CHAPTER

12 Measures of Central Tendency

Name: ..... Class: ..... Date: .....

12.1 MEAN

Key Skills Checklist	Confidence Level					Related Questions
	1	2	3	4	5	
Find the mean of a set of data						1, 2, 6
Find the mean of grouped data with class intervals						3, 7
Solve problems involving grouped data with individual values						3, 5, 14, 16
Solve problems involving mean of data						4, 8, 9, 10, 11, 12, 13, 15, 17

WORD TOOLBOX

mean

median

mode

measures of central tendency

averages

Mean, **median** and **mode** are the three **measures of central tendency**. They are also known as **averages** in statistics.

The mean of a set of data is derived by the formula:

$$\text{Mean} = \frac{\text{Sum of values}}{\text{Number of data}}$$

For example, the list shows the heights (in cm) of five boys.

172, 165, 177, 169, 180

To find the mean height of the boys, we take $\frac{172 + 165 + 177 + 169 + 180}{5} = 172.6$.

$\therefore$ mean height is 172.6 cm.

frequency table

When data is grouped into class intervals, we use a **frequency table** to represent the data.

The frequency table shows the Mathematics quiz scores of 40 students.

Quiz score (x)	Frequency
$0 < x \leq 5$	1
$5 < x \leq 10$	7
$10 < x \leq 15$	23
$15 < x \leq 20$	9

To find the mean of the grouped data with class intervals, we use the class mid-value to estimate the mean score of the class.

$$\text{Estimated mean score of 40 students} = \frac{(2.5)(1) + (7.5)(7) + (12.5)(23) + (17.5)(9)}{40}$$

$$= 12.5 \text{ marks}$$

BASIC MASTERY

1. Find the mean of each of the following set of numbers.

   (a) 1, 5, 8, 16, 20

   (b) 4.4, 2.8, 9.5, 11.8, 23.5

2. Find an expression for the mean of each of the following set of numbers.

   (a) $3x$, $9x$, $15x$, $24x$, $29x$

   (b) $4y - 1$, $4 + y$, $21 - 5y$, $3 - 3y$, $11 + 2y$

3. **(a)** The mean of six numbers is 17. Find the sum of these six numbers.

   (b) The sum of the values in a data set is 77. The mean of the data set is 5.5. Find the number of values in the data set.

4 Find the mean of x for each of the following distributions.

(a)

Age (x years)	10	11	12	13	14
Frequency	9	7	12	6	6

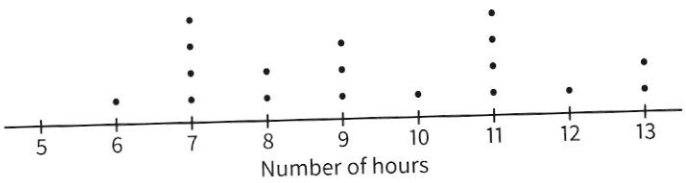
(b)

Travelling time (x min)	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$
Frequency	11	12	6	16	8

INTERMEDIATE

5 A survey is conducted on a group of students to find out the number of hours they spend on the Internet in a week. The dot diagram shows the results of the survey.

Number of hours spent on the Internet in a week



Find

(a) the number of students who took part in the survey,

(b) the total number of hours spent on the Internet,

(c) the mean number of hours spent on the Internet.

6 The stem-and-leaf diagram shows the heights (in cm) of plants in a nursery.

Heights of plants

Stem	Leaf
12	4 5 5 7 8
13	1 1 2 3 6 6 6 8 9 9
14	0 6 7 7 8

Key: 12 | 4 represents 124 cm.

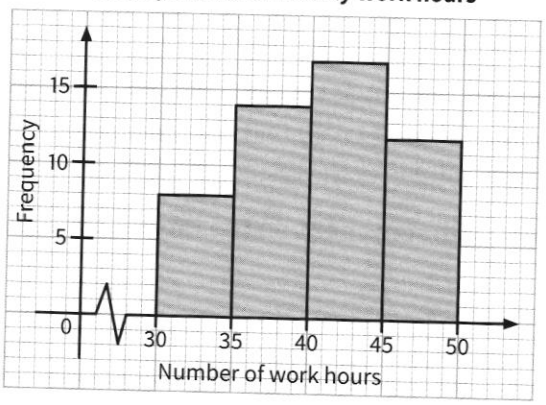
Find

(a) the number of plants,

(b) the mean height of the plants.

7 The histogram shows the survey results of the number of weekly work hours of a group of adults.

Survey results of weekly work hours



Find

(a) the number of adults who took part in the survey,

(b) an estimate of the mean number of work hours per week.

8 **(a)** The mean mass of six oranges is 320 g. Find the new mean mass when two oranges of masses 342 g and 308 g are added to the group.

(b) The mean height of eight adults is 175 cm. Find the new mean height when one of the adults, with a height of 180 cm, is excluded from the group.

9 The mean of $6 + x$, $2x - 3$, $2 + 4x$ and $x - 1$ is 11. Find the value of x.

10 Group *A* has 17 people and group *B* has 15 people. The mean ages of the people in groups *A* and *B* are 23 and 27 years respectively. Find the mean age of the people in groups *A* and *B* combined.

11 The mean score of Joshua's five quizzes is 13.8 marks. When another quiz score is included, the mean becomes 13. Find his score in the last quiz.

12 The mean of a set of six numbers is 37.5 and the mean of another set of eight numbers is x. If the combined mean of the 14 numbers is 27.5, find the value of x.

13 The mean of a set of six values is 62. The ratio of the values is 2 : 3 : 4 : 5 : 8 : 9. Find the largest value in the data set.

14 The table shows the number of library books borrowed by 40 students.

Number of books	4	5	6	7	8
Frequency	3	12	x	y	4

(a) Show that $x + y = 21$.

(b) The mean number of library books borrowed is 5.925.
Form an equation in x and y and show that it reduces to $6x + 7y = 133$.

(c) Solve the simultaneous equations in **(a)** and **(b)** to find the values of x and y.

ADVANCED

15 Sherwin completed four practice papers. If he obtains 69 marks for the next practice paper, the mean mark will increase by 1.5. Find Sherwin's current mean mark.

16 The table shows the number of days 50 students were late for school.

Number of days	0	1	2	3	4	5
Frequency	21	12	x	y	4	1

The mean number of days that the students were late is 1.22. Find the values of x and y.

17 a, b, c and d are positive integers. The mean of the numbers is x.
Express the new mean, in terms of x, when

(a) the value of a increases by 8,

(b) all four values increase by 3,

(c) all four values increase by 150%.

Name: ..... Class: ..... Date: .....

12.2 MEDIAN

Key Skills Checklist	Confidence Level					Related Questions
	1	2	3	4	5	
Find the median of a set of individual data						1, 5, 7, 11
Find the median of a set of grouped data						2, 3, 4, 9, 10, 12
Solve problems involving median of a set of data						6, 8, 10, 13, 14
Compare two sets of data						11, 12

WORD TOOLBOX

middle value
ascending order
descending order

The following shows the heights (in cm) of five boys, arranged in **ascending order**.

165 169 **172** 177 180

middle position

The **middle value** is the 3rd value.
∴ median height = 172 cm.

If the number of values is even, the median is the mean of the two middle values.
For example, the heights of six boys are as follows:

165 169 **172** **173** 177 180

middle position

∴ median height $= \frac{172 + 173}{2} = 172.5$ cm.

When the heights are arranged in **descending order**, the value(s) in the middle position does not change.

For grouped data with class intervals, the class which contains the middle value is the median class.

For example:

x	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$
Frequency	9	11	7	6	4

There are 37 values in total. Hence the median value is the 19th value.
∴ median class is $10 < x \leq 20$.

BASIC MASTERY

1. Find the median of each of the following sets of numbers.

(a) 10, 15, 18, 26, 40

(b) 4, 17, 19, 21, 25, 26, 32, 41

(c) 13, 43, 11, 25, 32, 26, 39

(d) 62, 54, 47, 32, 48, 53, 66, 41

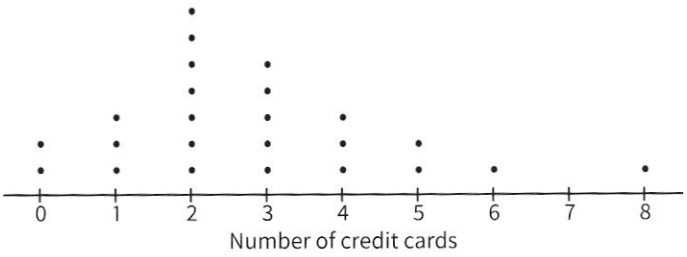
2. The table shows the number of books read by 30 children in a month.

Number of books read	0	1	2	3	4	5
Frequency	2	5	7	7	3	6

Find the median number of books read.

3 The dot diagram shows the number of credit cards owned by some adults.

Number of credit cards owned



Find

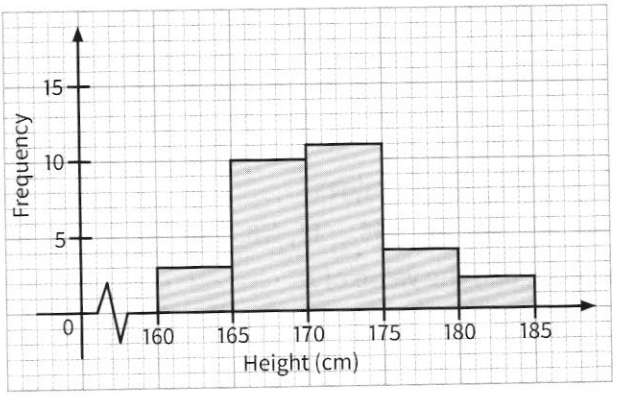
(a) the number of adults who took part in the survey,

(b) the median number of credit cards owned.

INTERMEDIATE

4 The stem-and-leaf diagram shows the heights of boys in a class.

Heights of boys



Find

(a) the number of boys in the class,

(b) the class interval where the median falls in.

5 The stem-and-leaf diagram shows the ages of 22 elderly at a community centre.

Ages of elderly

Stem	Leaf
6	7 7 7 8 8 9
7	0 0 1 2 3 5 6 7 8 9 9
8	2 2 4 6 6

Key: 6 | 7 represents 67 years old.

(a) Find the mean age of the elderly.

(b) Find the median age of the elderly.

(c) An elderly aged 70 enters the community centre. Find the new mean and median ages.

6 A data set consists of the following seven values.

$$11,\quad 6,\quad 15,\quad 16,\quad 8,\quad 19,\quad x$$

(a) Suppose the median is 12, find the value of x.

(b) Suppose that when the value 13 is included in the data set, the median is still 12. Find the value of x.

7 The table shows the household incomes (in $) of 12 families.

8750	3230	9470	6080	8420	5600
6420	9300	5710	6880	4720	19320

(a) Find the mean household income.

(b) Find the median household income.

(c) Suggest whether the mean or median is a better representation of the measure of central tendency for this set of data. Give a reason to support your answer.

8 **(a)** The median of five consecutive integers is 20. Find the largest integer.

(b) The median of eight consecutive even integers is 51. Find the smallest integer.

9 The following shows the Mathematics scores of 200 students.

Score (x)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	1	22	45	105	27

(a) Find the estimated mean score.

(b) Find the class interval which contains the median score.

10 The table shows the number of movies viewed by a class of students in a month.

Number of movies	0	1	2	3	4	5
Frequency	3	7	x	4	6	8

Given that the median is 3, find

(a) the largest possible value of x,

(b) the smallest possible value of x.

11 The Mathematics and Science test scores of 20 students are presented in a back-to-back stem-and-leaf diagram.

Test scores of students

Leaf for Mathematics	Stem	Leaf for Science
7 6	1	7 8 8
8 6 4 2 1	2	2 6 7 7 8 8 9
8 8 5 5 3 0	3	5 7 7 8 9
8 8 7 4 4 1 0	4	2 3 8
	5	0 0

Key: 6 | 1 | 7 represents 16 marks for Mathematics and 17 marks for Science.

(a) Find the mean score for each subject.

(b) Find the median score for each subject.

(c) Determine the subject in which the students performed better.

12 The table shows the number of siblings the students in Class *A* and Class *B* have.

Number of siblings	0	1	2	3	4
Frequency for Class A	12	19	6	2	1
Frequency for Class B	8	21	8	3	0

(a) Find the mean number of siblings for students in

(i) Class *A*,

(ii) Class *B*.

(b) Find the median number of siblings for students in

(i) Class *A*,

(ii) Class *B*.

(c) Explain whether the mean or median is a better representation of the measure of central tendency for this set of data.

ADVANCED

13 The table shows the number of exercise sessions attended by some adults in a week.

Number of exercise sessions	0	1	2	3	4	5	6
Frequency	3	x	8	5	8	4	2

(a) If $x = 4$,

(i) find the mean number of exercise sessions per week,

(ii) find the median number of exercise sessions per week.

(b) If the median is 2.5, find the value of x.

(c) If the median is 3, find the largest possible value of x.

14 The table shows the number of passengers boarding each taxi at a taxi stand.

Number of passengers	0	1	2	3	4
Frequency	$x+2$	8	10	9	$2x+3$

Given that x is a positive integer, find

(a) the value of x if there are 65 taxis,

(b) the value of x if the mean number of passengers is 2.3,

(c) the maximum and minimum values of x if the median number of passengers is 3.