1. **[RVHS 2020 Promos Q10]**

Functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{1}{x-2}, \quad x \in \mathbb{R},\ x \neq 2,$$
$$\mathrm{g}: x \mapsto x^2 - 4x, \quad x \in \mathbb{R}.$$

(i) Determine whether the composite functions fg and gf exist. Find the range of the composite function that exists. [4]

For the rest of the question, the domain of f is restricted to $x \in \mathbb{R},\ x < 2$.

(ii) Define f^{-1} in similar form. [2]

(iii) Sketch on a **single** clearly labelled diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, showing clearly the relationship between the graphs and stating the equations of any asymptotes and the coordinates of any axial intercepts. [2]

(iv) Write down the line in which the graph of $y = \mathrm{f}(x)$ must be reflected in order to obtain the graph of $y = \mathrm{f}^{-1}(x)$, and find the exact solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

2. **[NJC 2020 Promos Q9]**

(a) The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{1}{1+x} - 2, \quad \text{for } x \in \mathbb{R},\ x \geq 0,$$
$$\mathrm{g}: x \mapsto \left(x + \frac{4}{3}\right)^2 - \frac{1}{5}, \quad \text{for } x \in \mathbb{R}.$$

(i) Determine whether the composite function fg exists. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ and find the range of f. [2]

(iii) Find the exact range of gf. [2]

(b) The function h is defined by

$$\mathrm{h}: x \mapsto (x+1)^2 (x-2),\ x \in \mathbb{R}.$$

(i) Explain why the function h^{-1} does not exist. [1]

For the rest of the question, the domain of h is restricted to $x \in [-1, 1]$.

(ii) State the domain of h^{-1}. [1]

(iii) Sketch on the same diagram the graphs of $y = \mathrm{h}(x)$, $y = \mathrm{h}^{-1}(x)$ and $y = \mathrm{h}^{-1}\mathrm{h}(x)$. [3]

3. [**TMJC 2020 Promos Q7**]

The function f is defined by

$$\mathrm{f}: x \mapsto -\ln(x-2), \quad x \in \mathbb{R},\ 2 < x \leq 3.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state the domain and range of f^{-1}. [4]

(ii) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the *x*- and *y*- axes. [6]

(iii) Explain why the *x*-coordinate of the point of intersection of the graph of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ satisfies the equation

$$x + \ln(x-2) = 0,$$

and find the value of this *x*-coordinate. [2]

4. [**HCI 2020 Promos Q11**]

The function f is defined by

$$\mathrm{f}: x \mapsto \begin{cases} \cos\dfrac{\pi x}{2}, & 0 \leq x < 1 \\ 1 - x, & 1 \leq x \leq 2. \end{cases}$$

(i) Sketch the graph of f, indicating clearly the coordinates of the axial intercepts and endpoints. [3]

(ii) Show that f has an inverse, and define it in similar form. [5]

(iii) Find the range of values of *x* for which $\mathrm{f}\mathrm{f}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x)$. [2]

(iv) The function g is defined by

$$\mathrm{g}: x \mapsto \mathrm{e}^{x},\ 0 \leq x \leq \ln 2.$$

Find the range of fg. [2]

5. **[YIJC 2020 Promos Q8]**

The function h is defined by

$$\mathrm{h} : x \mapsto \mathrm{e}^{x-1}, \qquad \text{where } x \in \mathbb{R},\ x \geq -2.$$

(i) Find $\mathrm{h}^{-1}(x)$ and state the domain of h^{-1}. [2]

The function g is defined by

$$\mathrm{g} : x \mapsto x^2 - 1, \qquad \text{where } x \in \mathbb{R}, x \geq 0.$$

(ii) Explain why the composite function hg exists. [2]

(iii) Find the exact solution of hg(x) = 5. [3]

(iv) Find the exact range of hg. [1]

6. **[VJC 2020 Promos Q6]**

The functions f and g are defined by

$$\mathrm{f} : x \mapsto \left|3\mathrm{e}^{-x} - 8\right| - 2, \qquad x \in \mathbb{R}, x \geqslant \lambda,$$
$$\mathrm{g} : x \mapsto a \sin x, \qquad x \in \mathbb{R}, 0 < x < \pi,$$

where a is a positive constant.

(i) Find the smallest value of λ, in exact form, for f^{-1} to exist. [2]

For the rest of the question, λ takes the value found in part **(i)**.

(ii) Find $\mathrm{f}^{-1}(x)$. [3]

(iii) Find, in terms of a, the range of fg. [2]

7. **[TMJC 2020 Promos Q3]**

Functions f and g are defined by

$$\mathrm{f}: x \mapsto x-2, \qquad x \in \mathbb{R}, \quad x \leq 4,$$

$$\mathrm{g}: x \mapsto \frac{x}{3}, \qquad x \in \mathbb{R}, \quad x \leq 3.$$

(i) Explain why the composite function fg exists. [2]

(ii) Write down $\mathrm{fg}(x)$. [1]

(iii) State a sequence of transformations that will transform the graph of $y=\mathrm{fg}(x)$ onto the graph of $y=x$. The domain of fg need not be considered in this part of the question. [2]

The range of the function h is the set of all real numbers. The function hfg is defined by

$$\mathrm{hfg}: x \mapsto \ln\left(-\frac{x}{3}+6\right), \qquad x \in \mathbb{R}, \quad x \leq 3.$$

(iv) Find h in a similar form. [3]

Answers

1. (i) Since $R_g = [-4, \infty) \not\subseteq (-\infty, 2) \cup (2, \infty) = D_f$, fg does not exist.
   Since $R_f = (-\infty, 0) \cup (0, \infty) \subseteq (-\infty, \infty) = D_g$, gf exists.
   $R_{gf} = [-4, \infty)$.

   (ii) $f^{-1} : x \mapsto 2 + \frac{1}{x}, \quad x < 0$.

   (iv) $1 - \sqrt{2}$.

2. (a) i. Since $R_g = [-\frac{1}{5}, \infty) \not\subseteq [0, \infty)$, fg does not exist.

   ii. $R_f = (-2, 1]$.

   iii. $R_{gf} = [-\frac{1}{5}, \frac{11}{45})$.

   (b) i. The horizontal line $y = -1$ cuts the graph of $y = h(x)$ at more than one point. Hence h is not one-one. Therefore h^{-1} does not exist.

   ii. $D_{h^{-1}} = R_h = [-4, 0]$.

3. (i) $f^{-1}(x) = e^{-x} + 2$.
   $D_{f^{-1}} = R_f = [0, \infty)$.
   $R_{f^{-1}} = D_f = (2, 3]$.

   (iii) $x = 2.12$.

4. (ii) All horizontal line $y = k, k \in \mathbb{R}$ cuts the graph of $y = f(x)$ at most once. Hence f is one-one and f has an inverse.
   $f^{-1}x :\mapsto \begin{cases} \frac{2\cos^{-1} x}{\pi}, & 0 < x \leq 1 \\ 1 - x, & -1 \leq x \leq 0 \end{cases}.$

   (iii) $0 \leq x \leq 1$.

   (iv) $R_{fg} = [-1, 0]$.

5. (i) $h^{-1}(x) = \ln x + 1$.
   $D_{h^{-1}} = R_h = [e^{-3}, \infty)$.

   (ii) Since $R_g = [-1, \infty) \subseteq [-2, \infty) = D_h, hg$ exists.

   (iii) $x = \sqrt{2 + \ln 5}$.

   (iv) $R_{hg} = [e^{-2}, \infty)$.

6. (i) $-\ln \frac{8}{3}$.

   (ii) $f^{-1}(x) = \ln(2 - \frac{x}{3})$.

   (iii) $R_{fg} = (3, 6 - 3e^{-a}]$.

7. (i) $R_g = (-\infty, 1] \subseteq (-\infty, 4]$ so fg exists.

   (ii) $fg(x) = \frac{x}{3} - 2$.

   (iii) A: Scale by the factor of $\frac{1}{3}$ parallel to the x-axis.
   B: Translate by 2 units in the negative x-direction.

   (iv) $h : x \mapsto \ln(-x + 4), \quad x \in \mathbb{R}, x < 4$.