1. **[SRJC 18 MYE]**
The lines l_1 and the plane p_1 have equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} + t\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}, t \in \mathbb{R} \quad \text{and} \quad p_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 2.$$

It is given that the point A has the position vector $3\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$.

(a) Find the acute angle between l_1 and p_1 [2]

(b) Find the coordinates of the foot of the perpendicular from point A to p_1. [4]

The plane p_2 has equation $-2x + z = 6$. Given that the plane p_1 meets the plane p_2 at the line l_2,

(c) find the equation of l_2. [1]

2. **[TPJC 18 MYE]**
Referred to the origin O, points A, B and C have position vectors $\mathbf{a}, \mathbf{b}$ and $\frac{5}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ respectively. The point P on AB is such that $AP : PB = \lambda : 1 - \lambda$ and the point P on OC is such that $OP : PC = \mu : 1 - \mu$.

(a) Express $\overrightarrow{OP}$ in terms of $\lambda, \mathbf{a}$ and $\mathbf{b}$. [1]

(b) By expressing $\overrightarrow{OP}$ in terms of $\mu, \mathbf{a}$ and $\mathbf{b}$, find the values of λ and μ. Hence show that P is the midpoint of OC. [4]

(c) It is given that the position vectors of the points A and B are $2\mathbf{j} - \mathbf{k}$ and $-6\mathbf{i}+2\mathbf{j}+11\mathbf{k}$ respectively. The point Q lies on OA such that PQ is perpendicular to OA. Find the position vector of the point Q. [6]

3. **[JJC 18 MYE (modified)]**
A mine contains several underground tunnels beneath a hillside. All the tunnels are straight and the width of the tunnels may be neglected. A coordinate system is chosen with the z-axis pointing vertically upwards. The hillside contains points $A(10, -65, 15)$ and $B(-80, 95, 35)$.

The tunnel T_A starts at A and goes in the direction of the vector $15\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$.

(a) Write down a vector equation of T_A and find the shortest distance from B to T_A. [4]

(b) Another tunnel T_B starts at B and passes through the point $D(13, 133, p)$. T_A and T_B meets at the point Q. Find the coordinates of Q. [4]

4. **[AJC 18 J2 MYE]**
Two non-zero vectors, $\mathbf{a} + k\mathbf{b}$ and $\mathbf{a} - k\mathbf{b}$ are perpendicular to each other, where k is a positive constant and $\mathbf{a}$ is a unit vector.

Find the magnitude of $\mathbf{b}$ in terms of k. [3]

5. **[DHS J2 18 MYE]**
Relative to the origin O, two points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors. The points C and D lie on the midpoints of OA and AB respectively. The line segments OD and BC intersect at the point P such that $OP : PD = \lambda : 1 - \lambda$.

   (a) Find $\overrightarrow{OP}$ in terms of $\lambda, \mathbf{a}$ and $\mathbf{b}$. [1]

   (b) Show that the ratio of $BP : PC$ is $2 : 1$. [4]

6. **[TMJC 19 MYE]**
Last December, Mr. Stuart took his family on a vacation to Tamridian Island, where they embarked on a road trip. Points (x, y, z) are defined relative to the airport at $(0, 0, 0)$. They collected their vehicle at a car rental company whose location is represented by the coordinates $(1, 0, 2)$ and started driving on Pines Highway, a straight road in the direction $\begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}$. As they approached a junction located at $(-2, 1, 7)$, they exited the highway to continue on another straight road to a cafe located at $(-8, 8, 9)$.

   (a) Write down a vector equation of the line representing Pines Highway, which Mr. Stuart was driving on right after collecting the car. [1]

   (b) Find the exact length of projection of the path he took from the junction to the cafe onto Pines Highway. [3]

   (c) Hence find the exact shortest distance between the cafe and Pines Highway. [2]

   According to the map, there is a nice waterfall whose location is a reflection of the location of the cafe in the line reprsenting Pines Highway. If Mr. Stuart and his family had decided to vist the waterfall instead and followed another straight road at the junction that leads to the waterfall,

   (d) find a vector equation of the line representing the road which he would have travelled on. [6]

7. **[NJC 19 MYE]**
The plane Π has equation $x - 2y + 2z = 9$.

   (a) Find the cartesian equations of the planes such that the perpendicular distance from each plane to Π is 8. [3]

   The line L has equation $\dfrac{x-1}{6a} = \dfrac{y-2}{3a} = \dfrac{3-z}{3}$, where a is a constant.

   (b) If the angle between L and Π is 30°, find the possible exact values of a. [3]

Answers

1. (a) $6.3°$.
   (b) $(1, -4, 4)$.
   (c) $\mathbf{r} = \begin{pmatrix} -3 \\ 8 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \mu \in \mathbb{R}$.
2. (a) $(1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$.
   (b) $\lambda = \frac{1}{6}, \mu = \frac{1}{2}$.
   (c) $1.2\mathbf{j} - 0.6\mathbf{k}$.
3. (a) 180 units.
   (b) $Q(385, 285, -35)$.
4. $\frac{1}{k}$.
5. $\frac{\lambda}{2}(\mathbf{a} + \mathbf{b})$.
6. (a) $l : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}, \lambda \in \mathbb{R}$.
   (b) $\sqrt{35}$.
   (c) $3\sqrt{6}$.
   (d) $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} + \mu cvec0{-58}, \mu \in \mathbb{R}$.
7. (a) $x - 2y + 2z = 33$ and $x - 2y + 2 = -15$.
   (b) $a = \pm\sqrt{\frac{7}{45}}$.