MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9740/01

Paper 1

October/November 2011

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 Without using a calculator, solve the inequality

$$\frac{x^2+x+1}{x^2+x-2} < 0. \qquad [4]$$

2 It is given that $f(x) = ax^2 + bx + c$, where a, b and c are constants.

(i) Given that the curve with equation $y = f(x)$ passes through the points with coordinates $(-1.5, 4.5)$, $(2.1, 3.2)$ and $(3.4, 4.1)$, find the values of a, b and c. Give your answers correct to 3 decimal places. [3]

(ii) Find the set of values of x for which $f(x)$ is an increasing function. [2]

3 The parametric equations of a curve are

$$x = t^2, \quad y = \frac{2}{t}.$$

(i) Find the equation of the tangent to the curve at the point $\left(p^2, \frac{2}{p}\right)$, simplifying your answer. [2]

(ii) Hence find the coordinates of the points Q and R where this tangent meets the x- and y-axes respectively. [2]

(iii) Find a cartesian equation of the locus of the mid-point of QR as p varies. [3]

4 **(i)** Use the first three non-zero terms of the Maclaurin series for $\cos x$ to find the Maclaurin series for $g(x)$, where $g(x) = \cos^6 x$, up to and including the term in x^4. [3]

(ii) **(a)** Use your answer to part **(i)** to give an approximation for $\int_0^a g(x)\,dx$ in terms of a, and evaluate this approximation in the case where $a = \frac{1}{4}\pi$. [3]

(b) Use your calculator to find an accurate value for $\int_0^{\frac{1}{4}\pi} g(x)\,dx$. Why is the approximation in part **(ii)** **(a)** not very good? [2]

5 It is given that $f(x) = 2 - x$.

(i) On separate diagrams, sketch the graphs of $y = f(|x|)$ and $y = |f(x)|$, giving the coordinates of any points where the graphs meet the x- and y-axes. You should label the graphs clearly. [3]

(ii) State the set of values of x for which $f(|x|) = |f(x)|$. [1]

(iii) Find the exact value of the constant a for which $\int_{-1}^{1} f(|x|)\,dx = \int_1^a |f(x)|\,dx$. [3]

6 **(i)** Using the formulae for $\sin(A \pm B)$, prove that

$$\sin\left(r+\tfrac{1}{2}\right)\theta - \sin\left(r-\tfrac{1}{2}\right)\theta \equiv 2\cos r\theta \sin\tfrac{1}{2}\theta.$$ [2]

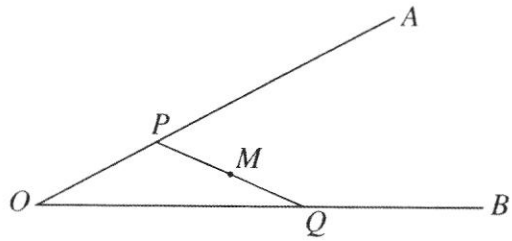
(ii) Hence find a formula for $\sum_{r=1}^{n} \cos r\theta$ in terms of $\sin\left(n+\tfrac{1}{2}\right)\theta$ and $\sin\tfrac{1}{2}\theta$. [3]

(iii) Prove by the method of mathematical induction that

$$\sum_{r=1}^{n} \sin r\theta = \frac{\cos\tfrac{1}{2}\theta - \cos\left(n+\tfrac{1}{2}\right)\theta}{2\sin\tfrac{1}{2}\theta}$$

for all positive integers n. [6]

7



Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P on OA is such that $OP : PA = 1 : 2$, and the point Q on OB is such that $OQ : QB = 3 : 2$. The mid-point of PQ is M (see diagram).

(i) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and show that the area of triangle OMP can be written as $k|\mathbf{a} \times \mathbf{b}|$, where k is a constant to be found. [6]

(ii) The vectors $\mathbf{a}$ and $\mathbf{b}$ are now given by

$$\mathbf{a} = 2p\mathbf{i} - 6p\mathbf{j} + 3p\mathbf{k} \qquad \text{and} \qquad \mathbf{b} = \mathbf{i} + \mathbf{j} - 2\mathbf{k},$$

where p is a positive constant. Given that $\mathbf{a}$ is a unit vector,

(a) find the exact value of p, [2]

(b) give a geometrical interpretation of $|\mathbf{a}.\mathbf{b}|$, [1]

(c) evaluate $\mathbf{a} \times \mathbf{b}$. [2]

8 **(i)** Find $\displaystyle\int \frac{1}{100 - v^2}\,\mathrm{d}v$. [2]

(ii) A stone is dropped from a stationary balloon. It leaves the balloon with zero speed, and t seconds later its speed v metres per second satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 0.1v^2.$$

(a) Find t in terms of v. Hence find the exact time the stone takes to reach a speed of 5 metres per second. [5]

(b) Find the speed of the stone after 1 second. [3]

(c) What happens to the speed of the stone for large values of t? [2]

9 **(i)** A company is drilling for oil. Using machine A, the depth drilled on the first day is 256 metres. On each subsequent day, the depth drilled is 7 metres less than on the previous day. Drilling continues daily up to and including the day when a depth of less than 10 metres is drilled. What depth is drilled on the 10th day, and what is the total depth when drilling is completed? [6]

(ii) Using machine B, the depth drilled on the first day is also 256 metres. On each subsequent day, the depth drilled is $\frac{8}{9}$ of the depth drilled on the previous day. How many days does it take for the depth drilled to exceed 99% of the theoretical maximum total depth? [4]

10 **Do not use a graphic calculator in answering this question.**

(i) The roots of the equation $z^2 = -8i$ are z_1 and z_2. Find z_1 and z_2 in cartesian form $x + iy$, showing your working. [4]

(ii) Hence, or otherwise, find in cartesian form the roots w_1 and w_2 of the equation

$$w^2 + 4w + (4 + 2i) = 0.$$ [3]

(iii) Using a single Argand diagram, sketch the loci

(a) $|z - z_1| = |z - z_2|$, [1]

(b) $|z - w_1| = |z - w_2|$. [1]

(iv) Give a reason why there are no points which lie on both of these loci. [1]

11 The plane p passes through the points with coordinates $(4, -1, -3)$, $(-2, -5, 2)$ and $(4, -3, -2)$.

(i) Find a cartesian equation of p. [4]

The line l_1 has equation $\frac{x-1}{2} = \frac{y-2}{-4} = \frac{z+3}{1}$ and the line l_2 has equation $\frac{x+2}{1} = \frac{y-1}{5} = \frac{z-3}{k}$, where k is a constant. It is given that l_1 and l_2 intersect.

(ii) Find the value of k. [4]

(iii) Show that l_1 lies in p and find the coordinates of the point at which l_2 intersects p. [4]

(iv) Find the acute angle between l_2 and p. [3]

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Section A: Pure Mathematics [40 marks]

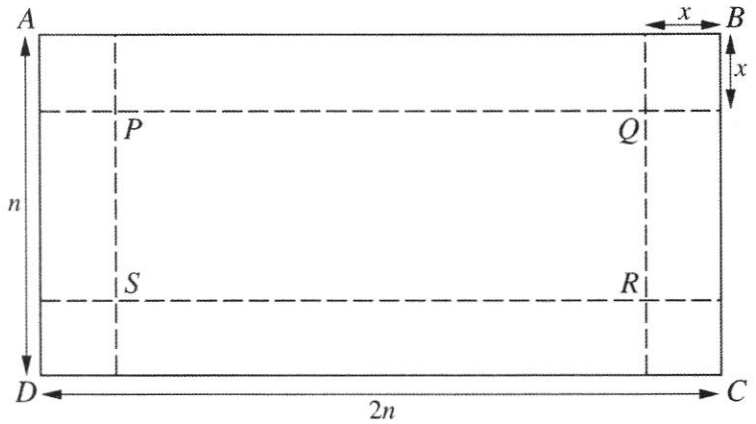
1 The complex number z satisfies $|z - 2 - 5i| \leqslant 3$.

(i) On an Argand diagram, sketch the region in which the point representing z can lie. [3]

(ii) Find exactly the maximum and minimum possible values of $|z|$. [2]

(iii) It is given that $0 \leqslant \arg z \leqslant \frac{1}{4}\pi$. With this extra information, find the maximum value of $|z - 6 - i|$. Label the point(s) that correspond to this maximum value on your diagram with the letter P. [3]

2



The diagram shows a rectangular piece of cardboard $ABCD$ of sides n metres and $2n$ metres, where n is a positive constant. A square of side x metres is removed from each corner of $ABCD$. The remaining shape is now folded along PQ, QR, RS and SP to form an open rectangular box of height x metres.

(i) Show that the volume V cubic metres of the box is given by $V = 2n^2x - 6nx^2 + 4x^3$. [3]

(ii) Without using a calculator, find in surd form the value of x that gives a stationary value of V, and explain why there is only one answer. [6]

3 The function f is defined by

$$\mathrm{f} : x \mapsto \ln(2x + 1) + 3, \quad x \in \mathbb{R},\ x > -\tfrac{1}{2}.$$

(i) Find $\mathrm{f}^{-1}(x)$ and write down the domain and range of f^{-1}. [4]

(ii) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the x- and y-axes. [4]

(iii) Explain why the x-coordinates of the points of intersection of the curves in part **(ii)** satisfy the equation

$$\ln(2x + 1) = x - 3,$$

and find the values of these x-coordinates, correct to 4 significant figures. [3]

4 **(a)** **(i)** Obtain a formula for $\int_0^n x^2 e^{-2x}\,dx$ in terms of n, where $n > 0$. [5]

(ii) Hence evaluate $\int_0^\infty x^2 e^{-2x}\,dx$. [1]

[You may assume that ne^{-2n} and $n^2e^{-2n} \to 0$ as $n \to \infty$.]

(b) The region bounded by the curve $y = \dfrac{4x}{x^2+1}$, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 2π radians about the x-axis. Use the substitution $x = \tan\theta$ to show that the volume of the solid obtained is given by $16\pi \int_0^{\frac{1}{4}\pi} \sin^2\theta\,d\theta$, and evaluate this integral exactly. [6]

Section B: Statistics [60 marks]

5 The continuous random variable X has the distribution $N(\mu, \sigma^2)$. It is known that $P(X < 40.0) = 0.05$ and $P(X < 70.0) = 0.975$. Calculate the values of μ and σ. [4]

6 It is desired to interview residents of a city suburb about the types of shop to be opened in a new shopping mall. In particular it is necessary to interview a representative range of ages.

(i) Explain how a quota sample might be carried out in this context. [2]

(ii) Explain a disadvantage of quota sampling in the context of your answer to part **(i)**. [1]

(iii) State the name of a method of sampling that would not have this disadvantage, and explain whether it would be realistic to use this method in this context. [2]

7 When I try to contact (by telephone) any of my friends in the evening, I know that on average the probability that I succeed is 0.7. On one evening I attempt to contact a fixed number, n, of different friends. If I do not succeed with a particular friend, I do not attempt to contact that friend again that evening. The number of friends whom I succeed in contacting is the random variable R.

(i) State, in the context of this question, two assumptions needed to model R by a binomial distribution. [2]

(ii) Explain why one of the assumptions stated in part **(i)** may not hold in this context. [1]

Assume now that these assumptions do in fact hold.

(iii) Given that $n = 8$, find the probability that R is at least 6. [1]

(iv) Given that $n = 40$, use an appropriate approximation to find $P(R < 25)$. State the parameters of the distribution you use. [4]

8 **(i)** Sketch a scatter diagram that might be expected for the case when x and y are related approximately by $y = a + bx^2$, where a is positive and b is negative. Your diagram should include 5 points, approximately equally spaced with respect to x, and with all x- and y-values positive. [1]

The table gives the values of seven observations of bivariate data, x and y.

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
y	18.8	16.9	14.5	11.7	8.6	4.9	0.8

(ii) Calculate the value of the product moment correlation coefficient, and explain why its value does not necessarily mean that the best model for the relationship between x and y is $y = c + dx$. [2]

(iii) Explain how to use the values obtained by calculating product moment correlation coefficients to decide, for this data, whether $y = a + bx^2$ or $y = c + dx$ is the better model. [1]

(iv) It is desired to use the data in the table to estimate the value of y for which $x = 3.2$. Find the equation of the least-squares regression line of y on x^2. Use your equation to calculate the desired estimate. [3]

9 Camera lenses are made by two companies, A and B. 60% of all lenses are made by A and the remaining 40% by B. 5% of the lenses made by A are faulty. 7% of the lenses made by B are faulty.

(i) One lens is selected at random. Find the probability that

(a) it is faulty, [2]

(b) it was made by A, given that it is faulty. [1]

(ii) Two lenses are selected at random. Find the probability that

(a) exactly one of them is faulty, [2]

(b) both were made by A, given that exactly one is faulty. [3]

10 In a factory, the time in minutes for an employee to install an electronic component is a normally distributed continuous random variable T. The standard deviation of T is 5.0 and under ordinary conditions the expected value of T is 38.0. After background music is introduced into the factory, a sample of n components is taken and the mean time taken for randomly chosen employees to install them is found to be $\bar{t}$ minutes. A test is carried out, at the 5% significance level, to determine whether the mean time taken to install a component has been reduced.

(i) State appropriate hypotheses for the test, defining any symbols you use. [2]

(ii) Given that $n = 50$, state the set of values of $\bar{t}$ for which the result of the test would be to reject the null hypothesis. [3]

(iii) It is given instead that $\bar{t} = 37.1$ and the result of the test is that the null hypothesis is not rejected. Obtain an inequality involving n, and hence find the set of values that n can take. [4]

11 A committee of 10 people is chosen at random from a group consisting of 18 women and 12 men. The number of women on the committee is denoted by R.

(i) Find the probability that $R = 4$. [3]

(ii) The most probable number of women on the committee is denoted by r. By using the fact that $P(R = r) > P(R = r + 1)$, show that r satisfies the inequality

$$(r+1)!(17-r)!(9-r)!(r+3)! > r!(18-r)!(10-r)!(r+2)!$$

and use this inequality to find the value of r. [5]

12 The number of people joining an airport check-in queue in a period of 1 minute is a random variable with the distribution Po(1.2).

(i) Find the probability that, in a period of 4 minutes, at least 8 people join the queue. [1]

(ii) The probability that no more than 1 person joins the queue in a period of t seconds is 0.7. Find an equation for t. Hence find the value of t, giving your answer correct to the nearest whole number. [4]

(iii) The number of people leaving the same queue in a period of 1 minute is a random variable with the distribution Po(1.8). At 0930 on a certain morning there are 35 people in the queue. Use appropriate approximations to find the probability that by 0945 there are at least 24 people in the queue, stating the parameters of any distributions that you use. (You may assume that the queue does not become empty during this period.) [5]

(iv) Explain why a Poisson model would probably not be valid if applied to a time period of several hours. [1]