm{P}(X \leq 1) < 0.5$

From GC

n	$\mathrm{P}(X \leq 1)$
8	$0.50332 > 0.5$
9	$0.43621 < 0.5$

Show the table clearly

Least $n = 9$

<u>Alternative</u>

$X' \sim \mathrm{B}(n, 0.8)$

$\mathrm{P}(X' \geq n-1) < 0.5$

$1 - \mathrm{P}(X' \leq n-2) < 0.5$

$\mathrm{P}(X' \leq n-2) > 0.5$

From GC

X	X'
0	n
1	$n-1$

n	$\mathrm{P}(X' \leq n-2)$
8	$0.49668 < 0.5$
9	$0.56379 > 0.5$

Least $n = 9$

2.5 Answer B,C

(ii) Let Y be the number of songs correctly identified by Tuck Wah in a sequence of 9 songs.

$Y \sim B(9, 0.8)$

$P(Y \geq 5) = 1 - P(Y \leq 4)$

$= 0.98042$ (5s.f.) $= 0.980$ (3 s.f.)

(iii) P(Tuck Wah is unable to correctly identify consecutive songs $\mid Y \geq 5$)

$= \dfrac{\text{P(Tuck Wah correctly identifies the 1st, 3rd, 5th, 7th and 9th songs)}}{P(Y \geq 5)}$

$= \dfrac{(0.8)^5(0.2)^4}{0.98042}$

$= 0.000535$ (3sf)

(iv) $E(Y) = 9(0.8) = 7.2$; $\quad \text{Var}(Y) = 9(0.8)(0.2) = 1.44$

$\overline{Y} = \dfrac{Y_1 + Y_2 + \ldots\ldots + Y_{120}}{120}$ Note that 120 is large

Since $n = 120$ is large, by the Central Limit Theorem,

$\overline{Y} \sim N\left(7.2, \dfrac{1.44}{120}\right)$ approximately

$P(\overline{Y} \geq 7) = 0.966$ (3sf)

<u>Alternative solution</u>

Let A be the number of songs that Tuck Wah is able to identify correctly out of 1080. $30 \times 4 \times 9 = 1080$

$A \sim B(1080, 0.8)$

$P(Y \geq 840) = 1 - P(Y \leq 839) = 0.968$

Show working clearly $7 \times 4 \times 30 = 840$

Comments:

(i) Students who use the complement method mostly get the inequality wrong. Some students did not show the table clearly.

(ii) This is well done.

(iii) Students generally knew that it was conditional probability, but they were not able to interpret the numerator correctly.

(iv) Many students could not understand the requirements of the question. Some wrote $A \sim B(120, 0.73819)$. Note that in this part, we are not interested in the number of sequences, but the number of songs.

3 Question D

8 A manufacturer wishes to find out the reasons for consumer complaints for his product. He conducted an analysis of 500 consumer complaints. The table below shows the results of the analysis in which the reason for complaints and the period of the complaint was lodged were recorded.

Reason for complaints	Appearance	Mechanical	Electrical
During warranty period	108	59	68
After warranty period	46	109	110

(a) One complaint is selected at random.

A, M and E are the events that the complaint selected is regarding appearance, mechanical and electrical respectively.

W is the event that the complaint selected occurs during warranty period.

(i) Calculate $\mathrm{P}(M' \cap W)$. [1]

(ii) Calculate $\mathrm{P}(A|W)$ and explain the meaning of this probability in the context of the question. [3]

(iii) Determine whether A and W are independent. Justify your conclusion. [2]

(b) Ten complaints are selected at random, one by one and with replacement. Find the largest value of n such that the probability that at most n complaints are regarding a product under warranty is less than 0.3. [2]

4 Question E,F,G

9 Two independent events A and B are such that $P(A|B) = P(B|A)$ and $P(A \cup B) = \frac{5}{9}$, find

(i) $P(A' \cap B')$, [1]

(ii) $P(A \cap B)$. [4]

10 A board of directors consists n men and $(10-n)$ women, where $3 \leq n \leq 7$. A committee consisting 3 randomly chosen people is to be formed. Let W be the number of women in the committee.

(i) Show that $P(W=1) = \dfrac{n(n-1)(10-n)}{240}$ and find, in terms of n, the probability distribution of W. [4]

(ii) Given $E(W) = 2.1$, find n. [2]

(iii) Hence, find Var(W). [1]

11 A shoe company manufactures a large number of sneakers every day. A small proportion, p, of these sneakers is defective. A check is carried out each day by taking a random sample of 15 sneakers and examining them for defects.

(i) State, in context, two assumptions needed for the number of defective sneakers in the sample to be well modelled by a binomial distribution. [2]

For the rest of this question, assume that the binomial law holds.

(ii) The probability that at least 14 of the sneakers in the sample are non-defective is 0.85553. Show that the probability that at most 2 sneakers in the sample are defective is 0.972, correct to 3 decimal places. [3]

If exactly 3 sneakers are defective, a further sample of 5 sneakers is taken. A day's production is accepted as satisfactory in either one of the following cases:

- The number of defective sneakers in the sample of 15 is at most 2;
- The number of defective sneakers in the sample of 15 is 3, **and** there is no defective sneaker in the sample of 5.

(iii) Find the probability that the day's production is accepted as satisfactory. [2]

Subsequently, the manufacturer realises a fault in the check for defective sneakers. If a sneaker is defective, there is a 90% chance that the check correctly identifies it as defective. If the sneaker is not defective, there is a 5% chance that the check incorrectly identifies it as defective.

(iv) Find the probability that a sneaker identified as not defective. [2]

4.1 Answer D

8ai	$\mathrm{P}(M' \cap W) = \dfrac{108+68}{500} = \dfrac{44}{125}$ (or 0.352)	$\mathrm{P}(M' \cap W) \neq \mathrm{P}(M') \times \mathrm{P}(W)$ If LHS = RHS, the events M' and W need to be independent.
ii	$\mathrm{P}(A \mid W) = \dfrac{\mathrm{n}(A \cap W)}{\mathrm{n}(W)}$ $= \dfrac{108}{108+59+68}$ $= \dfrac{108}{235} (\approx 0.460)$ It is the probability that a complaint of a product under warranty is regarding appearance.	
iii	Since $\mathrm{P}(A) = \dfrac{108+46}{500} = \dfrac{77}{250} (= 0.308) \neq \mathrm{P}(A \mid W)$, A and W are not independent. OR	

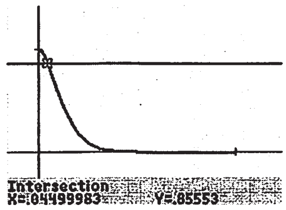
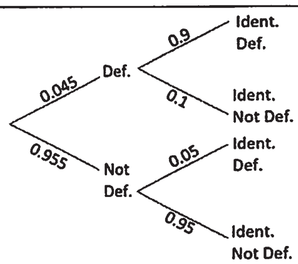
4.2 Answer D,E,F

Qn	Solution	Comments
	$\mathrm{P}(A)=\frac{108+46}{500}=\frac{77}{250}$, $\mathrm{P}(W)=\frac{108+59+68}{500}=\frac{47}{100}$, $\mathrm{P}(A\cap W)=\frac{108}{500}=\frac{27}{125}$ $\mathrm{P}(A)\mathrm{P}(W)=0.14476\neq \mathrm{P}(A\cap W)$ Hence, A and W are not independent.	
b	Let X be the number of complaints that are regarding a product under warranty, out of 10. $X\sim B(10,0.47)$ $\mathrm{P}(X\le n)<0.3$ By GC, n : $\mathrm{P}(X\le n)$ 2 : 0.07915 3 : 0.2255 4 : 0.45263 Largest value of $n=3$.	"at most n" means less than or equal to n.
9i	$\mathrm{P}(A'\cap B')=1-\frac{5}{9}=\frac{4}{9}$	
ii	$\mathrm{P}(A\mid B)=\mathrm{P}(B\mid A)$ $\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)}$ $\mathrm{P}(A)=\mathrm{P}(B)$ $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $\frac{5}{9}=\mathrm{P}(A)+\mathrm{P}(A)-\mathrm{P}(A)\mathrm{P}(A)$ $9[\mathrm{P}(A)]^2-18\mathrm{P}(A)+5=0$ $\mathrm{P}(A)=\frac{1}{3}$ or $\frac{5}{3}$(Reject $\because 0\le \mathrm{P}(A)\le 1$) $\mathrm{P}(A\cap B)=\left(\frac{1}{3}\right)^2=\frac{1}{9}$	
10i	$\mathrm{P}(W=1)=\mathrm{P}(MMW)+\mathrm{P}(MWM)+\mathrm{P}(WMM)$ $=\frac{10-n}{10}\times\frac{n}{9}\times\frac{n-1}{8}\times 3$ $=\frac{n(n-1)(10-n)}{240}$ Alternative Method:	This is not a binomial distribution. The probability of choosing a woman to be in the committee is not constant across the 3 selections.

4.3 Answer F,G

Qn	Solution	Comments
	$\mathrm{P}(W=1)=\dfrac{{}^{n}C_2\times{}^{10-n}C_1}{{}^{10}C_3}=\dfrac{\frac{n(n-1)}{2!}\times\frac{10-n}{1!}}{120}$ $=\dfrac{n(n-1)(10-n)}{240}$ $\begin{array}{\|c\|l\|}\hline w & \mathrm{P}(W=w) \\ \hline 0 & \dfrac{n(n-1)(n-2)}{720}\ \left(\text{or } \dfrac{{}^{n}C_3}{120}\right) \\ \hline 1 & \dfrac{n(n-1)(10-n)}{240} \\ \hline 2 & \dfrac{n(10-n)(9-n)}{240}\ \left(\text{or } \dfrac{n\left({}^{10-n}C_2\right)}{120}\right) \\ \hline 3 & \dfrac{(10-n)(9-n)(8-n)}{720}\ \left(\text{or } \dfrac{{}^{10-n}C_3}{120}\right) \\ \hline \end{array}$	
ii	$\mathrm{E}(W)=\dfrac{n(n-1)(10-n)}{240}+2\left(\dfrac{n(10-n)(9-n)}{240}\right)$ $+3\left(\dfrac{(10-n)(9-n)(8-n)}{720}\right)$ $2.1=\dfrac{10-n}{720}\left(3n(n-1)+6n(9-n)+3(9-n)(8-n)\right)$ $=\dfrac{10-n}{240}\left(n(n-1)+2n(9-n)+(9-n)(8-n)\right)$ $=\dfrac{10-n}{240}\left(n^2-n+18n-2n^2+72-17n+n^2\right)$ $=0.3(10-n)$ $0.3n=0.9$ $n=3$ Or By GC, $n=3$	
iii	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline w & 0 & 1 & 2 & 3 \\ \hline \mathrm{P}(W=w) & \dfrac{1}{120} & \dfrac{7}{40} & \dfrac{21}{40} & \dfrac{7}{24} \\ \hline \end{array}$ By GC, $\mathrm{Var}(W)=0.49$	
11i	1. The probability of a defective sneaker remains constant throughout the sample of 15. 2. Defective sneakers occur independently of each other.	There is no such thing as independent probabilities. It is incorrect to say "defective sneakers are independent of each other".

4.4 Answer G

Qn	Solution	Comments
ii	Let X be the number of defective sneakers, out of a sample of 15. $X \sim \mathrm{B}(15, p)$ $X + X' = 15 \Rightarrow X' = 15 - X$ $\mathrm{P}(X' \geq 14) = \mathrm{P}(15 - X \geq 14)$ $0.85553 = \mathrm{P}(X \leq 1)$ Using GC, $p = 0.045000$  $\mathrm{P}(X \leq 2) = 0.97239$ $\approx 0.972 (3\mathrm{dp})$	Read question carefully, "at least 14 sneakers are non-defective." You will need to define your variables clearly. For show questions, we show the rounding off to the required degree of accuracy.
iii	Let Y be the number of defective sneakers, out of a sample of 5. $X \sim \mathrm{B}(5, 0.045000)$ $\mathrm{P}(\text{Satisfactory}) = \mathrm{P}(X \leq 2) + \mathrm{P}(X = 3)\mathrm{P}(Y = 0)$ $= 0.991$	
iv	 $\mathrm{P}(\text{identified not defective}) = 0.0450 \times 0.1 + 0.955 \times 0.95$ $= 0.912$	

5 Question H,I

6 A survey conducted by the Careers Office of Kesamet University revealed that 35% of the graduates from the Class of 2016 found their first job within 6 months of graduation. The remaining graduates found their first job within 15 months of graduation. These graduates found either a full-time job, a part-time job or a temporary job.

Of the graduates who found their first job within 6 months of graduation, 80% found a full-time job, 15% found a part-time job and the rest found a temporary job.

Of the graduates who found their first job after 6 months but within 15 months of graduation, 20% found a full-time job, 40% found a part-time job and the rest found a temporary job.

(i) Show that the probability that a graduate found a part-time job is $\frac{5}{16}$.
Hence find the probability that a graduate who found a part-time job found it after 6 months but within 15 months of graduation. [4]

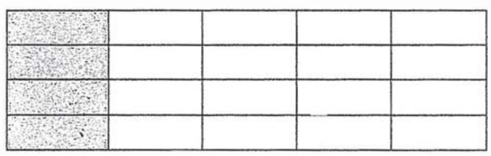
(ii) The events X and Y are defined by

X: A graduate finds a full-time job;

Y: A graduate finds the job within 6 months of graduation.

Determine, showing suitable calculations, whether X and Y are independent. [3]

7 As part of a military exercise, a rectangular piece of land is divided into 4 equal rows and 5 equal columns, forming 20 smaller plots as shown in the diagram below. The shaded plots are designated to be "No Man's Land".



3 identical simulated land mines are placed in 3 distinct plots of land. Find the number of ways this can be done if

(i) at least one mine is placed in "No Man's Land", [2]

(ii) all the mines are placed in the same column or in the same row, [3]

(iii) no two mines are placed in the same column and no two mines are placed in the same row. [3]

6 Question J

8 Robin participates in a game show with 2 preliminary rounds and 1 final round. To proceed to the final round, she needs to obtain a positive total score from the two preliminary rounds.

In the first preliminary round, Robin needs to answer 2 questions, each having 5 possible answers. Robin chooses an answer randomly for each question. For each question she answers correctly, she will score 50 points. However, she will lose any points she has scored if she answers the second question wrongly.

(i) Show that Robin's expected score in the first round is 12. [4]

In the second preliminary round, Robin needs to answer 3 questions, each also having 5 possible answers. Robin chooses an answer randomly for each question. For each question she answers correctly, she will score 12 points. For each question she answers wrongly, she will lose k points.

(ii) Find the largest possible integer value of k if Robin is expected to proceed to the final round. [5]

6.1 Answer H,I,J

Section B: Statistics [60 marks]

6 A survey conducted by the Careers Office of Kesamet University revealed that 35% of the graduates from the Class of 2016 found their first job within 6 months of graduation. The remaining graduates found their first job within 15 months of graduation. These graduates found either a full-time job, a part-time job or a temporary job.

Of the graduates who found their first job within 6 months of graduation, 80% found a full-time job, 15% found a part-time job and the rest found a temporary job.

Of the graduates who found their first job after 6 months but within 15 months of graduation, 20% found a full-time job, 40% found a part-time job and the rest found a temporary job.

(i) Show that the probability that a graduate found a part-time job is $\frac{5}{16}$.

Hence find the probability that a graduate who found a part-time job found it after 6 months but within 15 months of graduation. [4]

(ii) The events X and Y are defined by

X: A graduate finds a full-time job;

Y: A graduate finds the job within 6 months of graduation.

Determine, showing suitable calculations, whether X and Y are independent. [3]

[Solution]

(i) Let ***A* be the event where a graduate finds a part-time job.**

$$\mathrm{P}(A)=\frac{35}{100}\times\frac{15}{100}+\frac{65}{100}\times\frac{40}{100}=\frac{5}{16}$$

Let ***B* be the event where a graduate finds a job after 6 months but within 15 months of graduation**

$$\mathrm{P}(B\mid A)=\frac{\mathrm{P}(B\cap A)}{\mathrm{P}(A)}=\frac{\left(\frac{65}{100}\times\frac{40}{100}\right)}{\frac{5}{16}}=\frac{104}{125}$$

Comments for (i):

Well done for finding P(A). Badly done for finding $\mathrm{P}(B\mid A)$ with many:

- wrong interpretation of "Probability that a graduate **who found a part-time job (this is the "condition") found it after 6 months but within 15 months of graduation**"
- confusion with $\mathrm{P}(B\cap A)$: The question is not asking for probability that a graduate found a part time job after 6 months but within 15 months.

6.2 Answer H,I,J

(ii) $\mathrm{P}(X) = \frac{35}{100} \times \frac{80}{100} + \frac{65}{100} \times \frac{20}{100} = \frac{41}{100}$

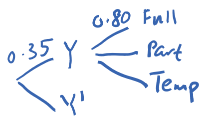
$\mathrm{P}(Y) = \frac{35}{100} = \frac{7}{20}$ **NOTE:** $\mathrm{P}(Y) \neq \frac{35}{100} \times \frac{80}{100}$ (need not be only full time job)

$\mathrm{P}(X \cap Y) = \frac{35}{100} \times \frac{80}{100} = \frac{7}{25}$

$\mathrm{P}(X) \times \mathrm{P}(Y) = \frac{41}{100} \times \frac{7}{20} = \frac{287}{200}$

Since $\mathrm{P}(X \cap Y) \neq \mathrm{P}(X) \times \mathrm{P}(Y)$, the events are not independent

Alternative: $\mathrm{P}(X|Y) = 0.8 \neq 0.41 = \mathrm{P}(X)$



Comments for (ii):

Generally well done except for confusion between mutually exclusive and independent.

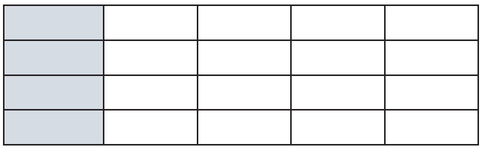
- $\mathrm{P}(X \cap Y) = \mathrm{P}(X) \times \mathrm{P}(Y) \Rightarrow X$ and Y are independent

  (or $\mathrm{P}(X|Y) = \mathrm{P}(X)$, $\mathrm{P}(Y|X) = \mathrm{P}(Y)$)

- $\mathrm{P}(X \cap Y) = 0 \Rightarrow$ X and Y are mutually exclusive (NOTHING to do with independent)
- Students must use X and Y as defined by question. Not allowed to use A and B in part **(ii).**

6.3 Answer H,I,J

7 As part of a military exercise, a rectangular piece of land is divided into 4 equal rows and 5 equal columns, forming 20 smaller plots as shown in the diagram below. The shaded plots are designated to be "No Man's Land".



3 identical simulated land mines are placed in 3 distinct plots of land. Find the number of ways this can be done if

(i) at least one mine is placed in "No Man's Land", [2]

(ii) all the mines are placed in the same column or in the same row, [3]

(iii) no two mines are placed in the same column and no two mines are placed in the same row. [3]

[Solution]

(i) No. of ways

= No. of ways w/o restriction – no. of ways with 0 mines in "No Man's Land"

= ${}^{20}C_3 - {}^{16}C_3$

= 1140 – 560

= 580

Alternative Solution

No. of ways

= ${}^{4}C_3 + {}^{4}C_2 \times {}^{16}C_1 + {}^{4}C_1 \times {}^{16}C_2$

= 4 + 96 + 480

= 580

(ii) No. of ways

= ${}^{5}C_1 \times {}^{4}C_3 + {}^{4}C_1 \times {}^{5}C_3$

= 60

(iii) No. of ways

= No. of ways to permute 3 distinct *x*-coordinates from 1 to 5, and permute 3 distinct *y*-coordinates from 1 to 4

$= \dfrac{{}^{5}P_3 \times {}^{4}P_3}{3!} = 240$

No. of ways

$$= \frac{\overset{1^{st}\text{ mine}}{(5 \times 4)} \times \overset{2^{nd}\text{ mine}}{(4 \times 3)} \times \overset{3^{rd}\text{ mine}}{(3 \times 2)}}{3!}$$

= 240

6.4 Answer H,I,J

Marker's Comments

(i) Students who worked on this part by finding the complementary case of no mine in "No Man's Land" were generally correct. Students who did not arrive at the answer thought that they had to arrange the mines and tried to do permutations instead. Note that it only matters that at least one mine is in "No Man's Land". It does not matter exactly which plots of land they are arranged in.

About half the candidates misinterpreted the question and found the case where exactly one mine is in No Man's Land. Note that **at least one can** be taken to mean one, two or three mines in No Man's Land in this case.

(ii) This part of the question is the best done out of the three parts. Most students were able to identify the two separate cases of either having the mines in the same column or in the same row.

(iii) Most students were unable to correctly solve this part of the question. Many students thought that the question was simply asking for the complement of the case in **(ii)**, which is not true.

Overall

There is a group of students who treated this question as a probability question. Note that when "number of ways", "how many ways" and equivalent phrasings should be taken as an indication that the topic is P & C, NOT PROBABILITY!!!

6.5 Answer H,I,J

8 Robin participates in a game show with 2 preliminary rounds and 1 final round. To proceed to the final round, she needs to obtain a positive total score from the two preliminary rounds.

In the first preliminary round, Robin needs to answer 2 questions, each having 5 possible answers. Robin chooses an answer randomly for each question. For each question she answers correctly, she will score 50 points. However, she will lose any points she has scored if she answers the second question wrongly.

(i) Show that Robin's expected score in the first round is 12. [4]

In the second preliminary round, Robin needs to answer 3 questions, each also having 5 possible answers. Robin chooses an answer randomly for each question. For each question she answers correctly, she will score 12 points. For each question she answers wrongly, she will lose k points.

(ii) Find the largest possible integer value of k if Robin is expected to proceed to the final round. [5]

[Solution]

(i) Let S be Robin's score in the first preliminary round.

s	0	50	100
$\mathrm{P}(S=s)$	$\left(\frac{4}{5}\right)^2+\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)=\frac{4}{5}$	$\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)=\frac{4}{25}$	$\left(\frac{1}{5}\right)^2=\frac{1}{25}$

$$\mathrm{E}(S)=0\left(\frac{4}{5}\right)+50\left(\frac{4}{25}\right)+100\left(\frac{1}{25}\right)$$
$$=12 \quad \text{(shown)}$$

(ii) Let T be Robin's score in the second preliminary round.

t	$-3k$	$12-2k$	$24-k$	36
$\mathrm{P}(T=t)$	$\left(\frac{4}{5}\right)^3=\frac{64}{125}$	$3\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^2=\frac{48}{125}$	$3\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)=\frac{12}{125}$	$\left(\frac{1}{5}\right)^3=\frac{1}{125}$

If Robin is expected to proceed to the next round,

$$\mathrm{E}(S)+\mathrm{E}(T)>0$$
$$12+(-3k)\left(\frac{64}{125}\right)+(12-2k)\left(\frac{48}{125}\right)+(24-k)\left(\frac{12}{125}\right)+36\left(\frac{1}{125}\right)>0$$
$$12-\frac{192k}{125}+\frac{576}{125}-\frac{96k}{125}+\frac{288}{125}-\frac{12k}{125}+\frac{36}{125}>0$$
$$\frac{300k}{125}<\frac{96}{5}$$
$$k<8$$

Largest integer value of k is 7

6.6 Answer H,I,J

Marker's Comments

(i) This part is generally very well done. Students need to be aware of the question tag **"Show"**. In such circumstances, it is important to show multiplication by 0, in this case $0\left(\frac{4}{5}\right)$, to demonstrate complete understanding of how the value for the expectation was derived.

(ii) Many students continued using the same variable as part **(i)**. Students should define a new variable to represent this scenario.

About half the candidature were unable to correctly interpret the final part of this question in which the total score needs to be positive. This led to errors where the most common one is letting $\mathrm{E}(Y) > 0$, where Y represents the score in the second round.

Instead, $\mathrm{E}(X) + \mathrm{E}(Y) > 0$ should be used, where X represents the score in the first round.

Another common error is the misconception that the expected scores can only occur as integer values. Note that the expectation is a statistical measure and may not necessarily be an integer as it is mathematically computed.