1. **[RVHS 2020 Promos Q12 (modified)]**

(a) Use the substitution $x=\cos\theta$ to show that $\int \sqrt{1-x^2}\,\mathrm{d}x=\frac{1}{2}\left(x\sqrt{1-x^2}-\cos^{-1}x\right)+c$, where c is an arbitrary constant. [4]

(b) Find $\int 2\sin kx\sin x\,\mathrm{d}x$, where $k\in\mathbb{Z}, k\geq 2$. [3]

(c) **(i)** Show that $1-x$ can be expressed as $A(2x-3)+B$, where A and B are constants to be determined. [1]

(ii) Hence find $\int \frac{1-x}{x^2-3x+1}\,\mathrm{d}x$. [4]

2. **[TMJC 2020 Promos Q2]**

By using the substitution $u=1+t^3$, find $\int_0^1 \frac{t^5}{\left(1+t^3\right)^3}\,\mathrm{d}t$ without using a calculator. [6]

3. **[TMJC 2020 Promos Q5]**

(a) Find $\int \frac{\mathrm{e}^x}{\left(1+2\mathrm{e}^x\right)^3}\,\mathrm{d}x$. [2]

(b) Find $\int \frac{x-1}{1+4x^2}\,\mathrm{d}x$. [3]

(c) Find $\int x\left(\ln x\right)^2\,\mathrm{d}x$. [4]

4. **[VJC 2020 Promos Q2]**

(a) Find

(i) $\int \frac{1}{x^2+2x+5}\,\mathrm{d}x$, [2]

(ii) $\int \frac{\left(\ln x\right)^3}{x}\,\mathrm{d}x$. [2]

(b) Find the exact value of $\int_0^{\frac{\pi}{2}} x\cos x\,\mathrm{d}x$. [3]

5. **[YIJC 2020 Promos Q7]**

(a) Find $\int x\tan^{-1}\left(2x^2\right)\,\mathrm{d}x$. [3]

(b) Use the substitution $x=\sin\theta$ to find the exact value of $\int_{0.5}^{1} x^2\sqrt{1-x^2}\,\mathrm{d}x$. [4]

6. **[YIJC 2020 Promos Q9]**

The region R is bounded by the curve $y=\dfrac{x}{\sqrt{a^2-x^2}}$, where $a>0$, the x-axis and the line $x=\dfrac{a}{2}$. Find in terms of a,

(i) the exact area of the region R, [4]

(ii) the exact volume of the solid obtained when R is rotated through 2π radians about the y-axis. [5]

7. **[NJC 2020 Promos Q10]**

A curve C has parametric equations

$$x=t^3\,,\quad y=1+\sqrt{1-t^2}\quad \text{where } -1\le t\le 1.$$

(i) Sketch C, labelling clearly the axial intercept. [2]

(ii) Use the substitution $t=\cos\theta$ to find the exact value of $\int_0^{\frac{\sqrt{3}}{2}} t^2\sqrt{1-t^2}\,\mathrm{d}t$. [5]

(iii) Hence, find the exact area bounded by C, the x-axis and the lines $x=0$ and $x=\dfrac{\sqrt{27}}{8}$.

[4]

Answers

1. (b) $\frac{5\sqrt{6}}{3}$ units.

   (c) $\frac{1}{k-1}\sin(k-1)x - \frac{1}{k+1}\sin(k+1)x + c$.

   (d) i. $A = -\frac{1}{2}, B = -\frac{1}{2}$.

   ii. $-\frac{1}{2}\ln|x^2-3x+1| - \frac{1}{2\sqrt{5}}\ln\left|\frac{2x-3-\sqrt{5}}{2x-3+\sqrt{5}}\right| + c$.

2. $\frac{1}{24}$.

3. (a) $-\frac{1}{4(1+2e^x)^2} + c$.

   (b) $\frac{1}{8}\ln(1+4x^2) - \frac{1}{2}\tan^{-1}(2x) + c$.

   (c) $(\ln x)^2\left(\frac{x^2}{2}\right) - (\ln x)\left(\frac{x^2}{2}\right) + \frac{x^2}{4} + c$.

4. (a) i. $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + c$.

   ii. $\frac{(\ln x)^4}{4} + c$.

   (b) $\frac{\pi}{2} - 1$.

5. (a) $\frac{x^2}{2} - \tan^{-1}(2x^2) - \frac{1}{8}\ln(1+4x^4) + c$.

   (b) $\frac{1}{8}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{8}\right) + c$.

6. (i) $a\left(1 - \frac{\sqrt{3}}{2}\right)$.

   (ii) $\pi a^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$.

7. (ii) $\frac{\sqrt{3}}{64} + \frac{\pi}{24}$.

   (iii) $\frac{27\sqrt{3}}{64} + \frac{\pi}{8}$.