1. **[YJC 2018 MYE]**

The current I in an electric circuit at time t satisfies the differential equation

$$4\frac{\mathrm{d}I}{\mathrm{d}t} = 2 + bI,$$

where b is a non-zero real constant. It is also given that $I = 2$ when $t = 0$.

(a) Find I in terms of b and t. [5]

(b) Describe what happens to the current in this circuit in the long run when

i. $b > 0$, [2]

ii. $b < 0$. [2]

2. **[TPJC 2018 MYE]**

In a chemical reaction, the mass, x grams, of a certain salt present at time t minutes satisfies the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(2 + x - x^2),$$

where $0 \leq x \leq 1$ and k is a constant. Initially, the mass of salt present is 1 gram and $\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{5}$.

(a) Show that $k = -\frac{1}{10}$. [1]

(b) By first expressing $2 + x - x^2$ in completed square form, find t in terms of x. [6]

(c) Find the time taken for there to be no salt present in the chemical reaction. [1]

(d) Express the solution of the differential equation in the form $x = f(t)$. Find the mass of salt present in the chemical reaction after 3 minutes. [4]

(e) Sketch the part of the curve with equation $x = f(t)$ which is relevant in this context. [2]

3. **[RVHS 2018 MYE]**

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference in the temperature of the object and its immediate surroundings.

Alfred heated his drink to 90°C and put it in a fridge. The temperature setting of the fridge is 5°C. After 5 minutes, he took it out and found that the drink is at 55°C.

Using Newton's Law of Cooling, find the additional time needed by Alfred to leave the drink in the fridge for it to cool to 20°C. [7]

Answers

1. (a) $I = \dfrac{(2+2b)e^{\frac{bt}{4}} - 2}{b}$.

   (b) When $b > 0$, the current will increase and approach infinity.
   When $b < 0$, the current will decrease and approach $-\frac{2}{b}$.

2. (b) $t = \dfrac{5}{3}\ln\left(\dfrac{-2(x-2)}{x+1}\right)$.

   (c) 2.31 minutes.

   (d) $x = \dfrac{4e^{-\frac{3}{5}t} - 1}{2e^{-\frac{3}{5}t} + 1}$.

3. 11.3 more minutes.