MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9758/01

Paper 1

October/November 2018

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

Singapore Examinations and Assessment Board

CAMBRIDGE
International Examinations

JC18 11_9758_01/5R

[Turn over

1 (i) Given that $y = \frac{\ln x}{x}$, find $\frac{dy}{dx}$ in terms of x. [2]

(ii) Hence, or otherwise, find the exact value of $\int_1^e \frac{\ln x}{x^2}\,dx$, showing your working. [4]

2 **Do not use a calculator in answering this question.**

A curve has equation $y = \frac{3}{x}$ and a line has equation $y + 2x = 7$. The curve and the line intersect at the points A and B.

(i) Find the x-coordinates of A and B. [2]

(ii) Find the exact volume generated when the area bounded by the curve and the line is rotated about the x-axis through $360°$. [4]

3 (i) It is given that $x\frac{dy}{dx} = 2y - 6$. Using the substitution $y = ux^2$, show that the differential equation can be transformed to $\frac{du}{dx} = f(x)$, where the function $f(x)$ is to be found. [3]

(ii) Hence, given that $y = 2$ when $x = 1$, solve the differential equation $x\frac{dy}{dx} = 2y - 6$, to find y in terms of x. [4]

4 (i) Find the exact roots of the equation $|2x^2 + 3x - 2| = 2 - x$. [4]

(ii) On the same axes, sketch the curves with equations $y = |2x^2 + 3x - 2|$ and $y = 2 - x$.

Hence solve exactly the inequality

$$|2x^2 + 3x - 2| < 2 - x.$$ [4]

5 Functions f and g are defined by

$$\mathrm{f} : x \mapsto \frac{x+a}{x+b} \quad \text{for } x \in \mathbb{R},\ x \neq -b,\ a \neq -1,$$
$$\mathrm{g} : x \mapsto x \quad \text{for } x \in \mathbb{R}.$$

It is given that ff = g.

Find the value of b.

Find $\mathrm{f}^{-1}(x)$ in terms of x and a. [5]

6 Vectors **a**, **b** and **c** are such that $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{a} \times 3\mathbf{b} = 2\mathbf{a} \times \mathbf{c}$.

(i) Show that $3\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$, where λ is a constant. [2]

(ii) It is now given that **a** and **c** are unit vectors, that the modulus of **b** is 4 and that the angle between **b** and **c** is 60°. Using a suitable scalar product, find exactly the two possible values of λ. [5]

7 A curve C has equation $\dfrac{x^2 - 4y^2}{x^2 + xy^2} = \dfrac{1}{2}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x - y^2}{2xy + 16y}$. [3]

The points P and Q on C each have x-coordinate 1. The tangents to C at P and Q meet at the point N.

(ii) Find the exact coordinates of N. [6]

8 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_{n+1} = 2u_n + An$, where A is a constant and $n \geqslant 1$.

(i) Given that $u_1 = 5$ and $u_2 = 15$, find A and u_3. [2]

It is known that the nth term of this sequence is given by

$$u_n = a(2^n) + bn + c,$$

where a, b and c are constants.

(ii) Find a, b and c. [4]

(iii) Find $\sum_{r=1}^{n} u_r$ in terms of n. (You need not simplify your answer.) [4]

9 A curve C has parametric equations

$$x = 2\theta - \sin 2\theta, \qquad y = 2\sin^2\theta,$$

for $0 \leqslant \theta \leqslant \pi$.

(i) Show that $\dfrac{dy}{dx} = \cot\theta$. [4]

(ii) The normal to the curve at the point where $\theta = \alpha$ meets the x-axis at the point A. Show that the x-coordinate of A is $k\alpha$, where k is a constant to be found. [4]

(iii) Do not use a calculator in answering this part.

The distance between two points along a curve is the arc-length. Scientists and engineers need to use arc-length in applications such as finding the work done in moving an object along the path described by a curve or the length of cabling used on a suspension bridge.

The arc-length between two points on C, where $\theta = \beta$ and $\theta = \gamma$, is given by the formula

$$\int_{\beta}^{\gamma} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta.$$

Find the total length of C. [5]

10 An electrical circuit comprises a power source of V volts in series with a resistance of R ohms, a capacitance of C farads and an inductance of L henries. The current in the circuit, t seconds after turning on the power, is I amps and the charge on the capacitor is q coulombs. The circuit can be used by scientists to investigate resonance, to model heavily damped motion and to tune into radio stations on a stereo tuner. It is given that R, C and L are constants, and that $I = 0$ when $t = 0$.

A differential equation for the circuit is $L\dfrac{dI}{dt} + RI + \dfrac{q}{C} = V$, where $I = \dfrac{dq}{dt}$.

(i) Show that, under certain conditions on V which should be stated,

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = 0.$$ [2]

It is now given that the differential equation in part **(i)** holds for the rest of the question.

(ii) Given that $I = Ate^{-\frac{Rt}{2L}}$ is a solution of the differential equation, where A is a positive constant, show that $C = \dfrac{4L}{R^2}$. [5]

(iii) In a particular circuit, $R = 4$, $L = 3$ and $C = 0.75$. Find the maximum value of I in terms of A, showing that this value is a maximum. [4]

(iv) Sketch the graph of I against t. [2]

11 Mr Wong is considering investing money in a savings plan. One plan, P, allows him to invest \$100 into the account on the first day of every month. At the end of each month the total in the account is increased by $a\%$.

(i) It is given that $a = 0.2$.

(a) Mr Wong invests \$100 on 1 January 2016. Write down how much this \$100 is worth at the end of 31 December 2016. [1]

(b) Mr Wong invests \$100 on the first day of each of the 12 months of 2016. Find the total amount in the account at the end of 31 December 2016. [3]

(c) Mr Wong continues to invest \$100 on the first day of each month. Find the month in which the total in the account will first exceed \$3000. Explain whether this occurs on the first or last day of the month. [5]

An alternative plan, Q, also allows him to invest \$100 on the first day of every month. Each \$100 invested earns a fixed bonus of \$$b$ at the end of every month for which it has been in the account. This bonus is added to the account. The accumulated bonuses themselves do not earn any further bonus.

(ii) **(a)** Find, in terms of b, how much \$100 invested on 1 January 2016 will be worth at the end of 31 December 2016. [1]

(b) Mr Wong invests \$100 on the first day of each of the 24 months in 2016 and 2017. Find the value of b such that the total value of all the investments, including bonuses, is worth \$2800 at the end of 31 December 2017. [3]

It is given instead that $\boldsymbol{a = 1}$ for plan P.

(iii) Find the value of b for plan Q such that both plans give the same total value in the account at the end of the 60th month. [3]

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MATHEMATICS

9758/02

Paper 2

October/November 2018

3 hours

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Section A: Pure Mathematics [40 marks]

1 The curve $y = \mathrm{f}(x)$ passes through the point $(0, 69)$ and has gradient given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\tfrac{1}{3}y - 15\right)^{\frac{1}{3}}.$$

(i) Find $\mathrm{f}(x)$. [4]

(ii) Find the coordinates of the point on the curve where the gradient is 4. [2]

2 **(a)** One of the roots of the equation $4x^4 - 20x^3 + sx^2 - 56x + t = 0$, where s and t are real, is $2 - 3\mathrm{i}$. Find the other roots of the equation and the values of s and t. [5]

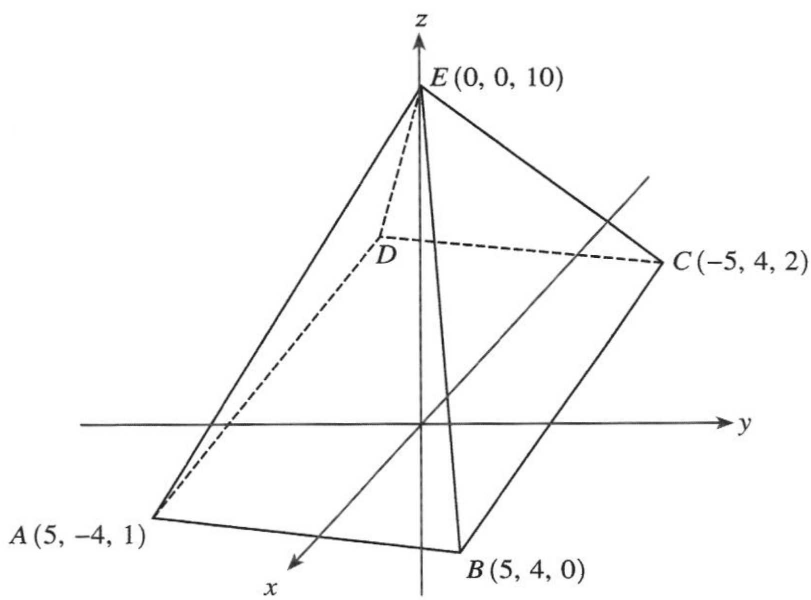
(b) The complex number w is such that $w^3 = 27$.

(i) Given that one possible value of w is 3, use **a non-calculator method** to find the other possible values of w. Give your answers in the form $a + \mathrm{i}b$, where a and b are exact values. [3]

(ii) Write these values of w in modulus-argument form and represent them on an Argand diagram. [2]

(iii) Find the sum and the product of all the possible values of w, simplifying your answers. [2]

3



An oblique pyramid has a plane base $ABCD$ in the shape of a parallelogram. The coordinates of A, B and C are $(5, -4, 1)$, $(5, 4, 0)$ and $(-5, 4, 2)$ respectively. The apex of the pyramid is at $E\,(0, 0, 10)$ (see diagram).

(i) Find the coordinates of D. [1]

(ii) Find the cartesian equation of face BCE. [3]

(iii) Find the angle between face BCE and the base of the pyramid. [3]

(iv) Find the shortest distance from the midpoint of edge AD to face BCE. [5]

4 In this question you may use expansions from the List of Formulae (MF26).

(i) Find the Maclaurin expansion of $\ln(\cos 2x)$ in ascending powers of x, up to and including the term in x^6. State any value(s) of x in the domain $0 \leqslant x \leqslant \frac{1}{4}\pi$ for which the expansion is **not** valid. [6]

(ii) Use your expansion from part **(i)** and integration to find an approximate expression for $\int \frac{\ln(\cos 2x)}{x^2}\,dx$. Hence find an approximate value for $\int_0^{0.5} \frac{\ln(\cos 2x)}{x^2}\,dx$, giving your answer to 4 decimal places. [3]

(iii) Use your graphing calculator to find a second approximate value for $\int_0^{0.5} \frac{\ln(\cos 2x)}{x^2}\,dx$, giving your answer to 4 decimal places. [1]

Section B: Probability and Statistics [60 marks]

5 The manufacturer of a certain type of fan used for cooling electronic devices claims that the mean time to failure (MTTF) is 65 000 hours. The quality control manager suspects that the MTTF is actually less than 65 000 hours and decides to carry out a hypothesis test on a sample of these fans. (An accelerated testing procedure is used to find the MTTF.)

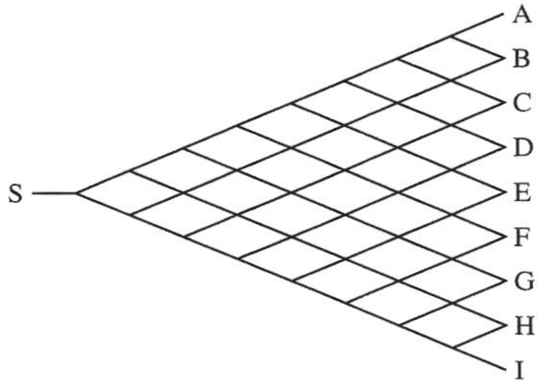
(i) Explain why the manager should take a sample of at least 30 fans, and state how these fans should be chosen. [2]

(ii) State suitable hypotheses for the test, defining any symbols that you use. [2]

The quality control manager takes a suitable sample of 43 fans, and finds that they have an MTTF of 64 230 hours.

(iii) Given that the manager concludes that there is no reason to reject the null hypothesis at the 5% level of significance, find the range of possible values of the variance used in calculating the test statistic. [3]

6



In a computer game, a bug moves from left to right through a network of connected paths. The bug starts at S and, at each junction, randomly takes the left fork with probability p or the right fork with probability q, where $q = 1 - p$. The forks taken at each junction are independent. The bug finishes its journey at one of the 9 endpoints labelled A – I (see diagram).

(i) Show that the probability that the bug finishes its journey at D is $56p^5q^3$. [2]

(ii) Given that the probability that the bug finishes its journey at D is greater than the probability that the bug finishes its journey at any one of the other endpoints, find exactly the possible range of values of p. [4]

In another version of the game, the probability that, at each junction, the bug takes the left fork is $0.9p$, the probability that the bug takes the right fork is $0.9q$ and the probability that the bug is swallowed up by a 'black hole' is 0.1.

(iii) Find the probability that, in this version of the game, the bug reaches one of the endpoints A – I, without being swallowed up by a black hole. [1]

7 The events A, B and C are such that $P(A) = a$, $P(B) = b$ and $P(C) = c$. A and B are independent events. A and C are mutually exclusive events.

(i) Find an expression for $P(A' \cap B')$ and hence prove that A' and B' are independent events. [2]

(ii) Find an expression for $P(A' \cap C')$. Draw a Venn diagram to illustrate the case when A' and C' are also mutually exclusive events. (You should not show event B on your diagram.) [2]

You are now given that A' and C' are **not** mutually exclusive, $P(A) = \frac{2}{5}$, $P(B \cap C) = \frac{1}{5}$ and $P(A' \cap B' \cap C') = \frac{1}{10}$.

(iii) Find exactly the maximum and minimum possible values of $P(A \cap B)$. [4]

8 A bag contains $(n+5)$ numbered balls. Two of the balls are numbered 3, three of the balls are numbered 4 and n of the balls are numbered 5. Two balls are taken, at random and without replacement, from the bag. The random variable S is the sum of the numbers on the two balls taken.

(i) Determine the probability distribution of S. [4]

(ii) For the case where $n = 1$, find $\mathrm{P}(S = 10)$ and explain this result. [1]

(iii) Show that $\mathrm{E}(S) = \dfrac{10n+36}{n+5}$ and $\mathrm{Var}(S) = \dfrac{\mathrm{g}(n)}{(n+5)^2(n+4)}$ where $\mathrm{g}(n)$ is a quadratic polynomial to be determined. [6]

9 Many electronic devices need a fan to keep them cool. In order to maximise the lifetime of such fans, the speed they run at is reduced when conditions allow. Running a fan at a lower speed reduces the power required. The following table gives details, for a particular type of fan, of the power required (P watts) at different fan speeds (R revolutions per minute).

Fan speed (R)	3600	4500	5400	6300	7200	8100	9000	9900
Power (P)	0.22	0.34	0.52	0.78	1.06	1.48	2.04	2.64

(i) Draw a scatter diagram of these data. Use your diagram to explain whether the relationship between P and R is likely to be well modelled by an equation of the form $P = aR + b$, where a and b are constants. [2]

(ii) By calculating the relevant product moment correlation coefficients, determine whether the relationship between P and R is modelled better by $P = aR + b$ or by $P = aR^2 + b$. Explain how you decide which model is better, and state the equation in this case. [5]

(iii) Use your equation to estimate the speed of the fan when the power is 0.9 watts. Explain whether your estimate is reliable. [2]

(iv) Use your equation to estimate the power used when the speed of the fan is 3300 revolutions per minute. Explain whether your estimate is reliable. [2]

(v) Re-write your equation from part **(ii)** so that it can be used when the speed of the fan, R, is given in revolutions per second. [1]

10 In this question you should state the parameters of any distributions that you use.

A manufacturer produces specialist light bulbs. The masses in grams of one type of light bulb have the normal distribution $N(50, 1.5^2)$.

(i) Sketch the distribution for masses between 40 grams and 60 grams. [2]

(ii) Find the probability that the mass of a randomly chosen bulb is less than 50.4 grams. [1]

Each light bulb is packed into a randomly chosen box. The masses of the empty boxes have the distribution $N(75, 2^2)$.

(iii) Find the probability that the total mass of 4 randomly chosen empty boxes is more than 297 grams. [2]

(iv) Find the probability that the total mass of a randomly chosen light bulb and a randomly chosen box is between 124.9 and 125.7 grams. [3]

In order to protect the bulbs in transit each bulb is surrounded by padding before being packed in a box. The mass of the padding is modelled as 30% of the mass of the bulb.

(v) The probability that the total mass of a box containing a bulb and padding is more than k grams is 0.9. Find k. [4]

(vi) Find the probability that the total mass of 4 randomly chosen boxes, each containing a bulb and padding, is more than 565 grams. [3]