1 Using algebraic method, solve the inequality $\frac{2}{x-3} < \frac{3}{x+2}$. [3]

Hence, solve the inequality

$$\frac{2}{|x-1|-3} < \frac{3}{|x-1|+2}$$

[3]

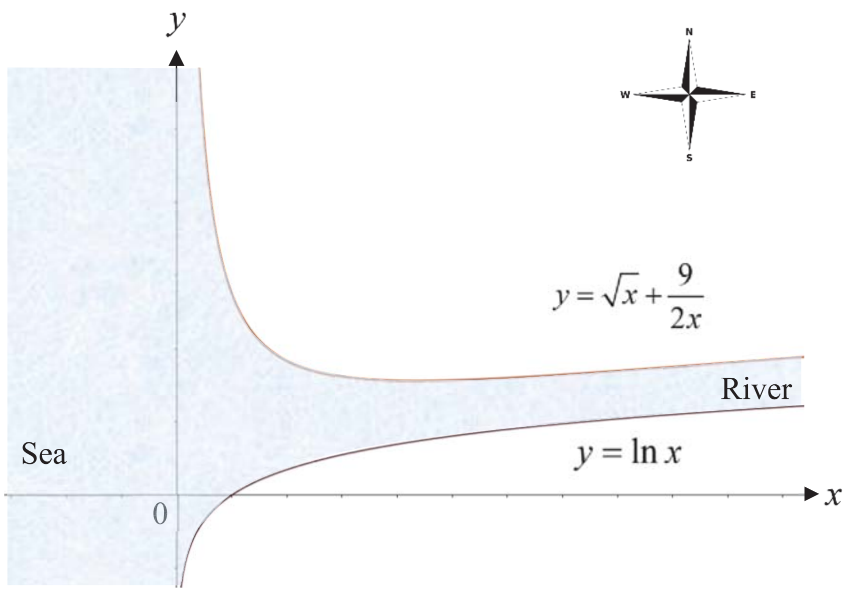
2 A sequence $u_1,\ u_2,\ u_3,\ \ldots$ is such that $u_1 = 6$ and $u_{n+1} = u_n + 3n^2 + 9n + 6$ where $n \geq 1$.

By considering $\sum_{r=1}^{n-1}(u_{r+1} - u_r)$, show that $u_n = n^3 + a\,n^2 + b\,n$ where a and b are integers to be determined. [You may use the result $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$.] [5]

3 Find the series expansion of $\frac{1}{\sqrt{1-4x^2}}$ in ascending powers of x, up to and including the term in x^4. [3]

Hence find the series expansion of $\sin^{-1} 2x$ in ascending powers of x, up to and including the term in x^5. [3]

4



The diagram shows a map depiction of a river opening up to the sea. The north bank of the river follows the curve $y=\sqrt{x}+\dfrac{9}{2x}$ and the south bank of the river follows the curve $y=\ln x$, where 1 unit depicts 1 kilometre.
An engineer wishes to build a bridge across the river, in a North-South direction. Using an **analytical method**, find the exact length of the shortest bridge the engineer can build. [8]

5 Using the substitution $y = z^2$, where $z > 0$, show that the differential equation $\dfrac{\mathrm{d}z}{\mathrm{d}x}=\dfrac{1+\cos\left(z^2\right)}{z}$ may be reduced to $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4\cos^2\dfrac{y}{2}$. [3]

Hence, find z in terms of x given that $z=\sqrt{\dfrac{\pi}{2}}$ when $x = 1$. [5]

6 The function f is defined by

$$\mathrm{f}(x)=\begin{cases}\left|(x-3)^2-9\right| & \text{for } x\geq 0,\\ 0 & \text{for } x<0.\end{cases}$$

(i) Sketch the graph of $y=\mathrm{f}\left(x\right)$, indicating clearly all axial intercepts. Explain why f^{-1} does not exist. [3]

The function g is defined by $\mathrm{g}:x\mapsto \mathrm{f}(x),\ 0\leq x\leq k$.

(ii) Given that g^{-1} exists, state the largest value of k and find $\mathrm{g}^{-1}(x)$. [4]

(iii) Taking the value of k found in part **(ii)**, solve the equation $\mathrm{g}\left(x\right)=\mathrm{g}^{-1}\left(x\right)$. [2]

7 The curve C_1 has the equation $y^2 - 2x^2 = 2$ and the curve C_2 has the equation $x^2+\left(y-1\right)^2=h^2$ where $1 < h < 1+\sqrt{2}$.

(i) Sketch C_1 and C_2, on the same diagram, stating the exact coordinates of any vertices and the equations of any asymptotes. [4]

(ii) Show that the y-coordinates of the points of intersection between C_1 and C_2 satisfy the equation $3y^2 - 4y - 2h^2 = 0$. [2]

(iii) Given that $h=\sqrt{2}$ and the region bounded by C_1 and C_2, in the first and second quadrants, is rotated through π radians about the y-axis. Find the exact volume of the solid obtained. 4]

8 Relative to the origin O, two points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. A line, l, passes through A and is parallel to $\mathbf{b}$. It is given that $\mathbf{b}$ is a unit vector.

(i) Write down a vector equation of l. Show that the position vector of the point N on l such that the length $\left|\overrightarrow{ON}\right|$ is the shortest is given by $\mathbf{a}-(\mathbf{a}.\mathbf{b})\mathbf{b}$. [4]

(ii) The point M is on AN produced such that $kAN = NM$, where k is a constant. Given that the position vector of M is $\mathbf{a}-5(\mathbf{a}.\mathbf{b})\mathbf{b}$, find k. [3]

It is given that $|\mathbf{a}| = 2$ and $\mathbf{a}.\mathbf{b} = \frac{1}{3}$.

(iii) Give a geometrical meaning of $|\mathbf{b}\times(\mathbf{a}-\mathbf{b})|$ and find its exact value. [3]

(iv) C is a point such that OC bisects the angle AOB. Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, a possible position vector of C. [1]

9 A curve C is defined by the parametric equations

$$x = \cos 2\theta + 2\cos\theta, \quad y = \sin 2\theta - 2\sin\theta$$

for $0 \le \theta \le 2\pi$.

(i) Use an analytical method to show that C is symmetrical about the x-axis. [1]

(ii) Sketch C, giving the coordinates of any points where the graph meet the x-axis. [You need not label the coordinates of vertices not lying on the x- and y-axes] [1]

(ii) Show that $\frac{dy}{dx} = \tan\frac{1}{2}\theta$. [4]

(iii) Given that the tangent to the curve at point A makes an angle of $\frac{\pi}{6}$ with the positive x–axis, find the exact coordinates of A and the equation of the tangent at A. [3]

(iv) Let A' be the reflection point of A in the x-axis. The tangent to the curve at A cuts the y-axis at P and the tangent to the curve at A' cuts the y-axis at P'. Find the exact area of triangle PRP', where R is the point of intersection of the tangents at A and A'. [3]

10 A rectangular water tank has a horizontal base with base area 800 cm^2 and a height of 45 cm. A ballcock valve controls the amount of water flowing into the tank, which is proportional to $(50 - x)$, where x cm is the depth of water in the tank.

Initially, the tank is empty and water enters the tank at a rate of 320 cm^3 per second.

(i) Show that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1}{125}(50 - x)$ and find x in terms of t. [6]

Once the depth of the water reaches the maximum allowable depth of 40 cm, a cut-off switch will trigger, which stops the flow of water into the tank.

(ii) Sketch the graph of x against t, for $0 \le t \le 300$, showing all relevant details. [2]

(iii) Explain mathematically what happens if the cut-off switch is faulty and does not trigger. [2]

The cut-off switch is replaced by an overflow outlet 40 cm above the base of the tank that allows any excess water to flow out of the tank into a trough with a volume of 15000 cm^3.

(iv) Find the time taken, starting from the instant the excess water flows out of the tank, for the trough to be completely filled with water, giving your answer correct to the nearest second. [2]

11 An animal welfare charity set up a shelter to house stray and abandoned animals. The charity engages two workers to build 40 new cages to house the animals at the shelter. The two workers are each tasked to complete the building of 20 cages in between 220 and 250 man hours inclusive.

(i) Worker A builds the first cage in h man hours and each cage takes 30 minutes more than the previous cage. Find the set of values of h which will enable A to complete his task within the stipulated time duration. [3]

(ii) Worker B builds the first cage in k man hours and the time for each subsequent cage is 5% more than the time for the previous cage. Find the set of values of k which will enable B to complete his task within the stipulated time duration. [2]

(iii) Assuming each worker completes his task in exactly 220 man hours, find the difference in the workers' time to build the 20^{th} cage, giving your answer to the nearest minute. [3]

On 1 January 2020 the charity opened an Operating Fund account of \$150 000 with a bank. On the first day of each subsequent month (starting from 1 February 2020), the charity made a withdrawal of \$2 000 from the account to purchase animal food and to pay for maintenance cost of the shelter. The bank pays a compound interest at the rate of p% per month on the last day of each month.

(iv) The charity wanted the amount in the Operating Fund account to be at least \$10 000 on 31 December 2030, after interest has been added. What interest rate per month, applied from January 2020, would achieve this? [5]

Temasek Junior College
2020 JC 2 JCT H2 Mathematics Paper 2

Section A: Pure Mathematics [40 marks]

1 The curve $y = \mathrm{f}(x)$, where $y < 0$, passes through the point $\left(0,\ -\frac{\sqrt{21}}{6}\right)$ and has gradient given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{4-3y^2}}{y}$.

(i) Find $\mathrm{f}(x)$. [4]

(ii) Find the exact coordinates of the point on the curve where the gradient is parallel to the x-axis. [2]

2 **(i)** Given that $y = \sec x + \tan x$, show that $\cos x \frac{\mathrm{d}y}{\mathrm{d}x} = y$.

By further differentiation of this result, find the Maclaurin expansion of $\sec x + \tan x$, in ascending powers of x, up to and including the term in x^3. [5]

Show that, to this degree of approximation, $\sec x + \tan x$ may be expressed in the form $a + b\ln(1-x)$, where a and b are constants to be determined. [2]

(ii) It is given that $\sec x + \tan x = \frac{3}{2}$, where $0 \le x < \frac{\pi}{2}$.

(a) By using the identity $1 + \tan^2 x = \sec^2 x$, or otherwise, show that $\tan x = \frac{5}{12}$. [2]

(b) Deduce, using the approximation obtained in **(i)**, that $\tan^{-1}\frac{5}{12} \approx m - \mathrm{e}^n$, where m and n are constants. [2]

3 A computer-controlled machine can be programmed to make cuts by entering the equation of the plane of the cut, and to drill holes by entering the equation of the drill line.

Figure 1 below shows a cuboid *OABCEFGH* (not drawn to scale) where $OA = 20$ cm, $OC = 30$ cm and $OE = 30$ cm. The point O is taken as the origin and unit vectors **i**, **j** and **k**, are taken along *OA*, *OC* and *OE* respectively. Take 1 unit to represent 1 cm.

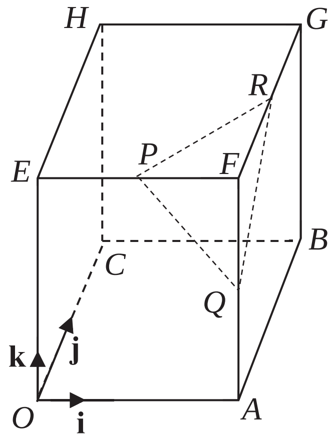


Figure 1

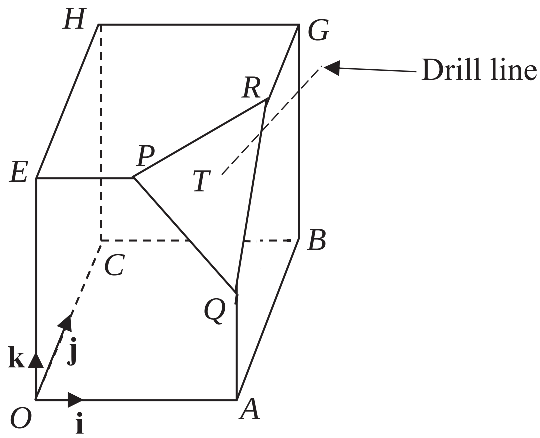


Figure 2

First, the machine makes a plane cut to the cuboid to remove the tetrahedron *PQRF*. The cut goes through the points *P*, *Q* and *R*, which are the midpoints of the sides *EF*, *FA* and *FG* respectively.

(i) Show that the cartesian equation of the plane p which contains the points *P*, *Q* and *R* is $3x - 2y + 2z = 90$. [3]

(ii) Find the acute angle between p and the base *OABC*. [2]

(iii) Two planes parallel to p are such that the distance from each plane to p is $\sqrt{17}$ cm. Find vector equations of the planes in scalar product form. [3]

Next, the machine drills a hole at a point *T* on p as shown in Figure 2. The drill line is perpendicular to p and passes through a point *S* with coordinates (5, 10, 15).

(iv) Find the coordinates of *T*. [3]

4 **(a)** Solve the simultaneous equations

$$z + \mathrm{i} = |w|,$$

$$\frac{w - \mathrm{i}}{z - 1} = 2,$$

giving your answers in cartesian form $x + \mathrm{i}y$. [5]

(b) The complex number u has modulus k and argument α, where $0 < \alpha < \frac{\pi}{6}$ and is represented by the point *P* on an Argand diagram. The line l passes through the origin *O* and makes an angle $\frac{\pi}{6}$ with the positive real axis. The point *Q* represents the complex number v such that *Q* is the reflection of *P* in l.

(i) Indicate the points *P* and *Q* and the line l on an Argand diagram. State the modulus of v and show that the argument of v is $\frac{\pi}{3} - \alpha$. [3]

(ii) Find $\sqrt{uv}$ in exponential form. [2]

(iii) Point *R* represents the complex number $\sqrt{uv}$. Indicate *R* on the Argand diagram in part **(i)**. Identify the geometrical shape of *OPRQ*. [2]

Section B: Statistics [60 marks]

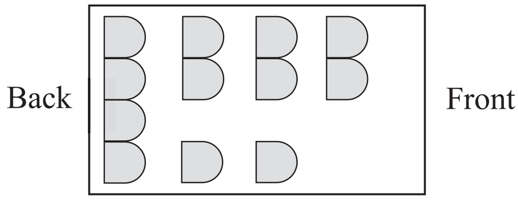
5 A researcher collects data on the number of passengers, x, on a public bus during peak hours in the Circuit Breaker period. He summarises his 18 readings as follows:

$$\sum x = 207, \qquad \sum x^2 = 2515.$$

(i) Find unbiased estimates of the population mean and variance of the number of passengers on a public bus during peak hours in the Circuit Breaker period. [3]

(ii) Taking the population mean to be 11.7, find the approximate probability that the mean number of passengers on 100 public buses during peak hours in the Circuit Breaker period is greater than 12. [3]

6



The diagram shows the seating plan for passengers in a minibus, which has 12 seats arranged in 4 rows. The back row has 4 seats and the front row has 2 seats on one side; the middle 2 rows have 2 seats on 1 side and 1 seat on the other side. 10 passengers get on the minibus.

(i) Find the number of different seating arrangements for the 10 passengers. [1]

(ii) Find the number of different seating arrangements if 2 particular passengers must not sit together. [3]

(iii) Find the number of different seating arrangements if there are 2 married couples among the 10 passengers and each couple must sit together. [4]

7 A high jumper estimates the probabilities that she will be able to clear the bar at various heights, based on her experience in training. These are given in the table below.

Height	Probability of success at each attempt
1.6 m	1
1.7 m	0.5
1.8 m	0.2
1.9 m	0

In a competition, she is allowed up to three attempts to clear the bar at each height. If she clears the bar, the bar is raised by 10 cm and she is allowed three attempts at the new height; and so on. It is assumed that she starts at height 1.6 m and the result of each attempt is independent of all her previous attempts.

(i) Show that the probability she will clear the bar at 1.7 m is 0.875. [2]

(ii) Given that she has cleared the bar at 1.7 m, find the probability that she will not clear the bar at 1.8 m. [1]

Hence find the probabilities that, in the competition, the greatest height she will clear at the bar is

(a) 1.6 m, [1]
(b) 1.7 m, [1]
(c) 1.8 m. [1]

The final result of her competition is the greatest height that she has cleared.

In a particular month, she participated in 3 competitions with the same rules.

(iii) Given that her worst result in the 3 competitions is 1.7 m, find the probability that she clears the height at 1.8 in exactly one competition. [3]

8 In a large population of Bichon Frises (a breed of dog), on average, one in three of them have blood type *Dal–*. Specimens of blood from the first five of this breed of dog attending a local veterinary clinic are to be tested. It can be assumed that these five Bichon Frises are a random sample from the population.

(i) State, in context, two assumptions needed for the number of Bichon Frises in the sample who are found to have blood type *Dal–* to be well modelled by a binomial distribution. [2]

Assume now that the number of Bichon Frises in the sample who are found to have blood type *Dal–* has a binomial distribution.

(ii) Find the probability that more than two of the Bichon Frises in the sample are found to have blood type *Dal–*. [2]

(iii) Three such samples of five Bichon Frises are taken. Find the probability that one of these three samples has exactly one Bichon Frise with blood type *Dal–*, another has exactly two Bichon Frises with blood type *Dal–*, and the remaining sample has more than two Bichon Frises with blood type *Dal–*. [3]

(iv) N such samples of five Bichon Frises are taken. Find the least value of N such that the probability that the number of these samples that contain two or fewer Bichon Frises with blood type *Dal–* will be at least 15 is more than 90%. [3]

9 A biased cubical die is thrown onto a horizontal table. The random variable X is the number on the face in contact with the table. The probability distribution of X is given by

$$P(X = x) = k(x - 1)!, \text{ where } x = 1, 2, 3, 4, 5, 6,$$

and k is a constant.

(i) Find the value of k. [2]

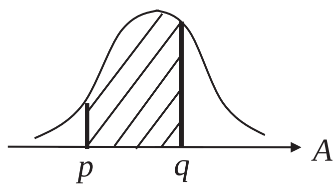
(ii) Show that $E(X) = \dfrac{873}{154}$ and hence, find Var(X). [3]

In a game, Antonio pays a stake of \$32 and throws two such dice onto a horizontal table. The amount Antonio receives is \$$Y$ where Y is the sum of the numbers on the faces (of both dice) **not in contact** with the table.

(iii) Find the probability that he receives more than \$38 in a particular game. [4]

(iv) Find Antonio's expected gain. [3]

10 **(a)** The diagram below shows the probability density function of a normal random variable A with mean 50 and variance 36.



The values p and q are such that $P(p < A < q) = \dfrac{1}{3}$.

Giving clear reasons, find the value of p such that $q - p$ is minimum. [2]

(b) A supermarket sells two varieties of watermelon. The weight of a Green Giant watermelon follows a normal distribution with mean 3.5 kg and variance 0.2 kg^2, while the weight of an Organic Emerald watermelon follows a normal distribution with mean 3.1 kg and variance 0.3 kg^2.

(i) Find the probability that 4 Green Giant watermelons have a greater total weight than 5 times the weight of an Organic Emerald watermelon. [3]

(ii) A packing basket can carry up to 50 kg of fruit. If the basket can carry n Green Giant watermelons at least 90% of the time, show that n satisfies the inequality

$$50 - 3.5n - 1.28155\sqrt{0.2n} \geq 0$$

and hence find the greatest value of n. [4]

Green Giant watermelons are sold for \$2.20 per kg and Organic Emerald watermelons are sold for \$2.60 per kg.

(iii) Find the probability that an Organic Emerald watermelon costs more than a Green Giant watermelon. [3]

(iv) Green Giant watermelons are displayed in the store in crates of 16 fruit each. Find the probability that, in a crate, at least 10 of the watermelons each cost at least \$8. [3]

Important things you need to take note:

1 **Read questions fully** (Eg, "in a North-South direction", "using an analytical method", "Hence",etc)

2 **Answer questions fully** (Eg, write "Area =", "No. of ways =", give coordinates as requested, etc)

3 **Write proper notations and give proper presentation** (Eg, use vectors $\underset{\sim}{a}$, $\overrightarrow{ON}$, give $\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$ where $\lambda \in \mathbb{R}$, use $\overline{X}$ to denote sample mean, define the random variables first, spell out Central Limit Theorem in full, etc)

4 **Simplify surds, logarithms, exponential forms** (Eg, $\left(\sqrt{2}\right)^3 = 2\sqrt{2}$, $\ln\left(e^{\frac{1}{2}}\right) = \frac{1}{2}$, etc)

5 **Sketch diagrams of suitable size** (at least 1/3 of sheet) **and accuracy**.

6 **Use at least 5 s.f. in intermediate answers** and **give final answer to 3 s.f.** (for non-exact answers. Exact answers such as 123888 or 2/7 can be left as such).

7 **Good time management - leave at least one hour for last 3 questions.**

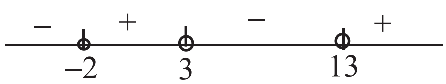
8 **Bring a compass for drawing circles.**

1 Using algebraic method, solve the inequality $\frac{2}{x-3}<\frac{3}{x+2}$. [3]

Hence, solve the inequality

$$\frac{2}{|x-1|-3}<\frac{3}{|x-1|+2}$$

[3]

Solution	Marker's Comments
$\frac{2}{x-3}<\frac{3}{x+2}$ $\Rightarrow \frac{2}{x-3}-\frac{3}{x+2}<0$ $\Rightarrow \frac{x-13}{(x-3)(x+2)}>0$ $\Rightarrow (x-13)(x-3)(x+2)>0,\quad x\neq -2,3$  Ans: $-2<x<3$ or $x>13$ ------ (*)	**Common Mistakes made:** ▪ Forget to change sign when multiplying by -1 on both sides ▪ Draw the wrong graph
Replace x by $\|x-1\|$, $-2<\|x-1\|<3$ or $\|x-1\|>13$ $\|x-1\|<3$ or $\|x-1\|>13$ $-2<x<4$ or $x<-12$ or $x>14$	Students who do it by graphical approach generally get the solutions right. **Many common misconceptions:** ▪ Confusion with 'and', 'or' $-2<\|x-1\|<3$ $\Rightarrow \|x-1\|>-2$ or $\|x-1\|<3$ × $\Rightarrow \|x-1\|>-2$ and $\|x-1\|<3$ ✓ ▪ $\|x-1\|>-2$ (Reject $\because$ no soln)× Instead, $x\in\mathbb{R}$ $\because$ always true✓ ▪ $\|x-1\|>13$ $\Rightarrow x-1>13$ or $x-1>-13$ × $\Rightarrow x-1>13$ or $x-1<-13$ ✓

2 A sequence $u_1,\ u_2,\ u_3,\ \ldots$ is such that $u_1 = 6$ and $u_{n+1} = u_n + 3n^2 + 9n + 6$ where $n \geq 1$.

By considering $\sum_{r=1}^{n-1}(u_{r+1} - u_r)$, show that $u_n = n^3 + a\,n^2 + b\,n$ where a and b are integers to be determined. [You may use the result $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$.] [5]

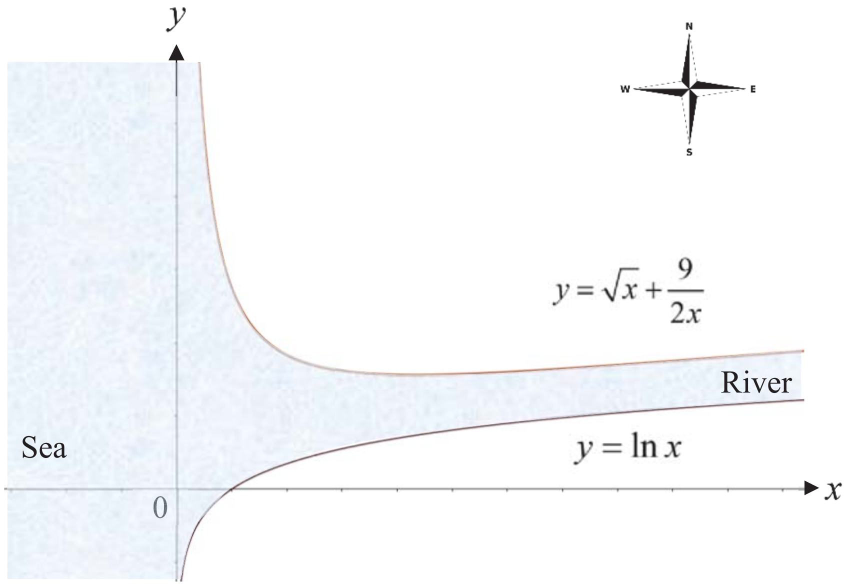
Solution	Marker's Comment
We have $u_{r+1} - u_r = 3r^2 + 9r + 6$ $\sum_{r=1}^{n-1}(u_{r+1} - u_r) = \sum_{r=1}^{n-1}\left(3r^2 + 9r + 6\right)$ $= 3\sum_{r=1}^{n-1} r^2 + 9\sum_{r=1}^{n-1} r + \sum_{r=1}^{n-1} 6$ $= 3 \times \frac{1}{6}(n-1)\left(n-1+1\right)\left(2\left(n-1\right)+1\right)$ $+9 \times \frac{1}{2}(n-1)\left(1+n-1\right) + 6(n-1)$ $= n^3 + 3n^2 + 2n - 6$ $\sum_{r=1}^{n-1}(u_{r+1} - u_r) = \cancel{u_2} - u_1$ $+\ \cancel{u_3} - \cancel{u_2}$ $+\ \cancel{u_4} - \cancel{u_3}$ $\vdots$ $+\ \cancel{u_{n-1}} - \cancel{u_{n-2}}$ $+\ u_n - \cancel{u_{n-1}}$ $=\ u_n - u_1$ $\therefore\ u_n - u_1 = n^3 + 3n^2 + 2n - 6$ $u_n = n^3 + 3n^2 + 2n$ Thus $a = 3$ and $b = 2$	It is wrong to write: $\sum_{r=1}^{n-1}\left(3n^2 + 9n + 6\right)$, must be r and not n. Students need to learn to recognise this method of replacement of n by n-1 in the given formula $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ instead of using this to subtract the n^{th} term. Students must show the MOD method and the cancellation to derive $\sum_{r=1}^{n-1}(u_{r+1} - u_r) = u_n - u_1$

3 Find the series expansion of $\dfrac{1}{\sqrt{1-4x^2}}$ in ascending powers of x, up to and including the term in x^4. [3]

Hence find the series expansion of $\sin^{-1} 2x$ in ascending powers of x, up to and including the term in x^5. [3]

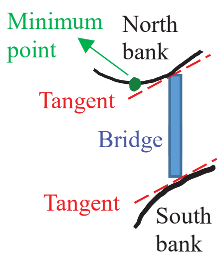
Solution	Marker's Comment
$\dfrac{1}{\sqrt{1-4x^2}} = \left(1-4x^2\right)^{-\frac{1}{2}}$ $= 1 + \left(-\dfrac{1}{2}\right)\left(-4x^2\right) + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-4x^2\right)^2 + \ldots$ $= 1 + 2x^2 + 6x^4 + \ldots$	• Common error: students used $4x^2$ instead of $-4x^2$ in the expansion. • **Inefficient approach**: students approached this as a Maclaurin Series problem resulting in very complicated and often incorrect working.
$\displaystyle\int \frac{1}{\sqrt{1-4x^2}}\,dx = \frac{1}{2}\sin^{-1} 2x + C$ $\displaystyle\sin^{-1} 2x = 2\int \frac{1}{\sqrt{1-4x^2}}\,dx$ $\displaystyle= 2\int (1 + 2x^2 + 6x^4 + \ldots)\,dx$ $= 2\left(x + \dfrac{2}{3}x^3 + \dfrac{6}{5}x^5 + \ldots\right) + C$ When $x = 0$, $\sin^{-1} 0 = 0$ $\therefore C = 0$ $\sin^{-1} 2x = 2x + \dfrac{4}{3}x^3 + \dfrac{12}{5}x^5 + \ldots$	• **Fundamental skills**: A large majority of students made some mistake integrating $\dfrac{1}{\sqrt{1-4x^2}}$ or differentiating $\dfrac{1}{2}\sin^{-1} 2x$, resulting in the loss of most marks in this part. • Even though the condition is not given explicitly, students must still notice that the value of C needs to be found (and working shown).

4



The diagram shows a map depiction of a river opening up to the sea. The north bank of the river follows the curve $y = \sqrt{x} + \dfrac{9}{2x}$ and the south bank of the river follows the curve $y = \ln x$, where 1 unit depicts 1 kilometre.

An engineer wishes to build a bridge across the river, in a North-South direction. Using an **analytical method**, find the exact length of the shortest bridge the engineer can build. [8]

Solution	Marker's Comment
Let L be the length of the bridge. $L = \sqrt{x} + \dfrac{9}{2x} - \ln x$ $\dfrac{dL}{dx} = \dfrac{1}{2\sqrt{x}} - \dfrac{9}{2x^2} - \dfrac{1}{x}$ To find minimum L, put $\dfrac{dL}{dx} = 0$:  $\dfrac{x^{\frac{3}{2}} - 9 - 2x}{2x^2} = 0$ $x^{\frac{3}{2}} - 9 - 2x = 0$ Let $u = x^{\frac{1}{2}}$ (use substitution to handle $x^{\frac{1}{2}}$) $u^3 - 2u^2 - 9 = 0$ (factorize cubic expression to solve) $(u-3)(u^2+u+3) = 0$ $u = 3$ or $u^2 + u + 3 = 0$ (No solution as discriminant < 0) $x = u^2 = 9$	- learn to let unknown length of bridge be variable (L) then find minimum L - note that "shortest" bridge does NOT mean it is perpendicular to north & south bank as bridge must be in north-south direction - finding the minimum point of the north bank does NOT give the shortest bridge as it does not take into account the south bank - **"analytical method"** means students must use techniques such as

$$\frac{\mathrm{d}^2L}{\mathrm{d}x^2}=-\frac{1}{4}x^{-\frac{3}{2}}+\frac{9}{x^3}+\frac{1}{x^2}$$ (2nd derivative test to show minimum)

When $x=9$,

$$\frac{\mathrm{d}^2L}{\mathrm{d}x^2}=-\frac{1}{4(27)}+\frac{1}{81}+\frac{1}{81}=\frac{5}{324}>0$$ (need to show value)

Therefore, L is minimum when $x=9$

Shortest $L=\left(\frac{7}{2}-\ln 9\right)$ km (answer question – find **exact** L)

Alternative (first derivative test)

	9^- e.g. 8.9	9	9^+ e.g. 9.1
Sign of $\frac{\mathrm{d}L}{\mathrm{d}x}$	– ve e.g. –0.00157	0	+ ve e.g. 0.00152
	\	—	/

Therefore, L is minimum when $x=9$

substitution and **factor theorem to factorize & solve cubic equation**. 'Exact' suggests that GC is not to be used.

- do remember to perform test for minimum unless question states otherwise

- notation : be careful how 1st derivative test is presented

5 Using the substitution $y = z^2$, where $z > 0$, show that the differential equation $\frac{dz}{dx} = \frac{1+\cos(z^2)}{z}$ may be reduced to $\frac{dy}{dx} = 4\cos^2\frac{y}{2}$. [3]

Hence, find z in terms of x given that $z = \sqrt{\frac{\pi}{2}}$ when $x = 1$. [5]

Solution

Given $\frac{dz}{dx} = \frac{1+\cos(z^2)}{z}$ --- (1)

Differentiate wrt x, $y = z^2 \Rightarrow \frac{dy}{dx} = 2z\frac{dz}{dx}$ --- (2)

Subst (1) into (2),

$$\frac{dy}{dx} = 2z\frac{dz}{dx}$$

$$= 2z\left(\frac{1+\cos(z^2)}{z}\right)$$

$$= 2(1+\cos y)$$

$$= 2\left(1+2\cos^2\frac{y}{2}-1\right)$$

Use $\cos y = 2\cos^2\frac{y}{2} - 1$

$\frac{dy}{dx} = 4\cos^2\frac{y}{2}$ (shown) --- (3)

$$\int \frac{1}{\cos^2\frac{y}{2}}\,dy = \int 4\,dx$$

$$\int \sec^2\left(\frac{y}{2}\right) dy = 4x + C$$

$$\frac{\tan\left(\frac{y}{2}\right)}{\frac{1}{2}} = 4x + C$$

$$y = 2\tan^{-1}(2x+C)$$

When $z = \sqrt{\frac{\pi}{2}}, y = \frac{\pi}{2}, x = 1 \Rightarrow C = -1$

Hence $z = \sqrt{2\tan^{-1}(2x-1)}$ $(z > 0)$

Answer questions fully (Hence, find z in terms of x)

Marker's Comments

OR Differentiate wrt y,

$y = z^2 \Rightarrow \frac{dy}{dz} = 2z$

$$\frac{dy}{dx} = \frac{dy}{dz}\times\frac{dz}{dx}$$

$$= 2z\left(\frac{1+\cos(z^2)}{z}\right) = 1+\cos(z^2)$$

$$= 1+\cos y$$

$$= 2\left(1+2\cos^2\frac{y}{2}-1\right)$$

$$= 4\cos^2\frac{y}{2}$$

DE (3) is easier to solve than DE (1)

Approach:

Solve DE (3) for *y* in terms of *x*.

Then subst back $y = z^2$ to get z in terms of x.

Need to memorise:

Since $\frac{d}{dx}\tan x = \sec^2 x$,

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

6 The function f is defined by

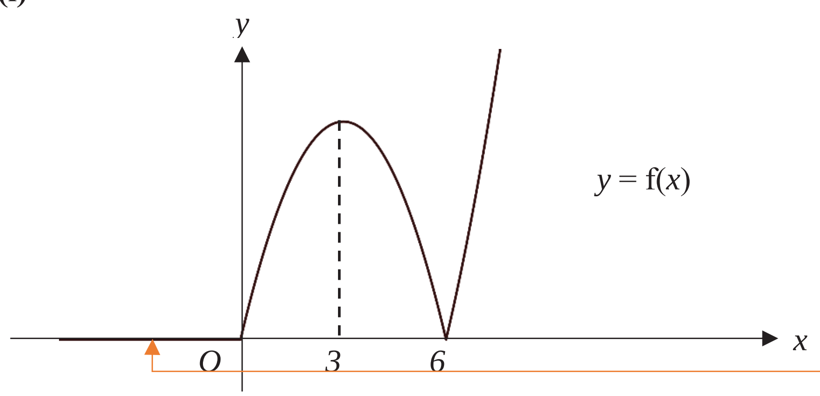
$$f(x)=\begin{cases}\left|(x-3)^2-9\right| & \text{for } x\geq 0,\\ 0 & \text{for } x<0.\end{cases}$$

(i) Sketch the graph of $y=\mathrm{f}\left(x\right)$, indicating clearly all axial intercepts. Explain why f^{-1} does not exist. [3]

The function g is defined by $\mathrm{g}:x\mapsto \mathrm{f}(x),\ 0\leq x\leq k$.

(ii) Given that g^{-1} exists, state the largest value of k and find $\mathrm{g}^{-1}(x)$. [4]

(iii) Taking the value of k found in part **(ii)**, solve the equation $\mathrm{g}\left(x\right)=\mathrm{g}^{-1}\left(x\right)$. [2]

Solution	Marker's Comment
(i)  Since the line $y=0$ (any value between 0 and 9 inclusive) cuts the graph of $y=\mathrm{f}\left(x\right)$ more than once, the function f is not one-one and f^{-1} does not exist.	$y=0$ for $x<0$ should be drawn Use a counter-example, a particular horizontal line, instead of writing '$y=b$, $b\in\mathbb{R}$ cuts more than once'. Function f is not one-one should be mentioned.
(ii) From the graph, $k=3$. For $0\leq x\leq 3$, graph of $y=\left\|(x-3)^2-9\right\|$ is a reflection of the graph of $y=(x-3)^2-9$. Let $y=\mathrm{g}(x)=-\left[(x-3)^2-9\right]=9-(x-3)^2$ $(x-3)=-\sqrt{9-y}$ or $\sqrt{9-y}$ $x=3-\sqrt{9-y}$ or $3+\sqrt{9-y}$ (reject since $x\leq 3$) $\mathrm{g}^{-1}(x)=3-\sqrt{9-x},\ 0\leq x\leq 9$	It is easier to identify and work on $y=-\left[(x-3)^2-9\right]$ for $0\leq x\leq 3$ instead of trying to find the inverse of $y=\left\|(x-3)^2-9\right\|$. Do not forget to find the $D_{\mathrm{g}^{-1}}$

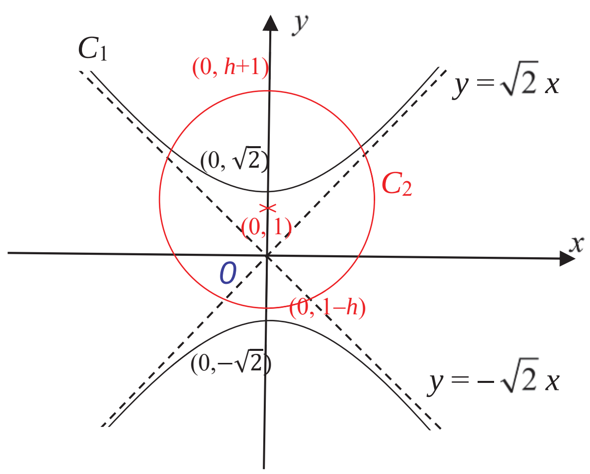
(iii) $g(x) = g^{-1}(x) \quad \Rightarrow g(x) = x$ $\Rightarrow 9-(x-3)^2 = x$ From GC, $x = 0$ or $x = 5$ (reject as $x \leq 3$)	Solve $g(x) = x$ instead of $g(x) = g^{-1}(x)$. Need to check whether the found x-value is within the range of x. If use graphical method, graphs of $g(x)$ and $g^{-1}(x)$ should be drawn symmetrical about $y = x$

7 The curve C_1 has the equation $y^2 - 2x^2 = 2$ and the curve C_2 has the equation $x^2 + (y-1)^2 = h^2$ where $1 < h < 1 + \sqrt{2}$.

(i) Sketch C_1 and C_2, on the same diagram, stating the exact coordinates of any vertices and the equations of any asymptotes. [4]

(ii) Show that the y-coordinates of the points of intersection between C_1 and C_2 satisfy the equation $3y^2 - 4y - 2h^2 = 0$. [2]

(iii) Given that $h = \sqrt{2}$ and the region bounded by C_1 and C_2, in the first and second quadrants, is rotated through π radians about the y-axis. Find the exact volume of the solid obtained. [4]

Solution	Marker's Comment
(i)  	**Accuracy in presentation** Common mistakes seen (i) 1. Wrong or unclear oblique asymptotes 2. Drawing C_1 and C_2 without accuracy – many students did not indicate the centre for C_2 clearly 3. Free-hand drawing of circle resulted in poor accuracy ! 4. C_1 and C_2 should not intersect below the x-axis (why?)
(ii) C_1 : $y^2 - 2x^2 = 2 \Rightarrow \frac{y^2}{2} - x^2 = 1$ ------ (1) C_2 : $x^2 + (y-1)^2 = h^2$ ------ (2) (1) + (2): $\frac{y^2}{2} + (y-1)^2 = h^2 + 1$ $y^2 + 2(y-1)^2 = 2(h^2+1)$ $\Rightarrow y^2 + 2(y^2 - 2y + 1) = 2(h^2+1)$ $\Rightarrow 3y^2 - 4y - 2h^2 = 0$ (Shown)	Observe, observe and observe – you should substitute x^2 from one equation into the other one – Avoid taking square roots.

(iii) Given that $h = \sqrt{2} \quad \Rightarrow 3y^2 - 4y - 4 = 0$

$(3y+2)(y-2) = 0$

Since $y > 0$, $y = 2$

Volume required $= \pi \int_{2}^{1+\sqrt{2}} 2-(y-1)^2 \, dy + \pi \int_{\sqrt{2}}^{2} \frac{y^2}{2} - 1 \, dy$

$$= \pi \left[2y - \frac{(y-1)^3}{3} \right]_2^{1+\sqrt{2}} + \pi \left[\frac{y^3}{6} - y \right]_{\sqrt{2}}^{2}$$

$$= \pi \left[\frac{4\sqrt{2}}{3} - \frac{5}{3} \right] + \pi \left[-\frac{2}{3} + \frac{2\sqrt{2}}{3} \right] = \left(2\sqrt{2} - \frac{7}{3} \right)\pi$$

Important : Draw a good and large diagram so that you can see clearly the region that is rotated about the y-axis

Quite a number of students made the following mistakes:

(a) Vol =

$$2\pi \int_{2}^{1+\sqrt{2}} (2-(y-1)^2)dy + 2\pi \int_{\sqrt{2}}^{2} (\frac{y^2}{2} - 1)dy$$

Where is the mistake?

(b) Vol =

$$\pi \int_{0}^{2} (2-(y-1)^2)dy - \pi \int_{\sqrt{2}}^{2} (\frac{y^2}{2} - 1)dy$$

(c) Many students did not simplify $\left(\sqrt{2}\right)^3$

(d) $\int_{2}^{1+\sqrt{2}} (2-(y-1)^2)dy = \int_{2}^{1+\sqrt{2}} (-y^2 + 2y + 1)\, dy$

[This is not wrong, but you need more time to evaluate the integral exactly]

8 Relative to the origin O, two points A and B have position vectors **a** and **b** respectively. A line, l, passes through A and is parallel to **b**. It is given that **b** is a unit vector.

(i) Write down a vector equation of l. Show that the position vector of the point N on l such that the length $|\overrightarrow{ON}|$ is the shortest is given by $\mathbf{a}-(\mathbf{a}.\mathbf{b})\mathbf{b}$. [4]

(ii) The point M is on AN produced such that $kAN = NM$, where k is a constant. Given that the position vector of M is $\mathbf{a}-5(\mathbf{a}.\mathbf{b})\mathbf{b}$, find k. [3]

It is given that $|\mathbf{a}| = 2$ and $\mathbf{a}.\mathbf{b} = \frac{1}{3}$.

(iii) Give a geometrical meaning of $|\mathbf{b}\times(\mathbf{a}-\mathbf{b})|$ and find its exact value. [3]

(iv) C is a point such that OC bisects the angle AOB. Write down, in terms of **a** and **b**, a possible position vector of C. [1]

- **Write proper notations and give proper presentation**

(Use vectors notations for vectors $\underset{\sim}{a}$, $\overrightarrow{ON}$, give $\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$ where $\lambda \in \mathbb{R}$

- Be clear if you are working with a vector $\underset{\sim}{a}$, or the magnitude of a vector, $|\underset{\sim}{a}|$.

Solution	Marker's Comment
(i) Equation of l: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\lambda \in \mathbb{R}$ Since N lies on the line, $\overrightarrow{ON} = \mathbf{a} + \lambda\mathbf{b}$ where $\lambda \in \mathbb{R}$ Since ON is perpendicular to the line, $\overrightarrow{ON}\bullet\mathbf{b} = 0$ $(\mathbf{a}+\lambda\mathbf{b})\bullet\mathbf{b} = 0$ $\mathbf{a}\bullet\mathbf{b} + \lambda\mathbf{b}\bullet\mathbf{b} = 0$ $\mathbf{a}\bullet\mathbf{b} + \lambda\|\mathbf{b}\|^2 = 0$ $\mathbf{a}\bullet\mathbf{b} + \lambda = 0$ since $\|\mathbf{b}\| = 1$ $\lambda = -\mathbf{a}\bullet\mathbf{b}$ Subst back: $\overrightarrow{ON} = \mathbf{a} - (\mathbf{a}\bullet\mathbf{b})\mathbf{b}$	Should write $\lambda \in \mathbb{R}$. Don't miss out **r** **Read question fully** - we want to find $\overrightarrow{ON}$, not $\|\overrightarrow{ON}\|$. N is the foot of perpendicular from O to l. Apply usual technique.

(ii) **By Ratio Theorem,** $\overrightarrow{ON}=\dfrac{k\overrightarrow{OA}+\overrightarrow{OM}}{k+1}$

Subst in given $\overrightarrow{ON}$ and $\overrightarrow{OM}$:

$$(k+1)\left[\mathbf{a}-(\mathbf{a}\bullet\mathbf{b})\mathbf{b}\right]=k\mathbf{a}+\mathbf{a}-5(\mathbf{a.b})\mathbf{b}$$
$$(k+1)\mathbf{a}-(k+1)(\mathbf{a}\bullet\mathbf{b})\mathbf{b}-k\mathbf{a}=\mathbf{a}-5(\mathbf{a.b})\mathbf{b}$$
$$-(k+1)(\mathbf{a}\bullet\mathbf{b})\mathbf{b}=-5(\mathbf{a.b})\mathbf{b}$$

Comparing coefficients of **b**:
$$k+1=5\Rightarrow k=4$$

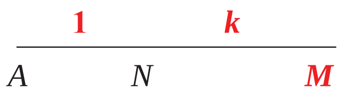
Alternative solution:

$$k\overrightarrow{AN}=\overrightarrow{NM}$$
$$k\left(\overrightarrow{ON}-\overrightarrow{OA}\right)=\left(\overrightarrow{OM}-\overrightarrow{ON}\right)$$
$$k\left(\mathbf{a}-(\mathbf{a.b})\mathbf{b}-\mathbf{a}\right)=\left(\mathbf{a}-5(\mathbf{a.b})\mathbf{b}-(\mathbf{a}-(\mathbf{a.b})\mathbf{b})\right)$$
$$-k(\mathbf{a.b})\mathbf{b}=-4(\mathbf{a.b})\mathbf{b}$$

Comparing coefficients of **b**: $k=4$

Get the ratio correct!

$kAN=NM \Rightarrow \dfrac{AN}{NM}=\dfrac{1}{k}$



It is wrong to divide by a vector!

$k=\dfrac{4(\mathbf{a.b})\mathbf{b}}{(\mathbf{a.b})\mathbf{b}}$ is undefined

$k=\dfrac{\left|-4(\mathbf{a.b})\mathbf{b}\right|}{\left|-(\mathbf{a.b})\mathbf{b}\right|}=4$ is correct

$|\mathbf{b}\times(\mathbf{a}-\mathbf{b})|=|\mathbf{a}\times\mathbf{b}|$ is

the **perpendicular (or shortest) distance**
from point *A* to the line passing through *O* and *B*

or the perpendicular distance from point *B* to the line *l*

or the area of the parallelogram with adjacent sides *OA* and *OB* (or *OB* and *AB*)

or 2 times the area of ΔOAB

Method 1

$|\mathbf{b}\times(\mathbf{a}-\mathbf{b})|=|\mathbf{a}\times\mathbf{b}|$

$=AF$

$=\sqrt{OA^2-OF^2}$

$=\sqrt{|\mathbf{a}|^2-|\mathbf{a.b}|^2}$

$=\sqrt{2^2-\left(\dfrac{1}{3}\right)^2}$

$=\dfrac{\sqrt{35}}{3}$

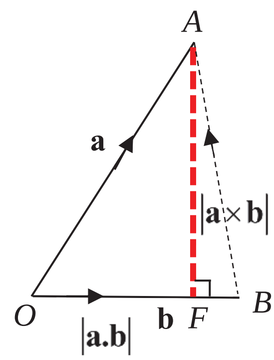
Important concepts to use:

$|\mathbf{a}\times\mathbf{b}|$ where **b** is a unit vector

is the perpendicular distance from point *A* to the line passing through *O* and *B*

or

$|\mathbf{a}\times\mathbf{b}|$ is the area of parallogram with adjacent sides *OA* and *OB*



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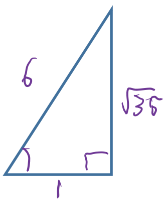
<u>Method 2</u>

$\cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \dfrac{1}{6}$

$\Rightarrow \sin\theta = \dfrac{\sqrt{35}}{6}$



$|\mathbf{b}\times(\mathbf{a}-\mathbf{b})|$
$= |\mathbf{b}\times\mathbf{a}-\mathbf{b}\times\mathbf{b}|$
$= |\mathbf{b}\times\mathbf{a}-\mathbf{0}|$
$= |\mathbf{a}\times\mathbf{b}|$
$= |\mathbf{a}||\mathbf{b}|\sin\theta$
$= 2(1)\dfrac{\sqrt{35}}{6} = \dfrac{\sqrt{35}}{3}$

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Note that:
$|\mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{b}| \neq |\mathbf{a}\times\mathbf{b}|-|\mathbf{b}\times\mathbf{b}|$
In general, $|\mathbf{u}-\mathbf{v}| \neq |\mathbf{u}|-|\mathbf{v}|$

Opposite vectors $\mathbf{b}\times\mathbf{a} = -(\mathbf{a}\times\mathbf{b})$
Same magnitude $|\mathbf{b}\times\mathbf{a}| = |\mathbf{a}\times\mathbf{b}|$

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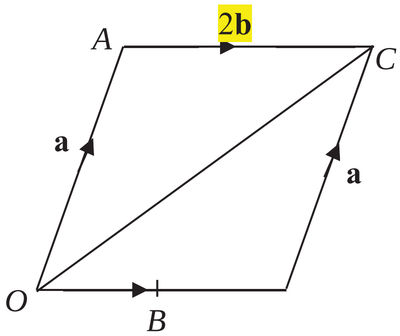
(iii) A possible $\overrightarrow{OC} = \mathbf{a}+2\mathbf{b}$



Given $|\mathbf{a}| = 2$ and $|\mathbf{b}| = 1$

Extend the vector $\overrightarrow{OB}$ such that it has the same length as OA. The shape above is a **rhombus** and thus OC bisects the angle AOB.

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Common mistake is to give the mid-point of *AB* as *C*.

Note that the diagonal of a parallelogram DOES NOT bisect the angle; only the diagonal of a rhombus bisects the angle.

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9 A curve C is defined by the parametric equations

$$x = \cos 2\theta + 2\cos\theta, \quad y = \sin 2\theta - 2\sin\theta$$

for $0 \le \theta \le 2\pi$.

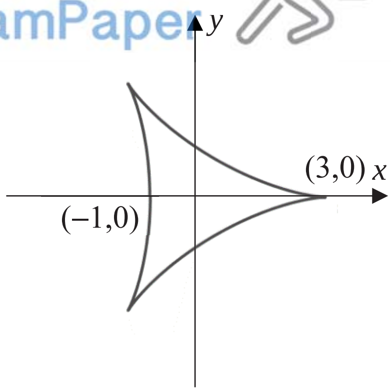
(i) Use an analytical method to show that C is symmetrical about the x-axis. [1]

(ii) Sketch C, giving the coordinates of any points where the graph meet the x-axis.
[You need not label the coordinates of vertices not lying on the x- and y-axes] [1]

(ii) Show that $\dfrac{dy}{dx} = \tan\dfrac{1}{2}\theta$. [4]

(iii) Given that the tangent to the curve at point A makes an angle of $\dfrac{\pi}{6}$ with the positive x–axis, find the exact coordinates of A and the equation of the tangent at A. [3]

(iv) Let A' be the reflection point of A in the x-axis. The tangent to the curve at A cuts the y-axis at P and the tangent to the curve at A' cuts the y-axis at P'. Find the exact area of triangle PRP', where R is the point of intersection of the tangents at A and A'. [3]

Solution	Marker's Comment
(i) Replace θ by $-\theta$, $x = \cos 2(-\theta) + 2\cos(-\theta) = \cos 2\theta + 2\cos\theta$, $y = \sin 2(-\theta) - 2\sin(-\theta). = -(\sin 2\theta - 2\sin\theta)$ Hence, x-axis **(y = 0)** is line of symmetry of C. Alternatively, Replace θ by $2\pi - \theta$, $x = 2\cos^2\theta - 1 + 2\cos\theta$ $= 2\cos^2(2\pi - \theta) - 1 + 2\cos(2\pi - \theta)$ $= \cos 2\theta + 2\cos\theta$ $y = 2\sin(2\pi - \theta)\cos(2\pi - \theta) - 2\sin(2\pi - \theta)$ $= 2(-\sin\theta)\cos\theta + 2\sin\theta$ $= -\sin 2\theta + 2\sin\theta$	Analytical method means you are not supposed to draw the graph using GC to show symmetry.
(ii) 	**Read question carefully** Students should indicate the coordinates of all x-intercepts.

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(iii) $\dfrac{dy}{d\theta} = 2\cos 2\theta - 2\cos\theta$

$\dfrac{dx}{d\theta} = -2\sin 2\theta - 2\sin\theta$

$\dfrac{dy}{dx} = \dfrac{2\cos 2\theta - 2\cos\theta}{-2\sin 2\theta - 2\sin\theta}$

$= -\dfrac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta}$

$= -\dfrac{-2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\sin\frac{3\theta}{2}\cos\frac{\theta}{2}}$

$= \tan\frac{1}{2}\theta$

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- While many could write down the expression for $\dfrac{dy}{dx}$ ☺, many could not show the result.
- Students should recognize that such forms, for example $\cos P - \cos Q, \sin P + \sin Q$ can be simplified by using factor formulas in MF26.
- Most students who tried to use the double angle formulas ended up having expressions more complicated, though it is possible to do so.

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(iv) Gradient at point A = $\tan\dfrac{\pi}{6} = \tan\dfrac{1}{2}\theta \;\Rightarrow\; \theta = \dfrac{\pi}{3}$

(Note: $0 \le \dfrac{\theta}{2} \le \pi$).

Therefore, $x = \cos\dfrac{2\pi}{3} + 2\cos\dfrac{\pi}{3} = \dfrac{1}{2}$, $y = \sin\dfrac{2\pi}{3} - 2\sin\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

i.e $A\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$

Therefore the equation of tangent:

$y - \left(-\dfrac{\sqrt{3}}{2}\right) = \tan\dfrac{\pi}{6}\left(x - \left(\dfrac{1}{2}\right)\right)$

$y = \dfrac{1}{\sqrt{3}}x - \dfrac{1}{2\sqrt{3}} - \dfrac{\sqrt{3}}{2}$ i.e. $y = \dfrac{1}{\sqrt{3}}x - \dfrac{2\sqrt{3}}{3}$

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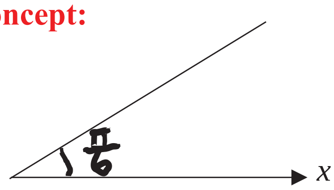
Common misconception:

Student thought $\theta = \dfrac{\pi}{6}$, and hence $\dfrac{dy}{dx} = \tan\dfrac{\pi}{12}$.

Concept:



Gradient of the tangent = $\tan\dfrac{\pi}{6}$

$= \dfrac{\text{opp}}{\text{adj}}$ (Think of the definition of gradient)

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(v) When $x = 0$, $y = -\dfrac{2\sqrt{3}}{3}$ $P\left(0, -\dfrac{2\sqrt{3}}{3}\right)$

When $y = 0$, $x = \dfrac{2\sqrt{3}}{3} \times \sqrt{3} = 2$ $R(2,0)$

Hence the area of triangle $PRP' = 2\left(\dfrac{1}{2} \times \dfrac{2\sqrt{3}}{3} \times 2\right) = \dfrac{4\sqrt{3}}{3}$

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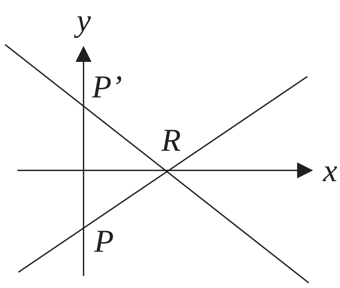
Draw a diagram



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10 A rectangular water tank has a horizontal base with base area 800 cm^2 and a height of 45 cm. A ballcock valve controls the amount of water flowing into the tank, which is proportional to $(50-x)$, where x cm is the depth of water in the tank.

Initially, the tank is empty and water enters the tank at a rate of 320 cm^3 per second.

(i) Show that $\frac{dx}{dt}=\frac{1}{125}(50-x)$ and find x in terms of t. [6]

Once the depth of the water reaches the maximum allowable depth of 40 cm, a cut-off switch will trigger, which stops the flow of water into the tank.

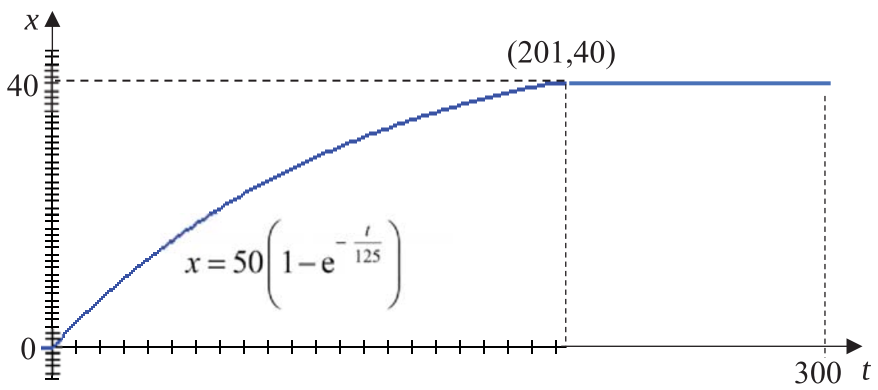
(ii) Sketch the graph of x against t, for $0 \le t \le 300$, showing all relevant details. [2]

(iii) Explain mathematically what happens if the cut-off switch is faulty and does not trigger. [2]

The cut-off switch is replaced by an overflow outlet 40 cm above the base of the tank that allows any excess water to flow out of the tank into a trough with a volume of 15000 cm^3.

(iv) Find the time taken, starting from the instant the excess water flows out of the tank, for the trough to be completely filled with water, giving your answer correct to the nearest second. [2]

Solution	Marker's Comment
(i) Let V cm^3 be the volume of water in the tank. Therefore $V = 800x$ $\frac{dV}{dt}=800\frac{dx}{dt}=k(50-x)$ When $x=0$, $\frac{dV}{dt}=320 \Rightarrow k=6.4$ Hence $\frac{dx}{dt}=\frac{1}{125}(50-x)$ (Shown) <u>Alternatively</u> $V=800x \Rightarrow \frac{dV}{dx}=800$ $\frac{dV}{dt}=k(50-x)$ When $x=0$, $\frac{dV}{dt}=320 \Rightarrow k=6.4$ $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt} \Rightarrow 6.4(50-x)=800\frac{dx}{dt}$ Hence $\frac{dx}{dt}=\frac{1}{125}(50-x)$ (Shown)	**Read the problem carefully**: A majority of students started with $\frac{dx}{dt}=k(50-x)$ which does not reflect what the question says ("amount of water flowing … proportional to $(50-x)$") As this is a "show" problem, all details must be correctly shown. Other misinterpretations include thinking that $\frac{dV}{dt}=320$ constantly, or that when $t=1$, $x=\frac{320}{800}$

$\int \frac{1}{50-x}\,\mathrm{d}x = \int \frac{1}{125}\,\mathrm{d}t$ $-\ln(50-x) = \frac{1}{125}t + C$, since $x \le 45$ $\Rightarrow 50 - x = Ae^{-\frac{t}{125}}$ where $A = e^{-C}$ When $t = 0$, $x = 0 \Rightarrow A = 50$ Hence $x = 50\left(1 - e^{-\frac{t}{125}}\right)$ (shown)	In contrast, almost all students were able to solve this standard differential equation. **Common error**: students forget to use $\lvert 50 - x \rvert$, or did not justify that $50 - x$ is positive.
(ii) Using GC, when $x = 40$, $t = 201.18$ 	**Read the problem carefully**: The required range of t is $0 \le t \le 300$. Students either forgot to end the graph at $t = 300$ or did not indicate the **relevant detail** (201, 40). Students need to realise that the cut-off switch stops the flow of water, which is represented by a **horizontal line** from $t = 201$ to $t = 300$.
(iii) As $t \to \infty, e^{-\frac{t}{125}} \to 0$, hence $x \to 50$. Therefore, if the cut-off switch is faulty and does not trigger, the water tank will overflow.	**Read the problem carefully**: Mathematical explanation is required – most students only described what would happen, without **justifying their claim** mathematically.
(iv) When $x = 40$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 6.4(50-40) = 64$ Hence time taken to fill trough = $\frac{15000}{64} = 235$ seconds (nearest second)	**Read the problem carefully**: the time required is measured from the point when $x = 40$, when $\frac{\mathrm{d}V}{\mathrm{d}t} = 320$ is no longer true. There is also no need to include the initial 201 seconds when the tank is filling up.

11 An animal welfare charity set up a shelter to house stray and abandoned animals. The charity engages two workers to build 40 new cages to house the animals at the shelter. The two workers are each tasked to complete the building of 20 cages in between 220 and 250 man hours inclusive.

(i) Worker A builds the first cage in h man hours and each cage takes 30 minutes more than the previous cage. Find the set of values of h which will enable A to complete his task within the stipulated time duration. [3]

(ii) Worker B builds the first cage in k man hours and the time for each subsequent cage is 5% more than the time for the previous cage. Find the set of values of k which will enable B to complete his task within the stipulated time duration. [2]

(iii) Assuming each worker completes his task in exactly 220 man hours, find the difference in the workers' time to build the 20$^{\text{th}}$ cage, giving your answer to the nearest minute. [3]

On 1 January 2020 the charity opened an Operating Fund account of \$150 000 with a bank. On the first day of each subsequent month (starting from 1 February 2020), the charity made a withdrawal of \$2 000 from the account to purchase animal food and to pay for maintenance cost of the shelter. The bank pays a compound interest at the rate of p% per month on the last day of each month.

(iv) The charity wanted the amount in the Operating Fund account to be at least \$10 000 on 31 December 2030, after interest has been added. What interest rate per month, applied from January 2020, would achieve this? [5]

Solution	Marker's Comment
(i) AP with first term h and common difference 0.5 For $220 \le S_{20} \le 250$, $220 \le \frac{20}{2}\left(2h+19\left(\frac{1}{2}\right)\right) \le 250$ $6.25 \le h \le 7.75$ Set of values of h is [6.25, 7.75] or $\{h : h \in \mathbb{R},\ 6.25 \le h \le 7.75\}$	**Common mistakes:** - Using 40 cages instead of 20 cages each. - Did not notice 'inclusive'. - Did not know how to express the answer in set form. - Wrong formula Eg: $S_{20} = h+19\left(\frac{1}{2}\right)$, $S_{20} = \frac{k\left(1.05^{19}-1\right)}{1.05-1}$
(ii) GP with first term k and common ratio 1.05 For $220 \le S_{20} \le 250$, $220 \le \frac{k\left(1.05^{20}-1\right)}{1.05-1} \le 250$ $6.65 \le k \le 7.56$ Set of values of k is [6.65, 7.56] or $\{k : k \in \mathbb{R},\ 6.65 \le k \le 7.56\}$	

(iii) For worker A, time to build the 20^{th} cage $= 6.25 + 19(0.5) = 15.75$ ---- (i) For worker B, time to build the 20^{th} cage $= 6.65337(1.05)^{19} = 16.81273$ ----(ii) $\therefore$ difference in the workers' time = 1.06273 h = 64 min (nearest min)	k and part (ii) are intermediate value. Need to have at least 5 decimal places for accuracy.

n	Account at start of *nth* month	Account at end of *n*th month
1	150 000	$\left(1+\frac{p}{100}\right)(150000)$
2	$\left(1+\frac{p}{100}\right)(150000)-2000$	$\left(1+\frac{p}{100}\right)\left(\left(1+\frac{p}{100}\right)(150000)-2000\right)$ $=\left(1+\frac{p}{100}\right)^2(150000)-2000\left(1+\frac{p}{100}\right)$
3	$\left(1+\frac{p}{100}\right)^2(150000)-2000\left(1+\frac{p}{100}\right)-2000$	$\left(1+\frac{p}{100}\right)\left(\left(1+\frac{p}{100}\right)^2(150000)-2000\left(1+\frac{p}{100}\right)-2000\right)$ $=\left(1+\frac{p}{100}\right)^3(150000)-2000\left(\left(1+\frac{p}{100}\right)^2+\left(1+\frac{p}{100}\right)\right)$
n		$\left(1+\frac{p}{100}\right)^n(150000)-2000\left(\left(1+\frac{p}{100}\right)^{n-1}+\left(1+\frac{p}{100}\right)^{n-2}+\ldots+\left(1+\frac{p}{100}\right)\right)$

(iv) Amount in the account at the end of nth month

$$=\left(1+\frac{p}{100}\right)^{132}\overset{(a)}{(150000)}-2000\left(\left(1+\frac{p}{100}\right)^{131}+\left(1+\frac{p}{100}\right)^{130}+\ldots+\overset{(b)}{\left(1+\frac{p}{100}\right)}\right)$$

$$=\left(1+\frac{p}{100}\right)^{132}(150\ 000)-2000\times\frac{\left(1+\frac{p}{100}\right)\left(\left(1+\frac{p}{100}\right)^{131}-1\right)}{\left(1+\frac{p}{100}\right)-1}$$

$$=\left(1+\frac{p}{100}\right)^{132}(150\ 000)-200\ 000\left(\frac{1}{p}+\frac{1}{100}\right)\left(\left(1+\frac{p}{100}\right)^{131}-1\right)$$

For $\left(1+\frac{p}{100}\right)^{132}(150\ 000)-200\ 000\left(\frac{1}{p}+\frac{1}{100}\right)\left(\left(1+\frac{p}{100}\right)^{131}-1\right)\geq 10\ 000$

Using GC, $p\geq 0.9795039$

The interest rate is 0.980%

Common Mistakes:

1. Deduct \$2000 at the end of first month before adding in interest which result value in (a) is \$148000

2. Did not add in interest at the end of Dec 2030 which result (b) is 1 instead of $\left(1+\frac{p}{100}\right)$

3. Did not add in the interest at the end of first month

4. Number of months from 01 Jan 2020 to 31 Dec 2030 counted wrongly.

5. Errors: $1+\frac{p}{100}=1.0p$ or $1.01p$

6. Use $1-\frac{p}{100}$ or $\frac{p}{100}$ instead of $1+\frac{p}{100}$

7. Wrong GP Sum formula.

8. Did not conclude and answer to the question on interest rate.

Section A: Pure Mathematics [40 marks]

1 The curve $y = \mathrm{f}(x)$, where $y < 0$, passes through the point $\left(0,\ -\frac{\sqrt{21}}{6}\right)$ and has gradient given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{4-3y^2}}{y}$.

(i) Find $\mathrm{f}(x)$. [4]

(ii) Find the exact coordinates of the point on the curve where the gradient is parallel to the x-axis. [2]

Suggested Solutions	**Marker's Comment**
(i) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{4-3y^2}}{y}$ $\int y\left(4-3y^2\right)^{-\frac{1}{2}}\,\mathrm{d}y = \int 1\,\mathrm{d}x$	Some students failed to see that the indefinite integral is of the standard form $\int \frac{f\,'(y)}{\sqrt{f(y)}}\,dy$.
$\frac{\left(4-3y^2\right)^{\frac{1}{2}}}{-6\times\frac{1}{2}} = x + C \Rightarrow -\frac{1}{3}\left(4-3y^2\right)^{\frac{1}{2}} = x + C$	Many numerical careless mistakes were spotted. Many students did not find the constant C using $-\frac{1}{3}\left(4-3y^2\right)^{\frac{1}{2}} = x + C$.
Curve passes through $\left(0,\ -\frac{\sqrt{21}}{6}\right)$: $-\frac{1}{3}\left(4-3\left(\frac{21}{36}\right)\right)^{\frac{1}{2}} = 0 + C \quad \Rightarrow \quad C = -\frac{1}{2}$ $-\frac{1}{3}\left(4-3y^2\right)^{\frac{1}{2}} = x - \frac{1}{2}$ $4-3y^2 = \left(\frac{3}{2}-3x\right)^2$ $y = -\sqrt{\frac{4}{3}-\frac{1}{3}\left(\frac{3}{2}-3x\right)^2}$ since $y < 0$ $\therefore\ \mathrm{f}(x) = -\sqrt{\frac{4}{3}-\frac{1}{3}\left(\frac{3}{2}-3x\right)^2}$ or $-\sqrt{\frac{4}{3}-\frac{3}{4}(1-2x)^2}$	Instead, they squared both sides and found the constant C from the equation such as : $4-3y^2 = (-3x+C)^2$, resulting in C having two values which is not correct ! Read the question carefully
(ii) Gradient is parallel to the x-axis, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{4-3y^2}}{y} = 0$	Given $y = \mathrm{f}(x) < 0$, quite a number of students did not give the required answer.

$\Rightarrow\ 4-3y^2=0$ $\Rightarrow\ y^2=\frac{4}{3}\ \Rightarrow\ y=-\frac{2}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}\ (\because\ y<0)$ Subst $y^2=\frac{4}{3}$ into $4-3y^2=\left(\frac{3}{2}-3x\right)^2$, $x=\frac{1}{2}$ Thus the coordinates are $\left(\frac{1}{2},\ -\frac{2\sqrt{3}}{3}\right)$	Read the question carefully Again, many students did not notice that $y<0$ is applied to this part of the question also

2 **(i)** Given that $y=\sec x+\tan x$, show that $\cos x\frac{\mathrm{d}y}{\mathrm{d}x}=y$.

By further differentiation of this result, find the Maclaurin expansion of $\sec x+\tan x$, in ascending powers of x, up to and including the term in x^3. [5]

Show that, to this degree of approximation, $\sec x+\tan x$ may be expressed in the form $a+b\ln(1-x)$, where a and b are constants to be determined. [2]

(ii) It is given that $\sec x+\tan x=\frac{3}{2}$, where $0\le x<\frac{\pi}{2}$.

(a) By using the identity $1+\tan^2 x=\sec^2 x$, or otherwise, show that $\tan x=\frac{5}{12}$. [2]

(b) Deduce, using the approximation obtained in **(i)**, that $\tan^{-1}\frac{5}{12}\approx m-\mathrm{e}^n$, where m and n are constants. [2]

Suggested Solution	**Markers' Comment**
(i) $y=\sec x+\tan x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\sec x\tan x+\sec^2 x$ $=\sec x(\tan x+\sec x)=y\sec x$ $\cos x\frac{\mathrm{d}y}{\mathrm{d}x}=y$ --- (1) Differentiating wrt x, $\cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\sin x\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}x}$ i.e. $\cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-(\sin x+1)\frac{\mathrm{d}y}{\mathrm{d}x}=0$ ---- (2)	Students should use implicit differentiation applied to (1) as the question stated "By further differentiation of this result". **Read Question Carefully.** Some students use quotient rule to differentiate $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{\cos x}$ which is a less efficient method.

$\cos x\frac{d^3y}{dx^3}-\sin x\frac{d^2y}{dx^2}-(\sin x+1)\frac{d^2y}{dx^2}-\cos x\frac{dy}{dx}=0$ i.e. $\cos x\frac{d^3y}{dx^3}-(2\sin x+1)\frac{d^2y}{dx^2}-\cos x\frac{dy}{dx}=0$ When $x=0,\ y=\sec 0+\tan 0=1$ $\frac{dy}{dx}=1,\ \frac{d^2y}{dx^2}=1,\ \frac{d^3y}{dx^3}=2$ $\therefore\ y=\sec x+\tan x$ $=1+(1)x+\frac{(1)}{2!}x^2+\frac{(2)}{3!}x^3+\ldots$ $=1+x+\frac{1}{2}x^2+\frac{1}{3}x^3+\ldots$	---- (3)
Using MF26, $\ln(1-x)=-x-\frac{1}{2}x^2-\frac{1}{3}x^3-\ldots$ $\sec x+\tan x$ $\approx 1+x+\frac{1}{2}x^2+\frac{1}{3}x^3$ from above $=1-\left(-x-\frac{1}{2}x^2-\frac{1}{3}x^3\right)\approx 1-\ln(1-x)$ $\therefore\ a=1,\ b=-1$	Some students fail to recognize that x is replaced by $-x$ using the following series expansion from MF26: $\ln(1+x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\ldots$ i.e. $\ln(1-x)=-x-\frac{1}{2}(-x)^2+\frac{1}{3}(-x)^3-\ldots$
(ii)(a) $\sec x+\tan x=\frac{3}{2}$, where $0\le x<\frac{\pi}{2}$ **Method 1** $\sec x=\frac{3}{2}-\tan x$ *Square both sides:* $\sec^2 x=\left(\frac{3}{2}-\tan x\right)^2$ $1+\tan^2 x=\frac{9}{4}-3\tan x+\tan^2 x$ $\tan x=\frac{5}{12}$	**Algebraic Manipulation Mistake** Many students made the algebraic mistake of squaring each term in the equation $\sec x+\tan x=\frac{3}{2}$, resulting in $\sec^2 x+\tan^2 x=\frac{9}{4}$ which is WRONG. They fail to realise that they should be expanding $(\sec x+\tan x)^2$ instead (See method 2)

Suggested Solution	Marker's Comment
Method 2 *Square both sides:* $\left(\sec x+\tan x\right)^2=\frac{9}{4}$ $\sec^2 x+2\sec x\tan x+\tan^2 x=\frac{9}{4}$ $1+\tan^2 x+2\sec x\tan x+\tan^2 x=\frac{9}{4}$ $2\tan x\left(\sec x+\tan x\right)=\frac{5}{4}$ Since $\sec x+\tan x=\frac{3}{2}$, $\therefore 2\left(\frac{3}{2}\right)\tan x=\frac{5}{4}\Rightarrow \tan x=\frac{5}{12}$	
(b) From **(a)**, $x=\tan^{-1}\frac{5}{12}$ Using **(i)**, $\sec x+\tan x=\frac{3}{2}\approx 1-\ln(1-x)$ $\ln(1-x)\approx-\frac{1}{2}$ $x\approx 1-\mathrm{e}^{-\frac{1}{2}}$ $\therefore \tan^{-1}\frac{5}{12}\approx 1-\mathrm{e}^{-\frac{1}{2}}$	Many fail to make the connection between the value of $\sec x+\tan x$ which is $\frac{3}{2}$ to $1-\ln(1-x)$. **See connection between different parts.**

3 A computer-controlled machine can be programmed to make cuts by entering the equation of the plane of the cut, and to drill holes by entering the equation of the drill line.

Figure 1 below shows a cuboid *OABCEFGH* (not drawn to scale) where $OA = 20$ cm, $OC = 30$ cm and $OE = 30$ cm. The point O is taken as the origin and unit vectors **i**, **j** and **k**, are taken along *OA*, *OC* and *OE* respectively. Take 1 unit to represent 1 cm.

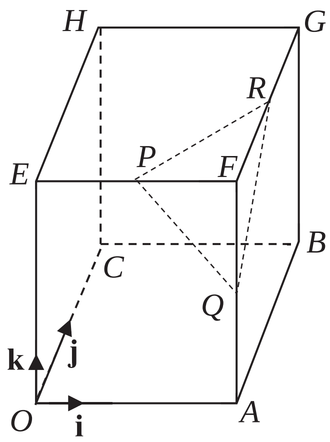


Figure 1

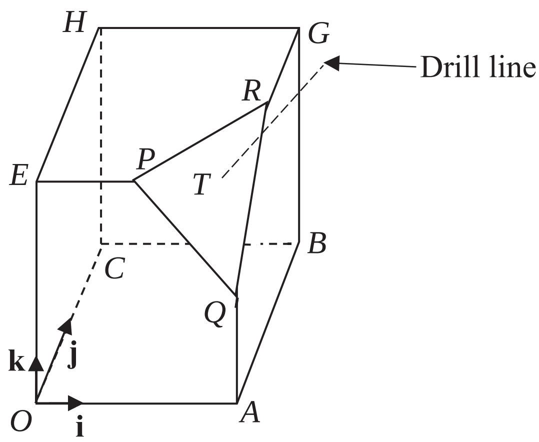


Figure 2

First, the machine makes a plane cut to the cuboid to remove the tetrahedron *PQRF*. The cut goes through the points *P*, *Q* and *R*, which are the midpoints of the sides *EF*, *FA* and *FG* respectively.

(i) Show that the cartesian equation of the plane p which contains the points P, Q and R is $3x - 2y + 2z = 90$. [3]

(ii) Find the acute angle between p and the base *OABC*. [2]

(iii) Two planes parallel to p are such that the distance from each plane to p is $\sqrt{17}$ cm. Find vector equations of the planes in scalar product form. [3]

Next, the machine drills a hole at a point T on p as shown in Figure 2. The drill line is perpendicular to p and passes through a point S with coordinates (5, 10, 15).

(iv) Find the coordinates of T. [3]

Suggested Solutions	**Marker's Comments**
(i) $\overrightarrow{OE} = \begin{pmatrix} 0 \\ 0 \\ 30 \end{pmatrix}$, $\overrightarrow{OF} = \begin{pmatrix} 20 \\ 0 \\ 30 \end{pmatrix}$, $\overrightarrow{OA} = \begin{pmatrix} 20 \\ 0 \\ 0 \end{pmatrix}$ By Ratio Theorem,	A small number of students found the cross product of vectors **OP** and **OQ** for the normal to the plane – which is conceptually wrong as we cannot assume the plane contains the O (the diagram is clear enough that O is not on the plane PQR)

$\overrightarrow{OP}=\frac{1}{2}\left(\overrightarrow{OE}+\overrightarrow{OF}\right)=\begin{pmatrix}10\\0\\30\end{pmatrix}$, $\overrightarrow{OQ}=\frac{1}{2}\left(\overrightarrow{OA}+\overrightarrow{OF}\right)=\begin{pmatrix}20\\0\\15\end{pmatrix}$, $\overrightarrow{OR}=\frac{1}{2}\left(\overrightarrow{OF}+\overrightarrow{OG}\right)=\begin{pmatrix}20\\15\\30\end{pmatrix}$ $\overrightarrow{PR}=\overrightarrow{OR}-\overrightarrow{OP}=\begin{pmatrix}20\\15\\30\end{pmatrix}-\begin{pmatrix}15\\0\\30\end{pmatrix}=\begin{pmatrix}10\\15\\0\end{pmatrix}=5\begin{pmatrix}2\\3\\0\end{pmatrix}$ $\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=\begin{pmatrix}20\\0\\15\end{pmatrix}-\begin{pmatrix}10\\0\\30\end{pmatrix}=\begin{pmatrix}10\\0\\-15\end{pmatrix}=5\begin{pmatrix}2\\0\\-3\end{pmatrix}$ $\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{pmatrix}2\\0\\-3\end{pmatrix}\times\begin{pmatrix}2\\3\\0\end{pmatrix}=\begin{pmatrix}9\\-6\\6\end{pmatrix}=3\begin{pmatrix}3\\-2\\2\end{pmatrix}$ Equation of p : $\mathbf{r}.\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}10\\0\\30\end{pmatrix}.\begin{pmatrix}3\\-2\\2\end{pmatrix}=90$ Cartesian equation of p : $3x-2y+2z=90$ (shown)	Many students found the normal using the cross product of vectors such as $\begin{pmatrix}20\\15\\30\end{pmatrix}$ and $\begin{pmatrix}20\\0\\15\end{pmatrix}$, they should just use the direction vectors that are parallel to the above vectors to make their working less tedious.
(ii) Let θ be the acute angle between the plane p and the base *ABCD*. $\cos\theta=\frac{\left\lvert\begin{pmatrix}3\\-2\\2\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}\right\rvert}{\sqrt{17}\sqrt{1}}$ $=\frac{2}{\sqrt{17}}$ $\theta=61.0^0$ or 1.06 rad	Quite a number of students did not notice that a normal to the base is $\begin{pmatrix}0\\0\\1\end{pmatrix}$ -they spent time using cross product to find this normal.

(iii) **<u>Method 1:</u>**

$$p:\ \mathbf{r}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}=90 \Rightarrow \mathbf{r}\bullet\frac{\begin{pmatrix}3\\-2\\2\end{pmatrix}}{\sqrt{17}}=\frac{90}{\sqrt{17}}$$

The 2 required planes are

$$\mathbf{r}\bullet\frac{\begin{pmatrix}3\\-2\\2\end{pmatrix}}{\sqrt{17}}=\frac{90}{\sqrt{17}}+\sqrt{17} \quad\text{and}\quad \mathbf{r}\bullet\frac{\begin{pmatrix}3\\-2\\2\end{pmatrix}}{\sqrt{17}}=\frac{90}{\sqrt{17}}-\sqrt{17}$$

$$\text{i.e. } \mathbf{r}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}=107 \quad\text{and}\quad \mathbf{r}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}=73$$

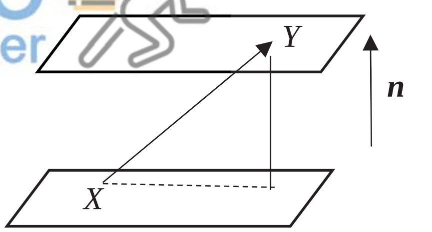
<u>Method 2:</u>

Let X be the point (30, 0, 0) on p.

Let equation of the required plane be $\mathbf{r}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}=b$ and let Y be the point $\left(\frac{b}{3},0,0\right)$ on it.

Distance between 2 planes = Length of projection of $\overrightarrow{XY}$ onto normal $=\sqrt{17}$

$$\frac{\left|\overrightarrow{XY}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}\right|}{\sqrt{17}}=\frac{\left|\begin{pmatrix}\frac{b}{3}-30\\0\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}\right|}{\sqrt{17}}=\sqrt{17}$$



$$|b-90|=17$$

$b-90=17$ or $b-90=-17$

$b=107$ or $b=73$

The 2 planes are $\mathbf{r}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}=107$ and $\mathbf{r}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}=73$

More than 50% of the students were not able to do this question.

They didn't know or understand that the distance between the origin and the plane, p, can be obtained by

$$\mathbf{r}\bullet\frac{\begin{pmatrix}3\\-2\\2\end{pmatrix}}{\sqrt{17}}=\frac{90}{\sqrt{17}}$$

Drawing appropriate diagram should help them to visualise that there are two possible answers

Students who got the answers usually did by Method 1

Students who attempted the question by Method 2 did not do well in general.

(iv) Drill line: $\mathbf{r}=\begin{pmatrix}5\\10\\15\end{pmatrix}+\alpha\begin{pmatrix}3\\-2\\2\end{pmatrix}$, $\alpha\in\mathbb{R}$ At T, $\begin{pmatrix}5+3\alpha\\10-2\alpha\\15+2\alpha\end{pmatrix}\bullet\begin{pmatrix}3\\-2\\2\end{pmatrix}=90$ $15+9\alpha-20+4\alpha+30+4\alpha=90$ $\Rightarrow\ \alpha=\frac{65}{17}$ $\therefore\ \overrightarrow{OT}=\frac{1}{17}\begin{pmatrix}280\\40\\385\end{pmatrix}$ The **coordinates** of T is $\left(\frac{280}{17},\frac{40}{17},\frac{385}{17}\right)$	**Wrong presentation of a vector equation of the drill line** Quite a number of students had the wrong concept of writing $\mathbf{ST}=\begin{pmatrix}5+3\alpha\\10-2\alpha\\15+2\alpha\end{pmatrix}$ instead of $\mathbf{OT}=\begin{pmatrix}5+3\alpha\\10-2\alpha\\15+2\alpha\end{pmatrix}$ **Read the question carefully** The question asks for coordinates of the T. Many students didn't answer to the question.

4 **(a)** Solve the simultaneous equations

$$z+\mathrm{i}=|w|,$$

$$\frac{w-\mathrm{i}}{z-1}=2,$$

giving your answers in cartesian form $x+\mathrm{i}y$. [5]

(b) The complex number u has modulus k and argument α, where $0<\alpha<\frac{\pi}{6}$ and is represented by the point P on an Argand diagram. The line l passes through the origin O and makes an angle $\frac{\pi}{6}$ with the positive real axis. The point Q represents the complex number v such that Q is the reflection of P in l.

(i) Indicate the points P and Q and the line l on an Argand diagram. State the modulus of v and show that the argument of v is $\frac{\pi}{3}-\alpha$. [4]

(ii) Find $\sqrt{uv}$ in exponential form. [2]

(iii) Point R represents the complex number $\sqrt{uv}$. Indicate R on the Argand diagram in part **(i)**. Identify the geometrical shape of $OPRQ$. [2]

[Solution]

(a)

For **complex number** w, $|w|$ should be interpreted as $\sqrt{x^2+y^2}$ if we let $w = x + yi$.
Students should not treat w like a **real number** by **wrongly** deducing that
since $|w| = z + \mathrm{i} \;\Rightarrow\; w = \pm(z + \mathrm{i})$. This result is incorrect for complex numbers.

$$z + \mathrm{i} = |w| \quad \Rightarrow \quad z = |w| - \mathrm{i} \quad \text{--- (1)}$$

$$\frac{w-\mathrm{i}}{z-1} = 2 \quad \Rightarrow \quad w - \mathrm{i} = 2z - 2 \quad \text{--- (2)}$$

Subst (1) into (2):

$$w - \mathrm{i} = 2(|w| - \mathrm{i}) - 2$$

$$w = 2|w| - 2 - \mathrm{i}$$

Substitution or elimination technique for solving simultaneous equations.

Let $w = x + y\,\mathrm{i}$, where $x, y \in \mathbb{R}$

$$x + y\,\mathrm{i} = 2\sqrt{x^2+y^2} - 2 - \mathrm{i}$$

Comparing real and imaginary parts:

Technique of letting complex no. be $x + yi$, then compare real and imaginary parts.

Imaginary: $\quad y\mathrm{i} = -\mathrm{i} \quad \Rightarrow \quad y = -1$

Real: $\quad x = 2\sqrt{x^2+1} - 2$

$$(x+2)^2 = 2^2(x^2+1)$$

$$3x^2 - 4x = 0$$

$$x(3x-4) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{4}{3}$$

Alternatively:

From GC, $x = 0$ or $x = \frac{4}{3}$

x can be 0 and answer should not be ignored or rejected.

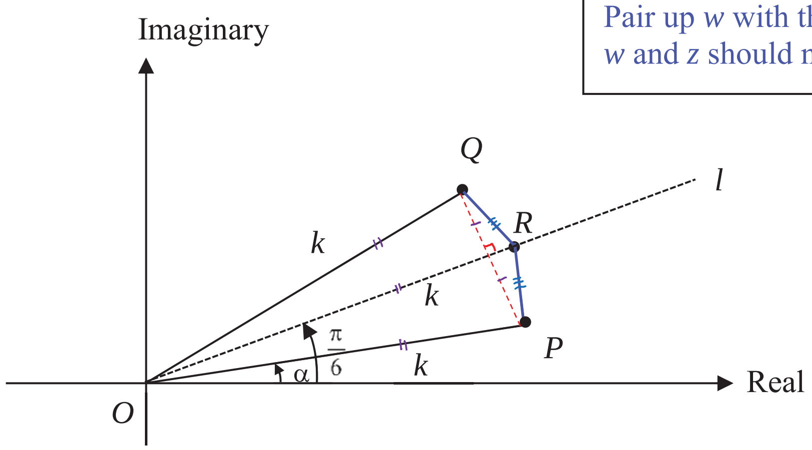
$$w = -\mathrm{i} \quad \text{or} \quad w = \frac{4}{3} - \mathrm{i}$$

$$\Rightarrow \quad |w| = 1 \quad \text{or} \quad |w| = \sqrt{\left(\frac{4}{3}\right)^2 + 1^2} = \frac{5}{3}$$

$$\Rightarrow \quad w = -\mathrm{i},\; z = 1 - \mathrm{i} \quad \text{or} \quad w = \frac{4}{3} - \mathrm{i},\; z = \frac{5}{3} - \mathrm{i}$$

(b)

Pair up w with the corresponding z.
w and z should not be



(i) By symmetry, $|v| = OQ = OP = |u| = k$

Also, $\angle QOR = \angle ROP = \dfrac{\pi}{6} - \alpha$

$\therefore \ \arg v = \dfrac{\pi}{6} + \left(\dfrac{\pi}{6} - \alpha\right) = \dfrac{\pi}{3} - \alpha$

(ii)
$$\begin{aligned}
\sqrt{uv} &= \sqrt{\left(k\,\mathrm{e}^{\alpha\,\mathrm{i}}\right)\left(k\,\mathrm{e}^{\left(\frac{\pi}{3}-\alpha\right)\mathrm{i}}\right)} \\
&= \sqrt{k^2 \ \mathrm{e}^{\alpha\mathrm{i}+\left(\frac{\pi}{3}-\alpha\right)\mathrm{i}}} \\
&= \sqrt{k^2 \ \mathrm{e}^{\frac{\pi}{3}\mathrm{i}}} \\
&= k\,\mathrm{e}^{\frac{\pi}{6}\mathrm{i}}
\end{aligned}$$

(iii) Figure $OPRQ$ forms a kite.

Section B: Statistics [60 marks]

5 A researcher collects data on the number of passengers, x, on a public bus during peak hours in the Circuit Breaker period. He summarises his 18 readings as follows:

$$\sum x = 207, \qquad \sum x^2 = 2515.$$

(i) Find unbiased estimates of the population mean and variance of the number of passengers on a public bus during peak hours in the Circuit Breaker period. [3]

(ii) Taking the population mean to be 11.7, find the approximate probability that the mean number of passengers on 100 public buses during peak hours in the Circuit Breaker period is greater than 12. [3]

[Solution]

(i) Let X be the number of passengers on a public bus during peak hours in the Circuit Breaker period.

An unbiased estimate of the population mean is $\bar{x} = \dfrac{207}{18} = 11.5$

An unbiased estimate of the population variance is

$$s^2 = \frac{1}{17}\left(2515 - \frac{207^2}{18}\right) = \frac{269}{34} \quad \text{or} \quad 7.91$$

Formula found in MF26:
$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$$

(ii)

Students need to spell out "CLT" in full and also state the reason why Central limit Theorem can be applied (n is large).

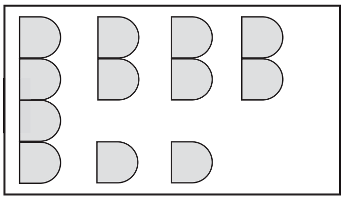
Since $n = 100$ is large, by Central Limit Theorem,

$$\bar{X} \sim \mathrm{N}\left(11.7, \frac{269}{3400}\right) \text{ approximately}$$

$$\mathrm{P}\left(\bar{X} > 12\right) = 0.143 \text{ (3 s.f.)}$$

$\bar{X}$ can be used to denote "mean number" since x is defined in (i) of question. No need to define a new variable.
Since σ^2 is unknown, we can use the unbiased estimate s^2 found in (i) to find $\mathrm{Var}(\bar{X})$.

6

Back  Front

The diagram shows the seating plan for passengers in a minibus, which has 12 seats arranged in 4 rows. The back row has 4 seats and the front row has 2 seats on one side; the middle 2 rows have 2 seats on 1 side and 1 seat on the other side. 10 passengers get on the minibus.

(i) Find the number of different seating arrangements for the 10 passengers. [1]

(ii) Find the number of different seating arrangements if 2 particular passengers must not sit together. [3]

(iii) Find the number of different seating arrangements if there are 2 married couples among the 10 passengers and each couple must sit together. [4]

[Solution]

(i) No. of arrangements = ${}^{12}P_{10} = 239500800$ $\left(\text{or } \dfrac{12!}{2!}\right)$	☺
(ii) No. of arrangements where 2 particular passengers must not sit together $= 239500800 - {}^{6}C_1 \times 2! \times {}^{10}P_8$ $= 239500800 - 21772800 = 217728000$ $\boxed{\left({}^{10}P_8 \text{ or } \dfrac{10!}{2!}\right)}$ <u>Explanation</u> ${}^{6}C_1$: number of ways which the 2 particular passengers can seat 2!: the 2 particular passengers can swop places ${}^{10}P_8$: arrange the rest of the 8 passengers	A more efficient approach will be the complement method.
(iii) Method 1 Case 1: One of the couple seats at the 2 middle seats at the back row Number of ways = ${}^{3}C_1 \times 2! \times 2! \times {}^{8}P_6 \times 2! = 483840$ $\boxed{\left({}^{8}P_6 \text{ or } \dfrac{8!}{2!}\right)}$ <u>Explanation</u> ${}^{3}C_1$: number of ways which the other couple can seat 2!2!: the 2 pairs of couples can arrange within themselves ${}^{8}P_6$: number of ways the rest of the 6 people can arrange 2!: the 2 couples can swop seats	Make sure you list your cases clearly, and explain what you have written

Case 2: None of the couples seat at the 2 middle seats at the back row Number of ways = ${}^{5}C_{2} \times 2! \times 2! \times 2! \times {}^{8}P_{6} = 1612800$ Explanation ${}^{5}C_{2}$: number of ways which the 2 couples can seat 2!: the 2 couples can swop seats $\}\ {}^{5}C_{1} \times {}^{4}C_{1}$ 2!2!: the 2 pairs of couples can arrange within themselves ${}^{8}P_{6}$: number of ways the rest of the 6 people can arrange **Method 2** Case 1: 2 couples sit in the back row No. of arrangements = ${}^{2}C_{1} \times 2! \times 2! \times {}^{8}P_{6} = 161280$ Case 2: 1 couple sits in the back row No. of arrangements = ${}^{2}C_{1} \times {}^{3}C_{1} \times 2! \times {}^{3}C_{1} \times 2! \times {}^{8}P_{6} = 1451520$ Case 3: No couple sits in the back row No. of arrangements = ${}^{3}C_{2} \times 2! \times 2! \times {}^{8}P_{6} \times 2! = 483840$ Total no. of arrangements = 2096640	

7 A high jumper estimates the probabilities that she will be able to clear the bar at various heights, based on her experience in training. These are given in the table below.

Height	Probability of success at each attempt
1.6 m	1
1.7 m	0.5
1.8 m	0.2
1.9 m	0

In a competition, she is allowed up to three attempts to clear the bar at each height. If she clears the bar, the bar is raised by 10 cm and she is allowed three attempts at the new height; and so on. It is assumed that she starts at height 1.60 m and the result of each attempt is independent of all her previous attempts.

(i) Show that the probability she will clear the bar at 1.7 m is 0.875. [2]

(ii) Given that she has cleared the bar at 1.7 m, find the probability that she will not clear the bar at 1.8 m. [1]

Hence find the probabilities that, in the competition, the greatest height she will clear at the bar is

(a) 1.6 m, [1]
(b) 1.7 m, [1]
(c) 1.8 m. [1]

The final result of her competition is the greatest height that she has cleared.

In a particular month, she participated in 3 competitions with the same rules.

(iii) Given that her worst result in the 3 competitions is 1.7 m, find the probability that she clears the height at 1.8 in exactly one competition. [3]

[Solutions]

(i) P(clears bar at 1.7 m) $= 0.5 + 0.5\times 0.5 + 0.5^2\times 0.5 = 0.875$ **OR** P(clears bar at 1.7 m) $= 1-0.5^3 = 0.875$	**Read the question carefully** Some students incorrectly identify this as a Binomial random variable. In fact the number of attempts can be 1, 2 or 3 depending on when the high jumper succeeds in her attempt.
(ii) P(does not clear bar at 1.8 m | clears bar at 1.7 m) $= 0.8^3 = 0.512$ (since the results of all the attempts are independent of one another)	Students who solve this using the usual method can also get it correct,but spend more time doing it.

(a) P(greatest height clear is 1.6 m) $= 1 - 0.875 = 0.125$ **(b)** P(greatest height clear is 1.7 m) $= 0.875 \times 0.512 = 0.448$ (c) P(greatest height clear is 1.8 m) $= 0.875 \times (1 - 0.512) = 0.427$	"**Find**": Working must be shown, either directly or by referencing a previous part. Simply writing down the answer will not suffice. Some students also assume that these are conditional probability questions.

(iii) P(clears 1.8 m in exactly 1 competition | worst result is 1.7 m in 3 competitions) $= \dfrac{\mathrm{P}(1.7,1.7,1.8)}{\mathrm{P}(1.7,1.8,1.8)+\mathrm{P}(1.7,1.7,1.8)+\mathrm{P}(1.7,1.7,1.7)}$ $= \dfrac{0.448^2 \times 0.427 \times 3}{0.448 \times 0.427^2 \times 3 \ + \ 0.448^2 \times 0.427 \times 3 \ + \ 0.448^3}$ $= \dfrac{0.257101824}{0.592067392} = 0.434$	**Read the question carefully** Many students did not notice, or ignored, the condition given in the question. Of those who realised that this is a conditional probability question, either: (a) cases were missing in the denominator, or (b) the number of permutations within each case was not considered.

8 In a large population of Bichon Frises (a breed of dog), on average, one in three of them have blood type *Dal*–. Specimens of blood from the first five of this breed of dog attending a local veterinary clinic are to be tested. It can be assumed that these five Bichon Frises are a random sample from the population.

(i) State, in context, two assumptions needed for the number of Bichon Frises in the sample who are found to have blood type *Dal*– to be well modelled by a binomial distribution. [2]

Assume now that the number of Bichon Frises in the sample who are found to have blood type *Dal*– has a binomial distribution.

(ii) Find the probability that more than two of the Bichon Frises in the sample are found to have blood type *Dal*–. [2]

(iii) Three such samples of five Bichon Frises are taken. Find the probability that one of these three samples has exactly one Bichon Frise with blood type *Dal*–, another has exactly two Bichon Frises with blood type *Dal*–, and the remaining sample has more than two Bichon Frises with blood type *Dal*–. [3]

(iv) N such samples of five Bichon Frises are taken. Find the least value of N such that the probability that the number of these samples that contain two or fewer Bichon Frises with blood type *Dal*– will be at least 15 is more than 90%. [3]

Suggested Solution	Markers' Comment
(i) For any Bichon Frises, the **probability** of having blood type *Dal*– is the **same**. The **blood type** of a Bichon Frises is **independent** of the blood type of any Bichon Frises.	The following "there are two outcomes, a randomly chosen Bichon Frises has either blood type *Dal*- or not blood type *Dal*-." is NOT an assumption. It is already implied by the given random variable "the number of Bichon Frises in the sample who are found to have blood type *Dal*–" It is wrong to say probability of having blood type Dal- is independent of one another as it is NOT the probability that is independent but the blood type of Bichon Frises. There are many students who assume that the selection of Bichon Frises need to be independent but question already mentioned that the five Bichon Frises forms a random sample which already implied that selection is independent.

<table>
<tr>
<td>(ii) Let X be the number of Bichon Frises (out of 5) who has blood type Dal–.
$X \sim \mathrm{B}(5, \frac{1}{3})$
$\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \le 2) = 0.20988 \approx 0.210$ (3 sf)</td>
<td>There are still quite a number of students who are unable to find $\mathrm{P}(X > 2)$ using the correct “binomcdf”. Common mistake include $\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \le 1)$.

Read Question Carefully:
Some students find $\mathrm{P}(X \ge 2)$ instead. Need to note that “more than 2” exclude 2.</td>
</tr>
<tr>
<td>(iii) Required probability
$= \mathrm{P}(X = 1)\,\mathrm{P}(X = 2)\,\mathrm{P}(X > 2) \times 3!$
$= 0.136$ (3 sf)</td>
<td>Many students did not multiply by 3! Or they multiply wrongly by 3 instead.

There are still some students who are confused on when to apply multiplication principle and when to apply addition principle.</td>
</tr>
<tr>
<td>(iv) Let W be the number of these samples (out of N) that contain two or fewer Bichon Frises with blood type Dal–.
$W \sim \mathrm{B}(N, \mathrm{P}(X \le 2))$ i.e $W \sim \mathrm{B}(N, 0.790123)$

$\mathrm{P}(W \ge 15) = 1 - \mathrm{P}(W \le 14) > 0.90$
or $\mathrm{P}(W \le 14) < 0.01$

Using GC:
<table>
<tr><th>N</th><th>$\mathrm{P}(W \ge 15)$</th></tr>
<tr><td>21</td><td>$0.8675 < 0.90$</td></tr>
<tr><td>22</td><td>$0.9284 > 0.90$</td></tr>
</table>
Therefore, the least value of N is 22.</td>
<td>Correct definition of Random Variable is needed.
There are some students who struggled to define accurately the random variable W. They thought that W is the number of Bichon Frises instead.

Higher Accuracy of working values needed:
Quite a number of students only use 0.790 in the computation of probabilities leading to poor accuracy.

Other mistakes:
$\mathrm{P}(W \ge 15) = 1 - \mathrm{P}(W \le 15)$
$\mathrm{P}(W > 15) = 1 - \mathrm{P}(W \le 15)$</td>
</tr>
</table>

9 A biased cubical die is thrown onto a horizontal table. The random variable X is the number on the face in contact with the table. The probability distribution of X is given by

$$\mathrm{P}(X = x) = k(x - 1)!, \text{ where } x = 1, 2, 3, 4, 5, 6,$$

and k is a constant.

(i) Find the value of k. [2]

(ii) Show that $\mathrm{E}(X) = \dfrac{873}{154}$ and hence, find $\mathrm{Var}(X)$. [3]

In a game, Antonio pays a stake of \$32 and throws two such dice onto a horizontal table. The amount Antonio receives is \$$Y$ where Y is the sum of the numbers on the faces (of both dice) **not in contact** with the table.

(iii) Find the probability that he receives more than \$38 in a particular game. [4]

(iv) Find Antonio's expected gain. [3]

[Solution]

(i)

x	1	2	3	4	5	6
$\mathrm{P}(X = x)$	k	k	$k(2!)$	$k(3!)$	$k(4!)$	$k(5!)$

$$\sum_{\text{all } x} \mathrm{P}(X = x) = k\left(1+1+2!+3!+4!+5!\right) = 1 \quad \Rightarrow \quad k = \frac{1}{154}$$

(ii) $\mathrm{E}(X) = \displaystyle\sum_{\text{all } x} x\mathrm{P}(X = x) = k\left(1!+2!+3!+4!+5!+6!\right) = \frac{873}{154}$

$$\mathrm{E}(X^2) = \sum_{\text{all } x} x^2\mathrm{P}(X = x) = k\left(1\times1!+2\times2!+3\times3!+4\times4!+5\times5!+6\times6!\right) = \frac{5039}{156}$$

$$\mathrm{Var}(X) = \mathrm{E}(X^2) - \left(\mathrm{E}(X)\right)^2 = \frac{5039}{154} - \left(\frac{873}{154}\right)^2 = 0.585 \text{ (3 sf)}$$

(iii) Let Y be the amount received by Antonio in a game.

Method 1

$Y = (21 - X_1) + (21 - X_2) = 42 - (X_1 + X_2)$

$\mathrm{P}(Y > 38) = \mathrm{P}(42 - (X_1 + X_2) > 38)$

$= \mathrm{P}(X_1 + X_2 < 4) = \mathrm{P}(\ (1,1), (1,2), (2,1)\ )$

$= \left(\dfrac{1}{154}\right)^2 + 2\left(\dfrac{1}{154}\right)^2$

$= 0.000126$ (3 sf)

(iv) $\mathrm{E}(Y) = 42 - 2\mathrm{E}(X) = \$30.66$

$\therefore$ Antonio's expected gain $= \$30.66 - \$32 = -\$1.34$

1. A lot of students find the probability distribution of Y and use it to solve part (iii) and part (iv). This working is tedious, time consuming and frequently result in inaccurate answer due to careless mistakes.

2. Some students $Y = X_1 + X_2$ or write $X_1 + X_2$ as $2X$.

3. Other common errors includes:
$\mathrm{P}(X_1 + X_2 < 4)$

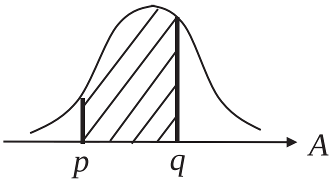
$= 2\left(\dfrac{1}{154}\right)^2 + 2\left(\dfrac{1}{154}\right)^2$ - Thinking need to x2 for P(1,1)

or

$= \left(\dfrac{1}{154}\right)^2 + \left(\dfrac{1}{154}\right)^2$ - Did not consider P(1,2) & P(2,1)

4. Some students find the expected gain as $32 - \mathrm{E}(Y)$ which likely because students think 'gain' mean positive value..

10 **(a)** The diagram below shows the probability density function of a normal random variable A with mean 50 and variance 36.



The values p and q are such that $P(p < A < q) = \frac{1}{3}$.

Giving clear reasons, find the value of p such that $q - p$ is minimum. [2]

(b) A supermarket sells two varieties of watermelon. The weight of a Green Giant watermelon follows a normal distribution with mean 3.5 kg and variance 0.2 kg^2, while the weight of an Organic Emerald watermelon follows a normal distribution with mean 3.1 kg and variance 0.3 kg^2.

(i) Find the probability that 4 Green Giant watermelons have a greater total weight than 5 times the weight of an Organic Emerald watermelon. [3]

(ii) A packing basket can carry up to 50 kg of fruit. If the basket can carry n Green Giant watermelons at least 90% of the time, show that n satisfies the inequality

$$50 - 3.5n - 1.28155\sqrt{0.2n} \geq 0$$

and hence find the greatest value of n. [4]

Green Giant watermelons are sold for \$2.20 per kg and Organic Emerald watermelons are sold for \$2.60 per kg.

(iii) Find the probability that an Organic Emerald watermelon costs more than a Green Giant watermelon. [3]

(iv) Green Giant watermelons are displayed in the store in crates of 16 fruit each. Find the probability that, in a crate, at least 10 of the watermelons each cost at least \$8. [3]

[Solution]

(a) Due to the **symmetrical bell-shaped curve**, minimum $q - p$ occurs when **p and q are symmetrical about the mean 50.**

Thus $P(A < p) = \frac{1}{3}$

From GC, $p = 47.4$ (3sf)

No need to find q or $q - p$

(b) (i) Let G be the weight of <u>a</u> Green Giant watermelon. $G \sim N(3.5, \underline{0.2})$

Let E be the weight of <u>an</u> Organic Emerald watermelon. $E \sim N(3.1, \underline{0.3})$

Let $S = G_1 + G_2 + G_3 + G_4 - 5E$

$E(S) = 4E(G) - 5E(E) = -1.5$

$Var(S) = 4Var(G) + 5^2\ Var(E) = 8.3$

$S \sim N(-1.5, 8.3)$

$P(S > 0) = 0.301$ (3 s.f.)

Define the random variables!!!
Avoid using letter O (looks like 0)
or letter Z (rep standard normal rv)

$G_1 + G_2 + G_3 + G_4 \neq 4G$

(i) Let T be the total weight of n Green Giant watermelons in a basket.

$T = G_1 + G_2 + G_3 + \ldots + G_n$

$G_1 + G_2 + G_3 + \ldots + G_n \neq nG$

$T \sim \mathrm{N}(3.5n, 0.2n)$

$\mathrm{P}(T \le 50) \ge 0.9$

$$\mathrm{P}\left(Z \le \frac{50-3.5n}{\sqrt{0.2n}}\right) \ge 0.9$$

From GC, $\dfrac{50-3.5n}{\sqrt{0.2n}} \ge 1.28155$

$\Rightarrow 50-3.5n-1.28155\sqrt{0.2n} \ge 0$

From GC (table),

n	$50-3.5n-1.28155\sqrt{0.2n}$
13	$2.4336 \ge 0$
14	$-1.144 < 0$

Hence greatest $n = 13$

(iii) Let $D = 2.6E - 2.2G$

$\mathrm{E}(D) = 2.6\mathrm{E}(E) - 2.2\mathrm{E}(G) = 0.36$

$\mathrm{Var}(D) = \mathbf{2.6^2}\ \mathrm{Var}(E) + \mathbf{2.2^2}\ \mathrm{Var}(G) = 2.996$

$D \sim \mathrm{N}(0.36, 2.996)$

$\mathrm{P}(D > 0) = 0.582$

Important result:
$\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$
if X and Y are independent.
Extension: Give one assumption in your calculations in part **(iii)**.

(iv) $\mathrm{P}(2.2G \ge 8) = \mathrm{P}\left(G \ge \dfrac{8}{2.2}\right) = 0.38021$ (5 s.f.)

Let X be the number of Green Giant watermelons (out of 16) that cost at least \$8.

$X \sim \mathbf{B}(16, \mathbf{0.38021})$

$\mathrm{P}(X \ge 10) = 1 - \mathrm{P}(X \le 9)$

$= 0.0412$ (3 s.f.)

Use at least 5 s.f. in intermediate answers and give final answer to 3 s.f.