

## 2.3 SOLVING SIMULTANEOUS LINEAR EQUATIONS BY ALGEBRAIC METHODS

Key Skills Checklist	Confidence Level					Related Questions
	1	2	3	4	5	
Solve simultaneous linear equations by the substitution method						1
Solve simultaneous linear equations by the elimination method						2
Solve simultaneous linear equations by either of the algebraic methods						3, 4, 5, 6, 7

### WORD TOOLBOX

<p><b>substitution method</b></p>	<p>A pair of linear equations in two variables <math>x</math> and <math>y</math> is given:</p> $y = x + 3$ $y + 2 = 2x$ <p>Using the <b>substitution method</b> to solve the simultaneous equations,</p> $y = x + 3 \dots\dots\dots \textcircled{1}$ $2x = y + 2 \dots\dots\dots \textcircled{2}$ <p>Substituting <math>\textcircled{1}</math> into <math>\textcircled{2}</math>,</p> $2x = (x + 3) + 2$ $2x = x + 5$ $x = 5$ <p>Substituting <math>x = 5</math> into <math>\textcircled{1}</math>,</p> $y = 5 + 3$ $= 8$ <p><math>\therefore</math> the solution is <math>x = 5</math> and <math>y = 8</math>.</p>
<p><b>elimination method</b></p>	<p>A pair of linear equations in two variables <math>x</math> and <math>y</math> is given:</p> $y + x = 1$ $y - x = 3$ <p>Using the <b>elimination method</b> to solve the simultaneous equations,</p> $y + x = 1 \dots\dots\dots \textcircled{1}$ $y - x = 3 \dots\dots\dots \textcircled{2}$ <p>Adding equations <math>\textcircled{1}</math> and <math>\textcircled{2}</math>, we eliminate the variable <math>x</math>.</p> $(y + x) + (y - x) = 1 + 3$ $2y = 4$ $y = 2$ <p>Substituting <math>y = 2</math> into <math>\textcircled{1}</math>,</p> $2 + x = 1$ $x = -1$ <p><math>\therefore</math> the solution is <math>x = -1</math> and <math>y = 2</math>.</p>

**BASIC MASTERY**

1 Solve the following simultaneous equations by the substitution method.

(a)  $y = x + 5$  and  $5x + y = 17$

(b)  $x = 7 + 2y$  and  $3x - 4y = 15$

(c)  $y = 3x - 4$  and  $5x - 3y = 20$

(d)  $x + 2y = 9$  and  $3x - 4y = -3$

(e)  $x - 3y = 25$  and  $x + 2y = -5$

(f)  $y + 7x = -15$  and  $5x + 4y = 9$

2 Solve the following simultaneous equations by the elimination method.

(a)  $x + y = 5$  and  $x + 4y = 14$

(b)  $-3x + 4y = 20$  and  $3x - 3y = -12$

(c)  $x - 3y = -16$  and  $5x + 3y = 46$

(d)  $x + 7y = -29$  and  $4x - 5y = 16$

(e)  $2x + 5y = 7$  and  $6x + 11y = 9$

(f)  $13x - 6y = 32$  and  $5x + 2y = 8$

3 Solve the following simultaneous equations.

(a)  $5y + 6x = 0$  and  $2x - 3y = 56$

(b)  $x - 2y = 11$  and  $3x - 4y = 21$

(c)  $3x + 4y = -18$  and  $5x - 2y = -4$

(d)  $9x - 4y = 10$  and  $3x + y = -13$

4 Solve the following simultaneous equations.

(a)  $8x - 3y - 12 = 0$  and  $2x + 5y - 26 = 0$

(b)  $4x + 5y = -9$  and  $-3x + 7y = -47$

5 Solve the following simultaneous equations.

(a)  $y = 5.7 - 3.5x$  and  $4y + 2.5x = 11.3$

(b)  $\frac{x}{3} - \frac{2y}{3} = -5$  and  $\frac{x}{2} + \frac{y}{3} = \frac{1}{2}$

**ADVANCED**

6 Given  $\sqrt{2x - 7y + 1} + (x + 2y + 6)^2 = 0$ , find the values of  $x$  and  $y$ .

7 Solve the simultaneous equations  $\frac{5}{x} + \frac{3}{y} = 4$  and  $\frac{2}{x} - \frac{1}{y} = \frac{2}{15}$ .

## 2.4 APPLICATIONS OF SIMULTANEOUS EQUATIONS

Key Skills Checklist	Confidence Level					Related Questions
	1	2	3	4	5	
Formulate a pair of simultaneous equations to solve word problems						1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

### WORD TOOLBOX

#### formulate

Given the word problem:

An apple costs \$ $x$  and an orange costs \$ $y$ .

3 apples and 3 oranges cost \$3.15 while 4 apples and 5 oranges cost \$4.80.

Find the cost of an apple and the cost of an orange.

To solve the word problem, we can **formulate** a pair of simultaneous equations:

$$3x + 3y = 3.15$$

$$4x + 5y = 4.80$$

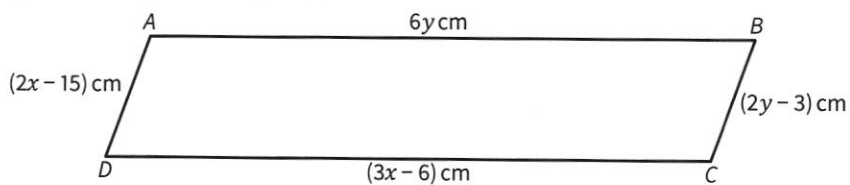
The solution of the simultaneous equations will be the cost of an apple and an orange respectively.

**BASIC MASTERY**

- 1 The sum of two numbers is 88 and their difference is 24.  
Let the larger number be  $x$  and the smaller number be  $y$ .
- (a) Using the given information, write down two simultaneous equations.

(b) Hence find the two numbers.

- 2 In the diagram,  $ABCD$  is a parallelogram.

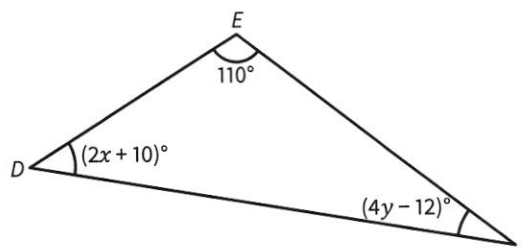
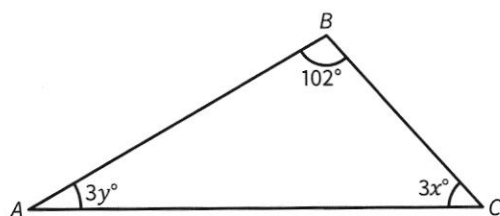


- (a) Write down two simultaneous equations and show that they can be reduced to  $x - y = 6$  and  $x - 2y = 2$ .

(b) Solve the simultaneous equations to find the values of  $x$  and  $y$ .



- 3 The diagrams show two distinct triangles,  $ABC$  and  $DEF$ .



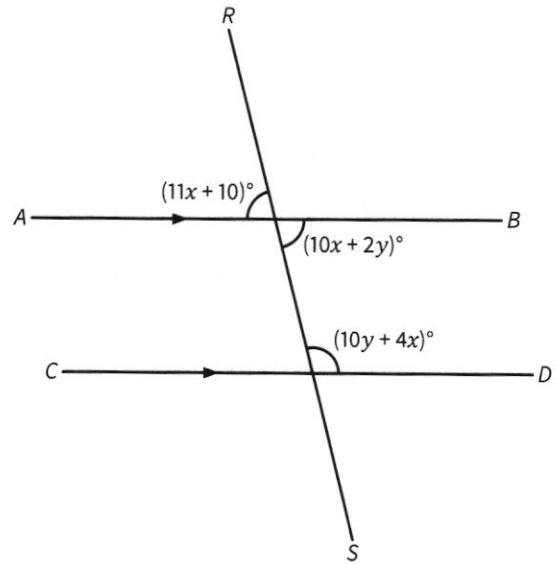
- (a) Write down two simultaneous equations and show that they can be reduced to  $x + y = 26$  and  $x + 2y = 36$ .

- (b) Solve the simultaneous equations to find the values of  $x$  and  $y$ .

#### INTERMEDIATE

- 4 An apple costs  $\$x$  and a pear costs  $\$(2y)$ .  
4 apples and 5 pears cost  $\$5.90$  while 7 apples and 3 pears cost  $\$6.30$ .  
Find the cost of an apple and the cost of a pear.

- 5 In the diagram, the line segment  $RS$  is the transversal of parallel lines  $AB$  and  $CD$ . Find the value of  $x$  and of  $y$ .




- 6 The ratio  $x : y$  is  $4 : 3$ .  
When the value of  $y$  decreases by 9, the new ratio is  $2 : 1$ .  
Find the value of  $x$  and of  $y$ .

- 7 Li Ting bought 25 stamps consisting of 20-cent and 50-cent stamps.  
The total cost of the stamps was \$9.20.  
Find the number of each type of stamp she bought.

- 8 The value of 40% of the sum of two integers  $x$  and  $y$  is 28.  
Given that  $y$  is 6 times of  $x$ , find the two integers  $x$  and  $y$ .

- 9 There are  $x$  boys and  $y$  girls in a guitar club. If 13 boys and 4 girls join the guitar club, the number of boys will be 50% more than the number of girls. If 1 boy joins and 12 girls leave the club, the number of boys will be twice the number of girls.  
Find the original number of boys and girls in the guitar club.

- 10 The tens digit of a two-digit number is half the ones digit.  
 When the digits are reversed, the number increases by 36.  
Find the number.

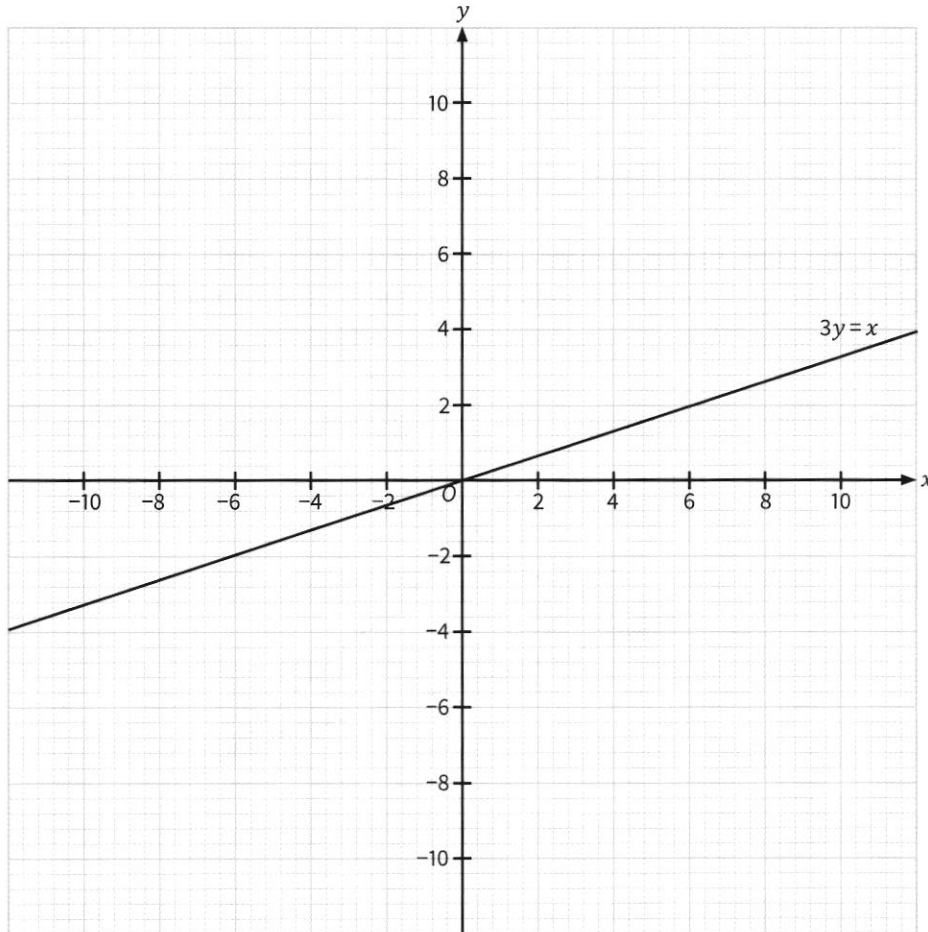
**ADVANCED**

- 11 Johnson and Siti are 100 m apart. Johnson walks at a speed of  $x$  m/s and Siti walks at a speed of  $y$  m/s. They will meet in 10 seconds if they walk towards each other.  
If both of them walk towards the right instead, Johnson will catch up with Siti in 50 seconds. Find the respective walking speeds of Johnson and Siti.
- 12 30 pens and 30 pencils were shared between Anwar and Brittney. Anwar received 9 more pens than pencils. The number of pencils Brittney received was twice the number of pens she received. Find the number of pens Anwar received.
- 13 When 3 is added to the numerator and denominator of a fraction, the new fraction is equivalent to  $\frac{4}{5}$ . When 3 is subtracted from the numerator and denominator of the original fraction, the new fraction is equivalent to  $\frac{5}{7}$ . Find the original fraction.

- 1 (a) Complete the table of values for the equation  $3x - 2y = 14$ . [1]

$x$	-2	0	2
$y$			

- (b) The diagram shows the graph of  $3y = x$ .  
On the same diagram, draw the graph of  $3x - 2y = 14$ . [1]



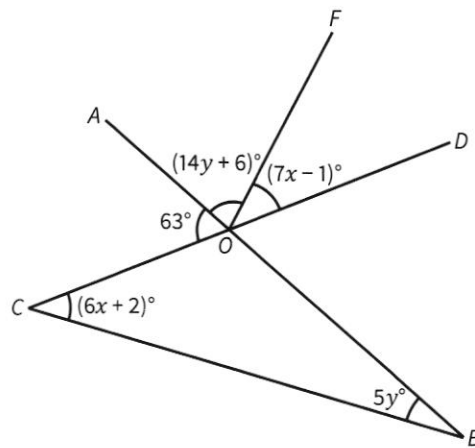
- (c) Hence solve the simultaneous equations  $3x - 2y = 14$  and  $3y = x$ .

..... [1]

- 2 Solve the simultaneous equations  $6x + 5y = 12$  and  $4x + 5y = 20$ .

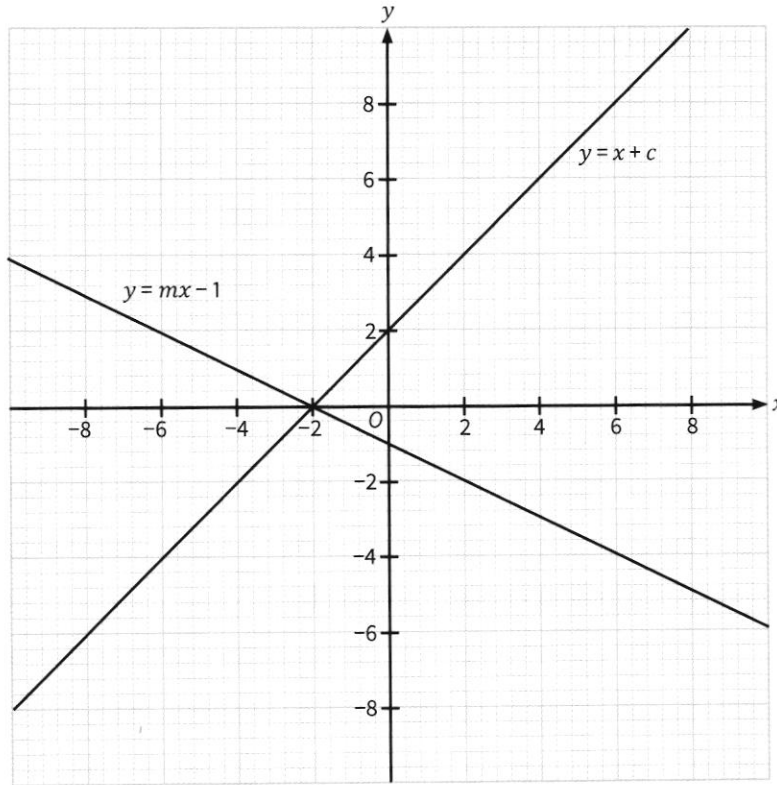
..... [3]

- 3 In the diagram, the lines  $AB$  and  $CD$  meet at the point  $O$ .  
By writing down two simultaneous equations, find the value of  $x$  and of  $y$ .



..... [4]

1 The diagram below shows the graphs of  $y = x + c$  and  $y = mx - 1$ , where  $m$  and  $c$  are constants.



(a) Using the above graph, write down the solution of the simultaneous equations  $y = x + c$  and  $y = mx - 1$ .

..... [1]

(b) State the value of

(i)  $m$ ,

..... [1]

(ii)  $c$ .

..... [1]

(c) Find the area of the triangle bounded by the two linear graphs and the  $y$ -axis.

..... [1]

- 2 Apples of equal mass are placed in a bowl on top of a weighing scale. The table shows the reading on the weighing scale,  $y$  grams, when  $x$  apples are placed in the bowl.

Number of apples ( $x$ )	5	8
Mass ( $y$ grams)	523	709

It is given that  $x$  and  $y$  are connected by the equation  $y = c + dx$ , where  $c$  and  $d$  are constants.

- (a) Write down two equations in  $c$  and  $d$ .

..... [2]

- (b) Solve the equations to find the value of  $c$  and of  $d$ .

..... [3]

- (c) What does the value of  $c$  represent?

.....

..... [1]